ACY-2021-22

INTERNAL ASSESSMENT MARKS

Date	Test No.	Max. Marks	Marks Obtained	Course Instructor Signature
21/5/22	01	30	21	G. H Rame
28/6/22	02	30	19	G. H Rame
15/7/22	03	30	18	G. H Rame
	Average		19 10	G. H Rame

29/40 G. H Rame

CERTIFICATE

This is to certify that Kum / Sri ....Manu. S. N........ with USN........1SV19EE007........ has satisfactorily completed the Internal Assessment tests in the subject ....Digital Signal processing.... with Subject Code ....18EE63........ as prescribed by the Visvesvaraya Technological University for the ....6th........ semester B.E. / M.Tech / MBA degree course in the year 20 -20

G. H Rame
Course Instructor

G. H Rame
Head of the Department

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TEST NO. 1

Q.No.	a	b	c	Total
Q1				
Q2				
Q3	6	10		16
Q4	10	2		12

Test - 1 Marks 28

TEST NO. 2

Q.No.	a	b	c	Total
Q1	10	8		18
Q2				
Q3	7			07
Q4				

Test - 2 Marks 25

TEST NO. 3

Q.No.	a	b	c	Total
Q1				
Q2	8			08
Q3	10	6		16
Q4				

Test - 3 Marks 24

REMARKS

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Internal Assessment - 1

Digital Signal processing

3a) $x(n) = \{1, 1, 0, 0\}$.

The DFT of the Sequence is,

$$x(n) = DFT[X(k)]$$

$$X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi}{N}kn}$$

$N = 4$,

$$x(k) = \sum_{n=0}^{4-1=3} x(n)\, e^{-j\frac{\cancel{2}\pi}{\cancel{4}2}kn}$$

$$x(k) = \sum_{n=0}^{3} x(n)\, e^{-j\frac{\pi}{2}kn}$$

$$x(k) = \overset{1}{x(0)}\, e^{-j\frac{\pi}{2}(0)k} + \overset{1}{x(1)}\, e^{-j\frac{\pi}{2}(1)k} + \overset{0}{x(2)}\, e^{-j\frac{\pi}{2}(2)k} + \overset{0}{x(3)}\, e^{-j\frac{\pi}{2}(3)k}.$$

$$x(k) = 1.\, e^{0} + 1.\, e^{-j\frac{\pi}{2}k}$$

$$x(k) = 1 + e^{-j\frac{\pi}{2}k} \rightarrow (A)$$

put, $k = 0$

$$x(0) = 1 + e^{-j\frac{\pi}{2}(0)}$$

$$x(0) = 1 + \overset{1}{e^{0}}$$

$$x(0) = 1 + 1$$

$$x(0) = 2$$

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put $k = 1$

$$x(1) = 1 + e^{-j\frac{\pi}{2}(1)}$$

$$x(1) = 1 + (\overset{0}{\cancel{\cos}\,90} - j\,\overset{1}{\cancel{\sin}\,90})$$

$$x(1) = 1 - j$$ //

put $k = 2$

$$x(2) = 1 + e^{-j\frac{\pi}{2}(2)}$$

$$x(2) = 1 + (\overset{-1}{\cancel{\cos}\,180} - j\,\overset{0}{\cancel{\sin}\,180})$$

$$x(2) = 1 - 1$$

$$x(2) = 0$$ //

3×180
$\frac{540}{2}$
270

put $k = 3$

$$x(3) = 1 + e^{-j\frac{\pi}{2}(3)}$$

$$x(3) = 1 + (\cos 270 - j \sin 270)$$

$$x(3) = 1 + j$$ //.

$$x(K) = \{\underset{x(0)}{2}, \underset{x(1)}{1 - j}, \underset{x(2)}{0}, \underset{x(3)}{1 + j}\}$$

~~Y(K) = {2~~ Midpoint. $\overset{\uparrow}{\frac{N}{2}} = \frac{4}{2} = 2$

$$Y(k) = \{\underset{y(0)}{2}, \underset{y(1)}{1 + j}, \underset{y(2)}{0}, \underset{y(3)}{1 - j}\}.$$

IDFT,

$$y(n) = \frac{1}{N}\sum_{k=0}^{N-1} Y(k)\, e^{j\frac{2\pi}{N}kn}$$

$N = 4.$

$$y(n) = \frac{1}{4}\sum_{k=0}^{4-1=3} Y(k)\, e^{j\frac{\overset{1}{\cancel{2}}\pi}{\cancel{4}_2}kn}$$

$$y(n) = \frac{1}{4} \sum_{K=0}^{3} Y(K)\, e^{j\frac{\pi}{2}Kn}$$

$$y(n) = \frac{1}{4}\left\{ \overset{2}{y(0)}\, e^{0} + \overset{1+j}{y(1)}\, e^{j\frac{\pi}{2}(1)n} + \overset{0}{y(2)}\, e^{j\frac{\pi}{2}(2)n} + \overset{1-j}{y(3)}\, e^{j\frac{\pi}{2}(3)n} \right\}$$

$$y(n) = \frac{1}{4}\left\{ 2 + (1+j)\, \cancel{\cos}\, e^{j\frac{\pi}{2}n} + (1-j)\, e^{j\frac{3\pi}{2}n} \right\}$$

put,

$n = 0$

$$y(0) = \frac{1}{4}\left\{ 2 + (1+j)\, e^{j\frac{\pi}{2}(0)} + (1-j)\, e^{0} \right\}$$

$$y(0) = \frac{1}{4}\left\{ 2 + (1+j)(1) + (1-j)(1) \right\}$$

$$y(0) = \frac{1}{4}\left\{ 2 + 1 + \cancel{j} + 1 - \cancel{j} \right\}$$

$$y(0) = \frac{\cancel{4}}{\cancel{4}} = 1$$

$$y(0) = 1$$

put $n = 1$

$$y(1) = \frac{1}{4}\left\{ 2 + (1+j)\, e^{j\frac{\pi}{2}(1)} + (1-j)\, e^{j\frac{3\pi}{2}(1)} \right\}$$

$$y(1) = \frac{1}{4}\left\{ 2 + (1+j)(\cos 90 + j \sin 90) + (1-j)(\cos 270 + j \sin 270) \right\}$$

$$y(1) = \frac{1}{4}\left\{ 2 + (1+j)(0 + j1) + (1-j)(0 + j(-1)) \right\}$$

$$y(1) = \cancel{1/2}\,\frac{1}{4}(2 + 1 + \cancel{j} + j^{2} + -\cancel{j} + j^{2})$$

$$= \frac{1}{4}(3\ (-1) + (-1)$$

$$= 1\cancel{4}, 0$$

put n=2,

$$y(2) = \frac{1}{4}\left\{2 + (1+j)e^{j\frac{\pi}{2}(2)} + (1-j)\,e^{j\frac{\pi}{2}(2)}\right\}$$

$$y(2) = \frac{1}{4}\left\{2 + (1+j)\,e^{j\pi} + (1-j)\,e^{j\pi}\right\}$$

$$y(2) = \frac{1}{4}\left\{2 + (1+j)(\cos 180 + j\sin 180) + (1-j)(\cos 180 + j\sin 180)\right\}$$

~~$y(2) = \frac{1}{4}\{2 + (1+j)($~~

$$y(2) = [illegible]\, 1$$

put, n=3,

$$y(3) = \frac{1}{4}\left\{2 + (1+j)e^{j\frac{\pi}{2}(3)} + (1-j)\,e^{j\frac{\pi}{2}(3)}\right\}$$

$$y(3) = \frac{1}{4}\left\{2 + (1+j)(\overset{0}{\cos 270} + j\overset{-1}{\sin 270}) + (1-j).(\overset{0}{\cos 270} + j\overset{-1}{\sin 270})\right\}$$

$$y(3) = \frac{1}{4}\left\{2 + (1+j)(-j) + (1-j)(-j)\right]$$

$$y(3) = \frac{1}{4}\left\{2 - j - \overset{-1}{j^2} - j + \overset{-1}{j^2}\right.$$

$$y(3) = \frac{1}{4}\left\{~~2 - 2j + 1 - 1~~\right\}$$

$$y(n) = \{1,$$

$$= 0 \quad = [illegible]$$

$$y(n) = \{1, 0, 1, 0\}$$

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3 b) $x(n) = \left(\frac{1}{2}\right)^n \{u(n) - u(n-4)]$

$N=4.$

i) If. $G(K) = W_4^{2K} X(K)$

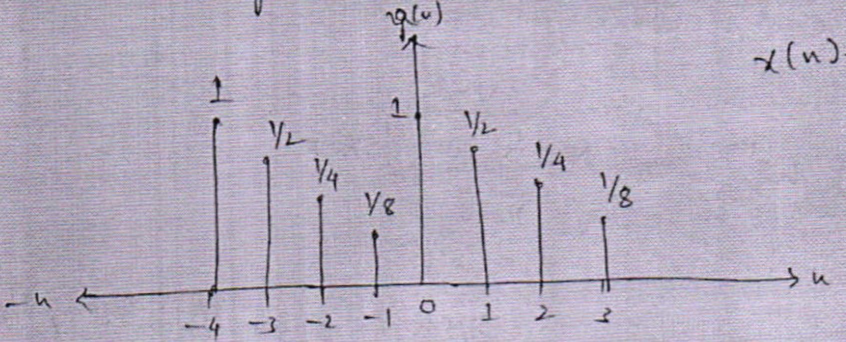
By using the time Shift method

$$x((n-m))_N = W_N^{mK} X(K)$$ (½)

$$x((n-2))_N = W_4^{2K} X(K)$$

$$g(n) = x((n-2))_N \quad -(1)$$

$x(n) = \{1, 1/2, 1/4, 1/8\}$



for put n=0,

$$g(0) = x((0-2))_4$$

$$g(0) = x((-2))_4$$

$$g(0) = 1/4.$$

put n=1

$$g(1) = x((1-2))_4$$

$$g(1) = x((-1))_4$$

$$g(1) = 1/8$$

put n=2

$$g(2) = x((2-2))_4$$

$$g(2) = x((0))_4$$

$$g(2) = 1,$$

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put, $n=3$

$$g(3) = x((3-2))_4$$

$$g(3) = x((1))_4$$

$$g(3) = 1/2$$

$$g(n) = \{ 1/4 ,\ 1/8 ,\ 1 ,\ 1/2 \}$$

ii). $\sum_{K=0}^{3} X(K)\, X^*(K)$

By using parseval's Theorem:

$$\sum_{K=0}^{N-1} x(n)\, W_N^{Kn} = N \sum_{K=0}^{N-1} |X(K)|^2$$

$$x(n) = \{ 1,\ 1/2,\ 1/4 ,\ 1/8 \}.$$

$$= 4 \sum_{K=0}^{4-1=3} |X(K)|^2$$

$$= 4 \left[x(0) + x(1) + x(2) + x(3) \right]$$

$$= 4 \left[1 + (1/2)^2 + (1/4)^2 + (1/8)^2 \right]$$

$$= 4 \left[1 + \frac{1}{4} + \frac{1}{16} + \frac{1}{64} \right]$$

$$\begin{array}{r|lll} 8 & 64 & 16 & 4 \\ \hline 2 & 8 & 2 & 4 \\ \hline 2 & 4 & 1 & 2 \\ \hline 2 & 2 & 1 & 1 \end{array}$$

$$= 4 \left[\frac{64 + 16 + 4 + 1}{64} \right]$$

$$= \cancel{4}^{1} \left[\frac{85}{\underset{16}{\cancel{64}}} \right] = \frac{85}{16} //$$

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iii) $X(0) + X(2)$

By putting Value of $K=0$, and 2.

$$x(n) = \sum_{n=0}^{N-1} x(K)\, e^{-j\frac{2\pi}{N}Kn}$$

$$x(n) = \sum_{n=0}^{4-1=3} x(K)\, e^{-j\frac{2\pi}{4}(0)}$$

$$x(n) = \sum_{n=0}^{3} x(K)\, e^{0}$$

$$x(n) = x(0) + x(1) + x(2) + x(3)$$

$$= 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8}$$

$$X(0) = \frac{8+4+2+1}{8} = \frac{15}{8}$$

$K=2$

$$x(n) = \sum_{n=0}^{N-1} x(K)\, e^{-j\frac{2\pi(2)n}{4}}$$

$$x(n) = \sum_{n=0}^{N-1} x(K)\, e^{-j\pi n}$$

$$= \left[x(0)\, e^{-j0} + x(1)\, e^{-j\pi(1)} + x(2)\, e^{-j\pi(2)} + x(3)\, e^{-j\pi(3)}\right]$$

$$= \left[1 + \frac{1}{2}(\cos\pi - j\sin\pi) + \frac{1}{4}(\cos 2\pi - j\sin 2\pi) + \frac{1}{8}(\cos 3\pi - j\sin 3\pi)\right]$$

$$X(2) = \frac{5}{8}$$

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$= x(0) + x(2)$

$= \frac{15}{8} + \frac{5}{8}$

$= \frac{20}{8}$

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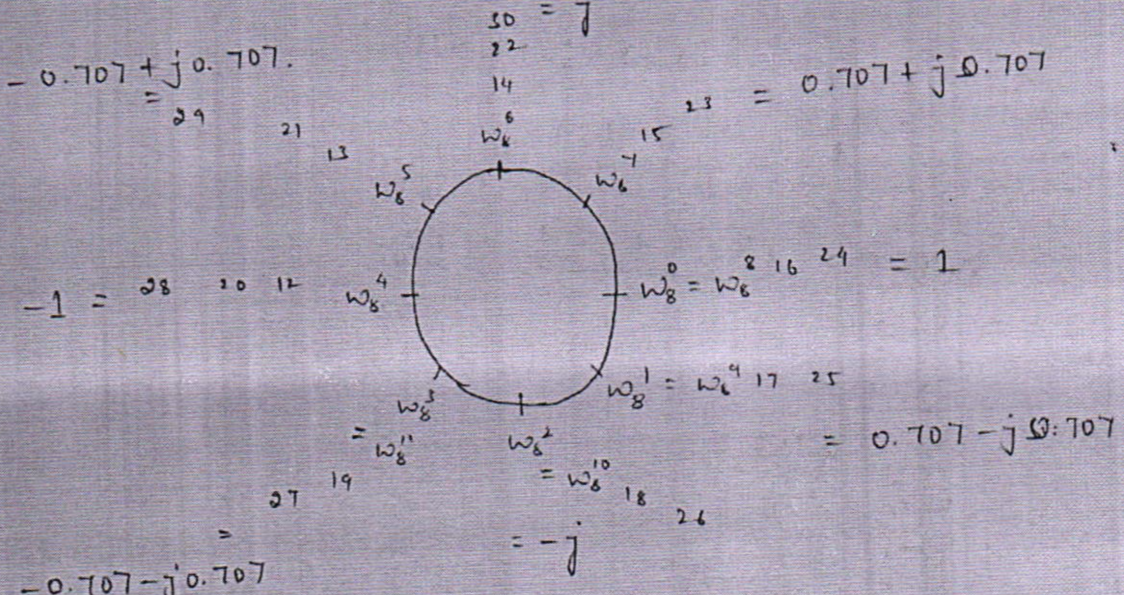
4a) $x(n) = \{1, 1, 1, 1\}$

$N = 8.$

fr The. 8 point DFT i.e.,

$x(n) = \{1, 1, 1, 1, 0, 0, 0, 0\}$



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$$\begin{bmatrix} X(0)\\ X(1)\\ X(2)\\ X(3)\\ X(4)\\ X(5)\\ X(6)\\ X(7) \end{bmatrix} = \begin{array}{c|cccccccc} & n=0 & 1 & 2 & 3 & 4 & 5 & 6 & 7\\ \hline k=0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0\\ 1 & W_8^0 & W_8^1 & W_8^2 & W_8^3 & W_8^4 & W_8^5 & W_8^6 & W_8^7\\ 2 & W_8^0 & W_8^2 & W_8^4 & W_8^6 & W_8^8 & W_8^{10} & W_8^{12} & W_8^{14}\\ 3 & W_8^0 & W_8^3 & W_8^6 & W_8^9 & W_8^{12} & W_8^{15} & W_8^{18} & W_8^{21}\\ 4 & W_8^0 & W_8^4 & W_8^8 & W_8^{12} & W_8^{16} & W_8^{20} & W_8^{24} & W_8^{28}\\ 5 & W_8^0 & W_8^5 & W_8^{10} & W_8^{15} & W_8^{20} & W_8^{25} & W_8^{30} & W_8^{35}\\ 6 & W_8^0 & W_8^6 & W_8^{12} & W_8^{18} & W_8^{24} & W_8^{30} & W_8^{36} & W_8^{42}\\ 7 & W_8^0 & W_8^7 & W_8^{14} & W_8^{21} & W_8^{28} & W_8^{35} & W_8^{42} & W_8^{49} \end{array} \cdot \begin{bmatrix} x(0)\\ x(1)\\ x(2)\\ x(3)\\ x(4)\\ x(5)\\ x(6)\\ x(7) \end{bmatrix}$$

$$\begin{bmatrix} X(0)\\ X(1)\\ X(2)\\ X(3)\\ X(4)\\ X(5)\\ X(6)\\ X(7) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1\\ 1 & \left(\frac{1}{\sqrt{2}}-j\frac{1}{\sqrt{2}}\right) & (-j) & \left(-\frac{1}{\sqrt{2}}-j\frac{1}{\sqrt{2}}\right) & \theta & \theta & -0 & 0\\ 1 & (-j) & (-1) & (j) & \theta & \theta & -0 & -0\\ 1 & \left(-\frac{1}{\sqrt{2}}-j\frac{1}{\sqrt{2}}\right) & (j) & \left(\frac{1}{\sqrt{2}}-j\frac{1}{\sqrt{2}}\right) & \theta & 0 & 0 & \theta\\ 1 & (-1) & (1) & (-1) & \theta & \theta & -0 & 0\\ 1 & \left(-\frac{1}{\sqrt{2}}+j\frac{1}{\sqrt{2}}\right) & (-j) & \left(\frac{1}{\sqrt{2}}+j\frac{1}{\sqrt{2}}\right) & - & - & - & -\\ 1 & (j) & (-1) & (-j) & - & - & - & -\\ 1 & \left(\frac{1}{\sqrt{2}}+j\frac{1}{\sqrt{2}}\right) & (j) & \left(-\frac{1}{\sqrt{2}}+j\frac{1}{\sqrt{2}}\right) & - & - & - & - \end{bmatrix} \cdot \begin{bmatrix} 1\\ 1\\ 1\\ 1\\ 0\\ 0\\ 0\\ 0 \end{bmatrix}$$

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$$\begin{bmatrix} x(0) \\ x(1) \\ x(2) \\ x(3) \\ x(4) \\ x(5) \\ x(6) \\ x(7) \end{bmatrix} = \begin{bmatrix} 1+1+1+1+0+0+0+0 \\ 1+\left(\cancel{\frac{1}{\sqrt{2}}}-j\frac{1}{\sqrt{2}}\right)+(-j)+\left(\cancel{-\frac{1}{\sqrt{2}}}+j\frac{1}{\sqrt{2}}\right)+0+0+0+0 \\ \cancel{1}+(\cancel{-j})+(\cancel{-1})+(\cancel{j})+0+0+0+0 \\ 1+\left(\cancel{-\frac{1}{\sqrt{2}}}-j\frac{1}{\sqrt{2}}\right)+(j)+\left(\cancel{\frac{1}{\sqrt{2}}}-j\frac{1}{\sqrt{2}}\right)+0+0+0+0 \\ \cancel{1}+(\cancel{-1})+(\cancel{1})+(\cancel{-1})+0+0+0+0 \\ 1+\left(\cancel{-\frac{1}{\sqrt{2}}}+j\frac{1}{\sqrt{2}}\right)+(-j)+\left(\cancel{\frac{1}{\sqrt{2}}}+j\frac{1}{\sqrt{2}}\right)+0+0+0+0 \\ \cancel{1}+\cancel{j}+(\cancel{-1})+(\cancel{-j})+0+0+0+0 \\ 1+\left(\cancel{\frac{1}{\sqrt{2}}}+j\frac{1}{\sqrt{2}}\right)+(j)+\left(\cancel{-\frac{1}{\sqrt{2}}}+j\frac{1}{\sqrt{2}}\right)+0+0+0+0 \end{bmatrix}$$

$$\begin{bmatrix} x(0) \\ x(1) \\ x(2) \\ x(3) \\ x(4) \\ x(5) \\ x(6) \\ x(7) \end{bmatrix} = \begin{bmatrix} 4 \\ 1-j2.4142 \\ 0 \\ 1-j0.4142 \\ 0 \\ 1+j0.4142 \\ 0 \\ 1+j2.4142 \end{bmatrix}$$

$$x(k) = \{4,\ 1-j2.4142,\ 0,\ 1-j0.4142,\ 0,\ 1+j0.4142,\ 0,\ 1+j2.4142\}.$$

<u>Magnitude</u>

$$|x(k)| = \sqrt{(x(k)_R)^2+(x(k))_I^2}$$

$$= \sqrt{(4)^2+(0)^2}$$

$$= 4$$

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$$\angle x(k) = \tan^{-1}\left[\frac{x(k)_I}{x(k)_R}\right]$$

$$= \tan^{-1}\left[\frac{0}{4}\right]$$

$$= \tan^{-1}(0) = 0$$

$$|x(k)| = \sqrt{(1)^2 + (-2.4142)^2}$$

$$= \sqrt{1 + 5.828}$$

$$= \sqrt{6.826}$$

$$|x(k)| = 2.6131$$

$$\angle x(k) = \tan^{-1}\left[\frac{-2.4142}{1}\right]$$

$$= -67.49 \times \frac{\pi}{180}$$

$$\angle x(k) = -1.1779$$

$$|x(k)| = \sqrt{(1)^2 + (-0.4142)^2}$$

$$= \sqrt{(1)^2 + 0.1715}$$

$$= \sqrt{1.1715}$$

$$|x(k)| = 1.0823$$

$$\angle x(k) = \tan^{-1}\left[\frac{-0.4142}{1}\right]$$

$$= -22.49 \times \frac{\pi}{180} = -0.3926.$$

$|x(k)| = \sqrt{(1)^2 + (0.4141)}$

$|x(k)| = 1.0823$

$= \tan^{-1}\left(\frac{0.4142}{1}\right)$

$= 0.3926$

$|x(k)| = \sqrt{(1)^2 + (2.4142)}$

$= \sqrt{1 + 5.828}$

$= \sqrt{6.828}$

$= 2.6131$

$\angle x(k) = -1.1779.$

$|x(k)| = \{4, 2.6131, 0, 1.0823, 0, 1.0823, 0, 2.6131\}$

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4b) $G(k) = \{1+j, -2.1+j3.2, -1.2+j2.4, 0, 0.9+j3.1, -0.3+j3\}$

$h(n) = g((n-4))_6.$ $N = 6$

$h(n) = g((n-4))_6$

$h(n) = g(0)\, e^{+j\frac{2\pi}{6}kn}$

$h(n) = (1+j)(1)$

$h(n) = 1.$

$h(n) = g(1)\ e^{-j\frac{\overset{1}{\cancel{2}}\pi}{\cancel{6}_3}(1)n}$

$h(n) = \cancel{g}\ (-2.1+j3.2)(\cos 60 - j\sin 60) = (-2.1+j3.2)(0.5-j0.866)$

$h(n) = -1.05 + j1..8186 + j1.6 - j^2 2.7712.$

$h(n) = -1.05 + j3.4186 + 2.7712$

$h(n) = 1.7212 + j3.4186$ //

$h(n) = g(2).\ e^{-j\frac{2\pi}{6}(2)}$

$= g(2).\ e^{-j\frac{\overset{2}{\cancel{4}}\pi}{\cancel{6}3}} = g(2).\ e^{-j2\pi/3}$

$= (-1.2+j2.4)(\cos 120 - j\sin 120) = (-1.2+j2.4)(-0.5-j0.866)$

$=$

02 ✓

$\boxed{\frac{28}{40}} = \boxed{\frac{21}{30}}$

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2nd internal Assessment

1 a) $h(n) = \{1, 1, 1\}$.

$x(n) = \{3, -1, 0, 1 / 3, 2, 0, 1 / 2, 1\}$.

$N = L + M - 1$ $\qquad M = 3 \qquad 2^3 = 8$.

$8 = L + 3 - 1$

$8 = L + 2$

$L = 6$.

0 0 3 -1
-1 3 0 1
0 1 3 2
3 2 0

$x_1(n) = \{0, 0, 3, -1, 0, 1\}$

$x_2(n) = \{0, 1, 3 \; 2 \; 0, 1\}$

$x_3(n) = \{0, 1, 2, 1, 0, 0\}$

$h(n) = \{1, 1, 1, 0, 0, 0\}$

$$\begin{bmatrix} y_1(0) \\ y_1(1) \\ y_1(2) \\ y_1(3) \\ y_4(4) \\ y_5(5) \\ \cancel{y_6(6)} \\ y_7(n) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & -1 & 3 & 0 \\ 0 & 0 & 1 & 0 & -1 & 3 \\ 3 & 0 & 0 & 1 & 0 & -1 \\ -1 & 3 & 0 & 0 & 1 & 0 \\ 0 & -1 & 3 & 0 & 0 & 1 \\ 1 & 0 & -1 & 3 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1+0 \\ 0+0+1 \\ 3+0+0 \\ -1+3+0 \\ -1+3 \\ \cancel{1-1} \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 3 \\ 2 \\ 2 \\ 0 \end{bmatrix}$$

$y_1(n) = \{1, 1, 3, 2, 2, 0\}$

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$$\begin{bmatrix} y_2(0) \\ y_2(1) \\ y_2(2) \\ y_2(3) \\ y_2(4) \\ y_2(5) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 2 & 3 & 1 \\ 1 & 0 & 1 & 0 & 2 & 3 \\ 3 & 1 & 0 & 1 & 0 & 2 \\ 2 & 3 & 1 & 0 & 1 & 0 \\ 0 & 2 & 3 & 1 & 0 & 1 \\ 1 & 0 & 2 & 3 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0+1+0 \\ 1+1 \\ 8 \\ 3+1+1 \\ 2+3+1 \\ 2+3 \\ 1+2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 4 \\ 6 \\ 5 \\ 3 \end{bmatrix}$$

$$y_2(n) = \{1, 2, 4, 6, 5, 3\}$$

$$\begin{bmatrix} y_3(0) \\ y_3(1) \\ y_3(2) \\ y_3(3) \\ y_3(4) \\ y_3(5) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 & 2 & 1 \\ 1 & 0 & 0 & 0 & 1 & 2 \\ 2 & 1 & 0 & 0 & 0 & 1 \\ 1 & 2 & 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1+0 \\ 2+1 \\ 1+2+1 \\ 1+2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 3 \\ 4 \\ 3 \\ 1 \end{bmatrix}$$

$$y_3(n) = \{0, 1, 3, 4, 3, 1\}$$

$$y_1(n) = \{\underbrace{\boxed{1}\,\boxed{1}}_{\substack{(D-1) \\ \text{discard}}}\,\boxed{3}\,\boxed{2}\,\boxed{2}\,\boxed{0}$$

$$y_2(n)\ \underbrace{\boxed{1}\,\boxed{2}}_{\substack{(D-1) \\ \text{discard}}}\,\boxed{4}\,\boxed{6}\,\boxed{5}\,\boxed{3}$$

$$y_3(n)\ \underbrace{\boxed{0}\,\boxed{1}}_{\substack{(D-1) \\ \text{discard}}}\,\boxed{3}\,\boxed{4}\,\boxed{3}\,\boxed{1}$$

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$$y(n) = \{3, 2, 2, 0, 4, 6, 5, 3, 3, 4, 3, 1\}$$

1b) For the Complex Multiplication and the Complex addition is having an Expressions, for addition. $N(N-1)$ and the $N\log_2 N$ and for Multiplication N^2 and $\frac{N}{2}\log_2 N$ 3

S.No N	Complex addition FFT $N(N-1)$	$N\log_2 N$	Complex Multiplication N^2	$\frac{N}{2}\log_2 N$
16	240	64	256	32
32	992	160	1024	80
128	16,256	896	16,384	448

$N=16$
$=N(N-1)$
$=16(16-1)$
$=16(15)$
240

$N=32$
$=32(32-1)$
$32(31)$
$=992$

$N\log_2 N$
$N.\frac{\log N}{\log 2}$
$16\frac{\log 16}{\log 2}$
$N=32$
$32.\frac{\log 32}{\log 2}$
$32(5)=160$

N^2
$(16)(16)$

$(32)(32)$

$\frac{N}{2}\log_2 N$
$\frac{N}{2}\frac{\log N}{\log 2}$
$\frac{16}{2}\frac{\log 16}{\log 2}$
$\frac{32}{2}\frac{\log 32}{\log 2}$
$(16)(5)$
$=80.$

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$N = 128$

$128(128-1)$

$128(127)$

$= 16,256$

$(128) . \frac{\log 128}{\log 2}$

$(128)(7)$

$= 896$

$(128)^2$

$= 16,384.$

$\frac{128}{2} \frac{\log 128}{\log 2}$

$(64)(7).$

$\Rightarrow 448$

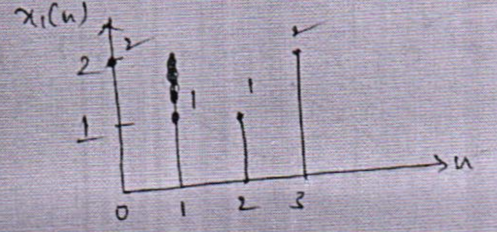
3a) ~~$X(0) = 0$, $X(1) = 2 + j2$, $X(2) = -j4$, $X(3) = 2 - j2$. $X(4) = 0$~~

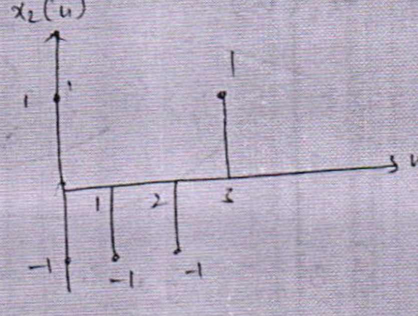
2b) ~~$x(n) = 2^{n}$; $N = 8$.~~

~~$n = 0, 1, 2, 3, 4 \ldots$~~

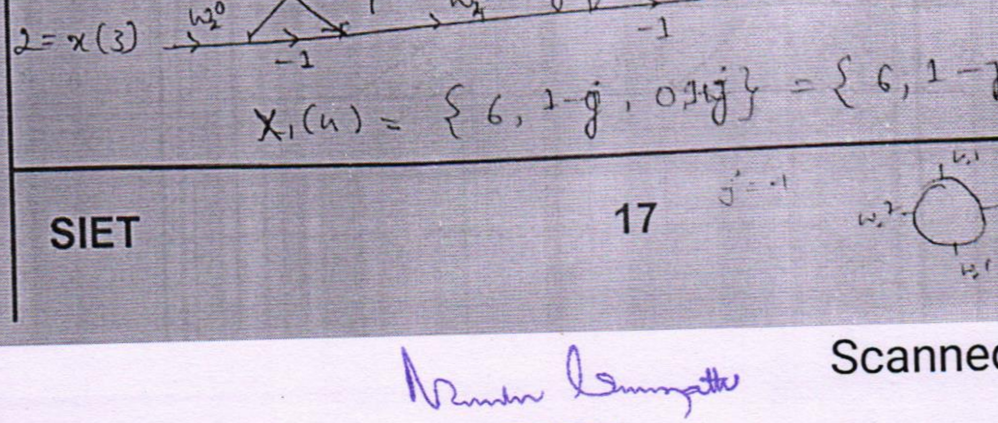
~~$N = k + M - 1$~~

3b) $x_1(n) = \{2, 1, 1, 2\}$. $x_2(n) = \{1, -1, -1, 1\}$





$x_1(n) \circledast x_2(n)$ for $N = 4$.



$X(0) = 6$

$X(1) = 01 - j$

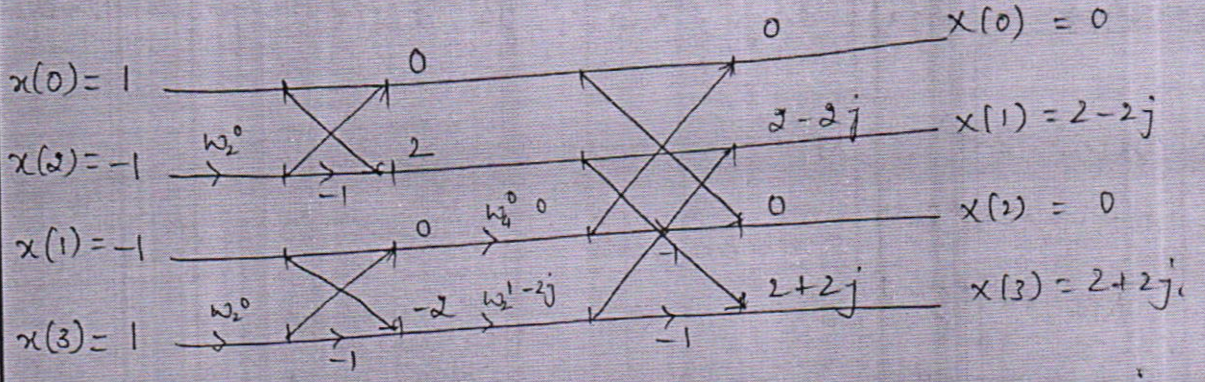
$X(2) = 0$

$X(3) = 21 + j$

$X_1(k) = \{6, 1-j, 0, 1+j\} = \{6, 1-j, 0, 1+j\}$.

$j = -1$

~~DIT~~ FFT	Bit reversal	
	2 1	
0 0	0 0	0
0 1	1 0	2
1 0	0 1	1
1 1	1 1	3



$x_3(n) = x_1(n) * x_2(n)$

$= \{6, 1-j, 0, 1+j\} * \{0, 2-2j, 0, 2+2j\}$

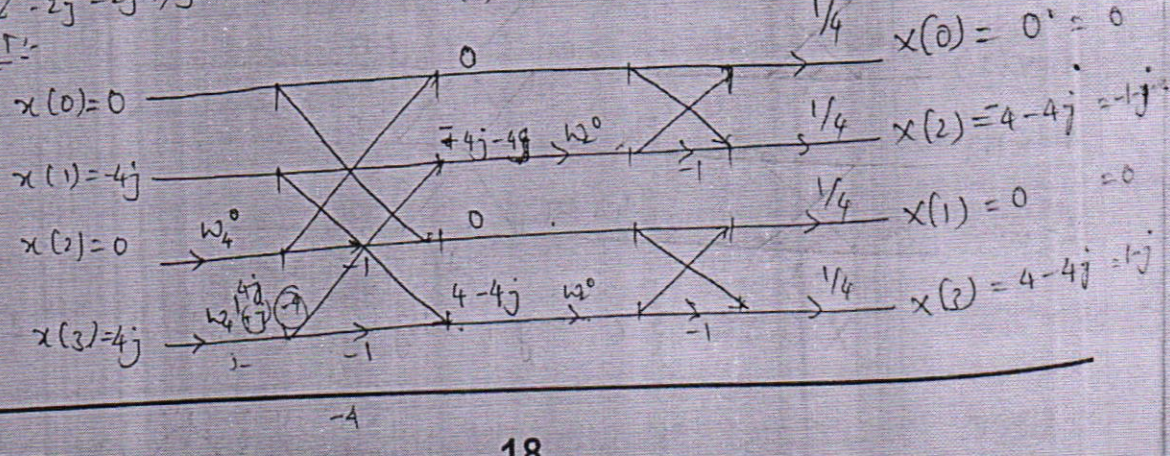
$x_3(n) = \{0, -4j, 0, 4j\}$

8

$(1-j)(2-2j)$ $(1+j)(2+2j)$

$\cancel{2} -2j -2j + \cancel{j}^2$ $2+2j+2j+2j^2_{(-1)}$

IDFT:-



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$x_3(n) = \{0, -1-j, 0, 1-j\}$.

$X(0) = 0, \ X(1) = 2+j2, \ X(2) = -j4, \ X(3) = 2-j2, \ X(4) = 0$

3a) Midpoint $\frac{8}{2} = 4$

$\{X(0), X(1), X(2), X(3), X(4), X(5), X(6), X(7)\}$

↑
midpoi

$X(1) = ~~X^*(7)~~ = +2~~+~~j2$

$X(2) = ~~X^*(6)~~ = -j4$

$X(3) = 2-j2$

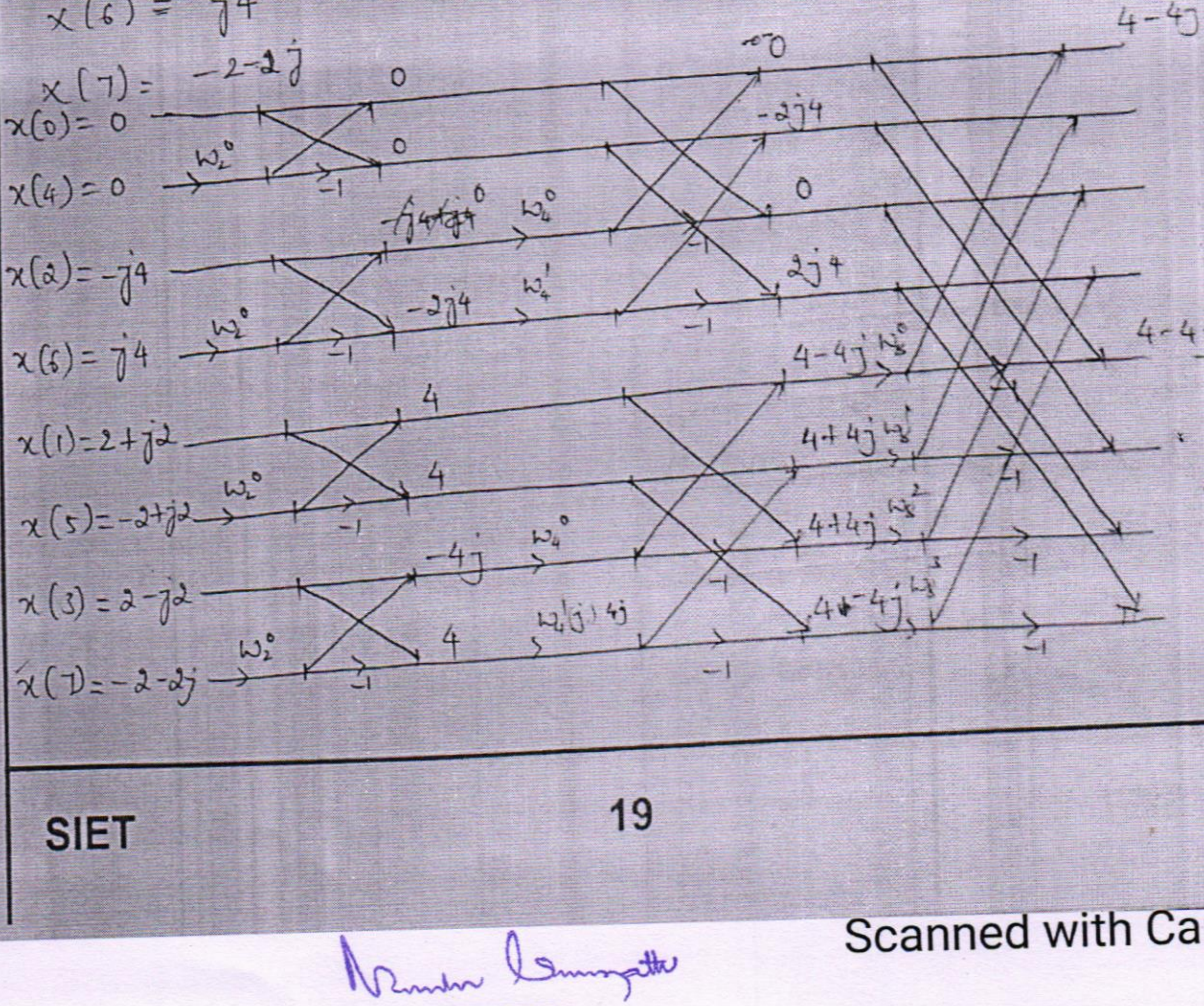
$X(4) = 0$

$X(5) = -2+j2$

$X(6) = j4$

$X(7) = -2-2j$

$4-4j-2j^0$
$4-6j$
$2+j^2+2+j^2$
$2-j^2$
$1-2j \ +2$
$2-j2+2+j^2$
$2-j2+2+4j$



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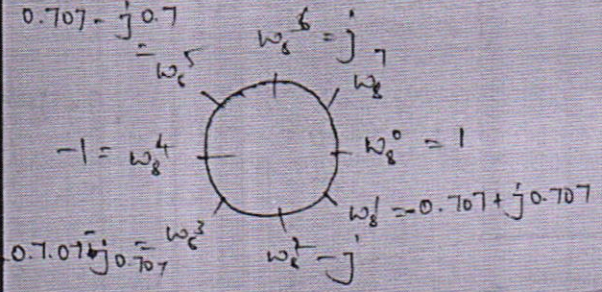

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Internal Assessment - III

3a) The bilinear transformation is used to Converting the Signals from analog Signal to digital Signal. and in the bilinear transformation the tropozoidal is Linked.

Consider the 1st order differential Equation,

$$\frac{d}{dt} y(t) = x(t) \quad — ①$$

Now, applying integration on both sides.

$$\int_{(n-1)T}^{nT} \frac{d}{dt} y(t)\, dt = \int_{(n-1)T}^{nT} x(t)\, dt$$

Now the above Equation becomes.

$$\left[y(t) \right]_{(n-1)T}^{nT} = \int_{(n-1)T}^{nT} x(t)\, dt \quad — ②$$

$$\left[y(nT) - y(n-1)T \right] = \int_{(n-1)T \; ⓐ}^{nT \; ⓑ} x(t)\, dt$$

By the tropozoidal Rule.

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$$\int_a^b x(t)\,dt = \frac{b-a}{2}\left[f(a) + f(b)\right]$$

$$\left[y(nT) - y(n-1)T\right] = \frac{\left[nT - (n-1)T\right]}{2}\left[x(nT) + x(n-1)T\right]$$

$$\left[y(nT) - y(n-1)T\right] = \frac{n\cancel{T} - \cancel{n}T + T}{2}\left[x(nT) + x(n-1)T\right]$$

$$\left[y(nT) - y(n-1)T\right] = \frac{T}{2}\left[x(nT) + x(n-1)T\right]$$

$$y(n) = y(t)\Big|_{t=nT}$$

$$y(n) = y(nT)$$

By applying Z- transform on to the above equation

$$\left[y(n) - y(n-1)\right] = \frac{T}{2}\left[x(n) + x(n-1)\right]$$

$$\left[y(z) - z^{-1}y(z)\right] = \frac{T}{2}\left[x(z) + z^{-1}x(z)\right]$$

$$\left[y(z) - z^{-1}y(z)\right]\frac{2}{T} = \left[x(z) + z^{-1}x(z)\right]$$

~~$\frac{2}{T}\left[y(z) - z^{-1}y(z) + z^{-1}x(z)\right] = x(z)$~~

$$y(z)\left[1 - z^{-1}\right]\frac{2}{T} = x(z)\left[1 + z^{-1}\right]$$

$$x(z) = \frac{2}{T}\,y(z).\,\frac{1-z^{-1}}{1+z^{-1}} \quad — ③$$

Now. By applying Laplace transform.

Considering Eq^n ①

$$\frac{d}{dt} y(n) = x(n)$$

$$s\,Y(s) = X(s)$$

$$s\,Y(s) = \frac{2}{T}\,\frac{1-z^{-1}}{1+z^{-1}}\,y(z) \quad - ④$$

Now. Comparing Equation. ③ & ④

$$s = \frac{2}{T}\,\frac{1-z^{-1}}{1+z^{-1}}\,y(z). \quad - ⑤$$

//.

The. above. Equation is Know as. the. Bilinear Transformation.

Now. Consider. Eq^n ⑤. i.e.,

$$\cancel{s = \frac{2}{T}\,\frac{1-z^{-1}}{1+z^{-1}}\,y(z)}$$

$$\cancel{s\frac{T}{2}\,(1+z^{-1}) = (1-z^{-1})\,y(z)}$$

$$s\frac{T}{2}\,\frac{1+z^{-1}}{1\mp z^{-1}} = y(z)$$

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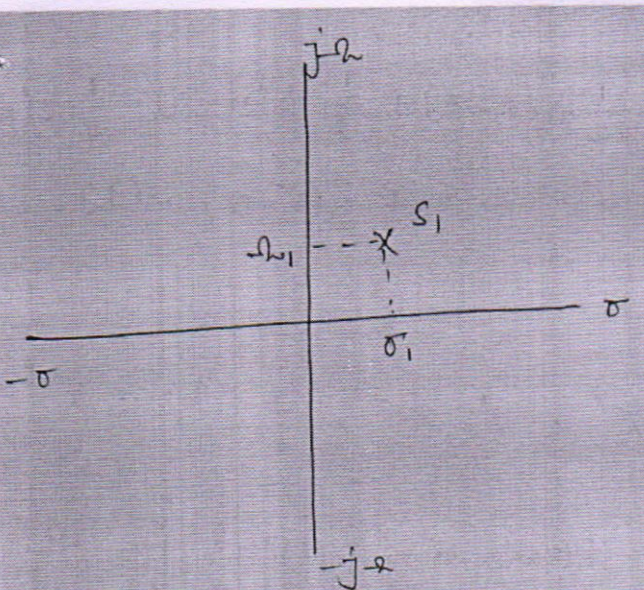

$$S = e^{S_1(\sigma_1 + \Omega_1)} \qquad e^{p_k t} \quad t = nT$$

$$e^{p_k nT}$$

$$S_1 = e^{(p_1\sigma_1 + p_1\Omega_1)S_1}$$

$$S_1 = e^{p_1\sigma_1 S_1} \cdot e^{p_1\Omega_1 S_1}$$

3)

$$S = \frac{2}{T}\,\frac{1-z^{-1}}{1+z^{-1}}\,y(z)$$

$$2\,\frac{ST}{2}(1+z^{-1}) = 1 - z^{-1}\,y(z)$$

$$\frac{ST}{2}\left(1 + \cancel{z}\frac{1}{z}\right) = \left(1 - \frac{1}{z}\right) y(z)$$

$$\frac{ST}{2}\left(\frac{z+1}{\cancel{z}}\right) = \left(\frac{z-1}{\cancel{z}}\right) y(z)$$

$$\boxed{\frac{ST}{2}\,\frac{(z+1)}{(z-1)} = y(z)}$$

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3 b) $h(n) = \delta(n) - \frac{1}{4}\delta(n-1) + \frac{1}{2}\delta(n-2) + \frac{1}{2}\delta(n-3) - \frac{1}{4}\delta(n-4)$ 2a)

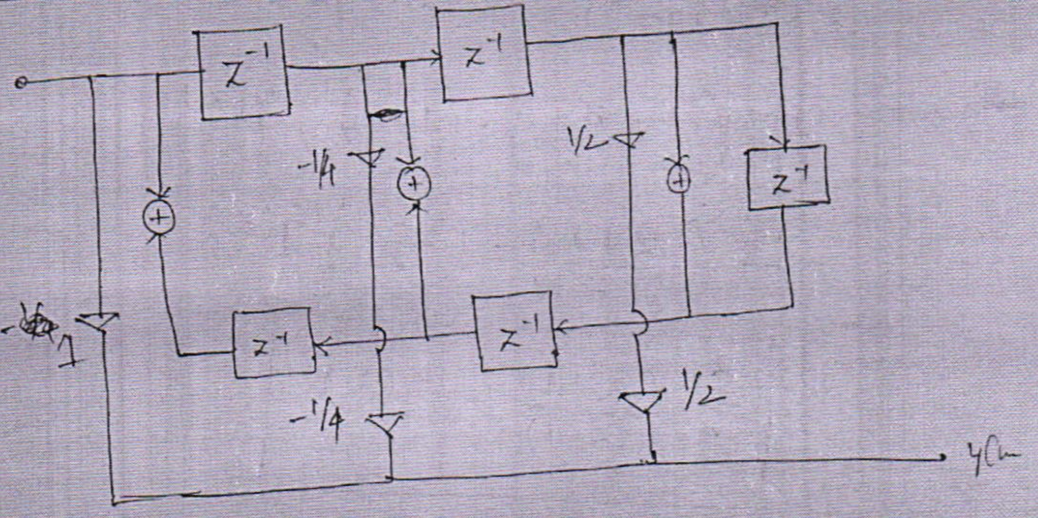
$+ \delta(n-5)$ —①

Direct form I :-

$$h(z) = 1 - \frac{1}{4}z^{-1} + \frac{1}{2}z^{-2} + \frac{1}{2}z^{-3} - \frac{1}{4}z^{-4} + 1 . z^{-5}$$

The odd No/- of z^{-1} term's are present in the above Equation.

DIrect form I :



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2a) Impulse invarient Method :-

The impulse invariant Method. it is used to Converting the analog Signal to the digital Signal. and in the transformation from the analog Signal to the digital Signal.

$$h_a(t)$$

$$h(n) = h_a(t) \quad — ⑧$$

$$\text{where} \quad t = nT$$

$$h(n) = h_a(nT)$$

for the impulse invariant Method. Considering the partial differential Equation by the Consideration of uniform values.

$$h(n) = \sum_{K=1}^{N} \frac{A_K}{S-P_K} \quad — ④$$

$$h(n) = \frac{A_1}{S-P_1} + \frac{A_2}{S-P_2} + \cdots \frac{A_N}{S-P_N}$$

where.-

A_K is the gain. of the. K^{th} pole.

P_K is the. pole of K^{th} pole.

Now by Applying the. Laplace Transform for the above Eqn we get.

$$h(s) = \sum_{K=1}^{N} A_K e^{P_K t} \quad — ②. \Rightarrow A_K \frac{1}{1-e^{P_K nT}}$$

Now.

$$\boxed{h(n) = \sum_{k=1}^{M} \frac{A_k}{s-p_k}} \quad -③$$

$$h(n) = h(t)\Big|_{t=nT}$$

$$h(n) = h(nT)$$

Consider.

$$h(n) = Z\left[h(t)\right]$$

$$h(t) = \sum_{k=1}^{n} \frac{A_k}{s-p_k} z^{-n}$$

$$h(t) = \sum_{k=-\infty}^{\infty} \sum_{k=1}^{n} \frac{A_k}{s-p_k} z^{-n}$$

$$h(t) = \sum_{k=0}^{\infty} A_k \sum_{k=1}^{n} \frac{1}{s-p_k} z^{-n}$$

$$h(t) = \sum_{k=1}^{n} A_k \sum_{n=0}^{\infty} \frac{1}{s-p_k} z^{-n}$$

By Z- transforming.

$$h(z) = A_k \sum_{n=0}^{\infty} \left(\cancel{A_k} e^{+p_k n} z^{-n}\right)$$

$$h(z) = A_K \sum_{n=0}^{\infty} \left(e^{p_K} z^{-1} \right)^n$$

$$h(z) = \frac{A_K}{1 - e^{p_K} z^{-1}} \quad — ④ \qquad \sum a^n = \frac{1}{1-a}$$

Now Comparing the above Equation. with Laplace transform Equation

$$\frac{A_K}{s - p_K} \longrightarrow \frac{A_K}{1 - e^{p_K} z^{-1}}$$

$$\frac{1}{s - p_K} \longrightarrow \frac{1}{1 - e^{p_K} z^{-1}} \, //$$

8 The above Equation the. Convertion of the Signal from. analog to digital. transformation Equation.

~~2b)~~

2b) $T = 1$ Sec

$$H(z) = \frac{1}{s^2 + \sqrt{2}\, s + 1}$$

$$f = \frac{1}{T} \Rightarrow f = \frac{1}{1} \Rightarrow 1 \text{ HZ}. \qquad = 2\pi f$$

$$= 2\pi(1)$$

$$=$$

00

$\frac{24}{40}$

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