ACY 2020-21

SHRIDEVI

SHRIDEVI INSTITUTE OF ENGINEERING & TECHNOLOGY, TUMKUR-06
DEPARTMENT OF MATHEMATICS
III-semester: II-Internal assessment Test: November -2020
18MAT31: Transform calculus, Fourier series and numerical techniques
(Common to all branches)

Time: 90 min] [Max marks: 40

NOTE: ANSWER ALL THE QUESTIONS

1. a) Solve by Using Z-transformation: $y_{n+1}+\frac{1}{4}y_n=\left(\frac{1}{4}\right)^n (n\geq 0), y_0=0$ **(6 Marks – CO 3)**

b) Find the Complex Fourier transformation of the function $f(x)=\begin{cases}1 & for\ |x|\leq a\\ 0 & for\ |x|\geq a\end{cases}$

Hence evaluate $\int_0^\infty \frac{\sin x}{x}dx$ **(7 Marks – CO 3)**

c) If $f(x)=\begin{cases}1-x^2 & for\ |x|<1\\ 0 & for\ |x|\geq 1\end{cases}$, Find the Fourier transform of f(x) and hence find the value of

(i) $\int_0^\infty \frac{x\cos x-\sin x}{x^3}dx$ (ii) $\int_0^\infty \frac{x\cos x-\sin x}{x^3}\cos\left(\frac{x}{2}\right)dx$ **(7 Marks - CO3)**

2. a) Given $\frac{dy}{dx}=x^2(1+y)$ and y(1) = 1, y(1.1) = 1.233, y(1.2) = 1.548, y(1.3) = 1.979. Evaluate y(1.4) by Adams-Bashforth method. **(6 Marks – CO 5)**

b) By Runge-Kutta method, Solve $\frac{d^2y}{dx^2}=x\left(\frac{dy}{dx}\right)^2-y^2$ for x = 0.2 correct to four decimal places, using the initial conditions y = 1 and $y'=0$ when x = 0. **(7 Marks – CO 5)**

c) Apply Milne's method to compute y(0.8) given that $\frac{d^2y}{dx^2}=1-2y\frac{dy}{dx}$ and the following table of initial values. **(7 Marks – CO 5)**

x	0	0.2	0.4	0.6
y	0	0.02	0.0795	0.1762
y'	0	0.1996	0.3937	0.5689

26/11/2020

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18MAT31 : TRANSFORM CALCULUS, FOURIER SERIES AND NUMERICAL TECHNIQUES

III - Semester : II - Internal Assesment Test : November-2020

SCHEME OF Evaluation (Common to All branches)

1] (a)

$$Z_T(y_{n+1}) + \frac{1}{4} Z_T(y_n) = Z_T\left[\left(\frac{1}{4}\right)^n\right]$$

$$z\left[\bar{y}(z) - y_0\right] + \frac{1}{4}\bar{y}(z) = \frac{z}{z-\frac{1}{4}}$$

$$\frac{\bar{y}(z)}{z} = \frac{1}{\left(z-\frac{1}{4}\right)\left(z+\frac{1}{4}\right)} = \frac{A}{z-\frac{1}{4}} + \frac{B}{z+\frac{1}{4}} \longrightarrow \text{2 Marks}$$

put $z = \frac{1}{4}$; $A = 2$.

$z = -\frac{1}{4}$; $B = -2$ → 2 Marks

$$Z_T^{-1}\left[\bar{y}(z)\right] = 2\left\{Z_T^{-1}\left[\frac{z}{z-\frac{1}{4}}\right] - Z_T^{-1}\left[\frac{z}{z+\frac{1}{4}}\right]\right\}$$

$$\boxed{y_n = 2\left[\left(\frac{1}{4}\right)^n - \left(-\frac{1}{4}\right)^n\right]} \longrightarrow \text{2 Marks}$$

(b)

$$F(u) = \int_{x=-\infty}^{\infty} f(x)\, e^{iux}\,dx = \int_{x=-a}^{a} 1 \cdot e^{iux}\,dx$$

$$\boxed{F(u) = \frac{2\sin au}{u}} \longrightarrow \text{2 Marks}$$

Inverse fourier transform is

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} F(u)\, e^{-iux}\,du = f(x) \longrightarrow \text{1 Mark}$$

$$f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{2\sin au}{u}\, e^{-iux}\,du \quad \text{put } x=0 \longrightarrow \text{2 Marks}$$

$$\frac{1}{\pi}\int_{-\infty}^{\infty} \frac{\sin au}{u}\,du = 1$$

$$\frac{2}{\pi}\int_{0}^{\infty} \frac{\sin au}{u}\,du = 1$$

$$\boxed{\int_{0}^{\infty} \frac{\sin x}{x}\,dx = \frac{\pi}{2}} \longrightarrow \text{2 Marks}$$

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(c) $F(u) = \int_{-\infty}^{\infty} f(x)\, e^{iux} dx = \int_{-1}^{1} (1-x^2)\, e^{iux} dx$ ⟶ 1 Mark

$$F(u) = -\frac{2}{u^2}\left(e^{iu} + e^{-iu}\right) - \frac{2i}{u^3}\left(e^{iu} - e^{-iu}\right)$$

$$F(u) = -4\frac{\cos u}{u^2} + 4\frac{\sin u}{u^3}$$

$$\boxed{F(u) = 4\left(\frac{\sin u - u\cos u}{u^3}\right)}$$ ⟶ 2 Marks

(i) $f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(u) \cdot e^{-iux} dx$

$$\int_0^{\infty} \frac{x\cos x - \sin x}{x^3}\, dx = -\frac{\pi}{4}$$ ⟶ 2 Marks

(ii) put $x = 1/2$

$$\int_0^{\infty} \frac{x\cos x - \sin x}{x^3} \cos\frac{x}{2}\, dx = -\frac{3\pi}{16}$$ ⟶ 2 Marks

2] (a) $y_0' = 2$, $y_1' = 2.70193$, $y_2' = 3.66912$, $y_3' = 5.03451$

$$y_4^{(P)} = y_3 + \frac{h}{24}\left[55y_3' - 59y_2' + 37y_1' - 9y_0'\right]$$

$$\boxed{y_4^{(P)} = 2.5722}$$ ⟶ 3 Marks

$$y_4' = f(x_4, y_4) = 7.0015$$

$$y_4^{(C)} = y_3 + \frac{h}{24}\left[9y_4' + 19y_3' - 5y_2' + y_1'\right]$$

$$\boxed{y_4^{(C)} = 2.5749}$$ ⟶ 2 Marks

→ $y_4' = 7.0068$

$$\boxed{y_4^{(C)} = 2.5751}$$

Thus $\boxed{y(1.4) = 2.5751}$ ⟶ 1 Mark

(b)

$$\frac{d^2y}{dx^2} = x\left(\frac{dy}{dx}\right)^2 - y^2$$

$$\frac{dz}{dx} = xz^2 - y^2 \quad \text{with } y=1,\ z=0 \text{ at } x=0 \longrightarrow 1 \text{ Mark}$$

$K_1 = 0$, $K_2 = -0.02$, $K_3 = -0.01998$,

$K_4 = -0.03916 \longrightarrow$ 2 Marks

$l_1 = -0.2$, $l_2 = -0.1998$, $l_3 = -0.1958$,

$l_4 = -0.19055 \longrightarrow$ 2 Marks

$$y(x_0+h) = y_0 + \frac{1}{6}[K_1 + 2K_2 + 2K_3 + K_4]$$

Thus $\boxed{y(0.2) = 0.9801}$ $\longrightarrow$ 2 Marks

(c) $z' = 1 - 2yz$

$z_0' = 1$, $z_1' = 0.992$, $z_2' = 0.9374$, $z_3' = 0.7995 \longrightarrow$ 2 Marks

$$y_4^{(P)} = y_0 + \frac{4h}{3}(2z_1 - z_2 + 2z_3)$$

$$z_4^{(P)} = z_0 + \frac{4h}{3}(2z_1' - z_2' + 2z_3')$$

$\boxed{y_4^{(P)} = 0.3049}$ & $\boxed{z_4^{(P)} = 0.7055}$ $\longrightarrow$ 2 Marks

$$y_4^{(C)} = y_2 + \frac{h}{3}[z_2 + 4z_3 + z_4]$$

$$z_4^{(C)} = z_2 + \frac{h}{3}[z_2' + 4z_3' + z_4']$$

$z_4' = 0.5698$

$\boxed{y_4^{(C)} = 0.3045}$ & $\boxed{z_4^{(C)} = 0.7074}$ $\longrightarrow$ 2 Marks

Again $\boxed{y_4^{(C)} = 0.3046}$

Thus $\boxed{y(0.8) = 0.3046}$ $\longrightarrow$ 1 Mark

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