AY - 2021-22

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SHRIDEVI INSTITUTE OF ENGINEERING AND TECHNOLOGY

(Recognised by Govt. of Karnataka, Affiliated to VTU, Belagavi and Approved by AICTE, New Delhi. An ISO 9001:2015 Certified Institution)

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SHRIDEVI
EDUCATION

ASSIGNMENT BOOK

USN : 1 S V 1 9 E E 0 0 7

Name of the Student : Manu. S. N

Course : Digital Signal processing Code : 18EE63

Semester : 6th Branch : EEE

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ASSIGNMENT MARKS

Date	Assignment No.	Max. Marks	Marks Obtained	Course Instructor Signature
20/5/22	1	10	10	G. H Rame
27/6/22	2	10	10	G H Rame
14/7/22	3	10	10	G. H Rame
	4			
	5			
	Average		10	G. H Rame

CERTIFICATE

This is to certify that Kum/ Sri ....Manu. S. N..........

with USN ....1SV19EE007........has satisfactorily completed the

Assignments in the subject ...Digital Signal processing........

with Subject Code ....18EE63........as prescribed by the

Visvesvaraya Technological University for the ....6th....... semester

B.E. / M.Tech / MBA degree course in the year 20 -20

G. H Rame
Course Instructor

G. H Rame
Head of the Department

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Assignment - 01

Digital Signal processing

3 a) Find the DFT of a Sequence $x(n) = \{1, 1, 0, 0\}$ and find the IDFT of $Y(k) = \{2, 1+j, 0, 1-j\}$.

Ans:

$x(n) = \{1, 1, 0, 0\}$

$N = 4.$

$$X(k) = \text{DFT}[x(n)]$$

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N}kn}$$

$$X(k) = \sum_{n=0}^{4-1=3} x(n) e^{-j\frac{2\pi}{4_2}kn}$$

$$X(k) = \sum_{n=0}^{3} x(n) e^{-j\frac{\pi}{2}kn}$$

$$X(k) = \overset{1}{x(0)}\, \overset{1}{e^0} + \overset{1}{x(1)}\, e^{-j\frac{\pi}{2}k(1)} + \overset{0}{x(2)}\, e^{-j\frac{\pi}{2}k(2)} + \overset{0}{x(3)}\, e^{-j\frac{\pi}{2}k(3)}$$

$$X(k) = 1 + 1 \cdot e^{-j\frac{\pi}{2}k}$$

put, $k = 0$, $X(0) = 1 + 1e^0$

$X(0) = 2$

put, $k = 1$, $X(1) = 1 + 1 \cdot e^{-j\frac{\pi}{2}(1)}$

$= 1 + \overset{0}{\cos\frac{\pi}{2}} - j \overset{1}{\sin\frac{\pi}{2}}$

$X(1) = 1 - j$

put $k=2$,

$$x(2) = 1 + 1e^{-j\frac{\pi}{2}(2)}$$

$$= 1 + e^{-j\pi}$$

$$= 1 + \cos\pi - j\sin\pi$$

$$= 1 - 1 = 0$$

$k=3$,

$$x(3) = 1 + 1 \cdot e^{-j\frac{\pi}{2}(3)}$$

$$= 1 + e^{-j\frac{3\pi}{2}}$$

$$= 1 + \cos\frac{3\pi}{2} - j\sin\frac{3\pi}{2}$$

0, -1

$$= 1 - j(-1)$$

$$x(3) = 1 + j$$

$$x(k) = \{2, 1-j, \underset{\uparrow}{0}, 1+j\}$$

Midpoint

$$= \frac{N}{2} = \frac{4}{2} = 2$$

1 Compute 4-point DFT of $x(n) = \sin^2\left(\frac{n\pi}{2} + \frac{\pi}{3}\right)$ from $X(K)$ obtain the DFT of $\cos^2\left(\frac{n\pi}{2} + \frac{\pi}{3}\right)$.

$\Rightarrow$ $N = 4$

$$x(n) = \sin^2\left(\frac{n\pi}{2} + \frac{\pi}{3}\right) \qquad 0 \le n \le N-1$$

$$0 \le n \le 4-1$$

$$0 \le n \le 3$$

when,

$n = 0$, $x(0) = \sin^2\left(\frac{(0)\pi}{2} + \frac{\pi}{3}\right)$

$x(0) = \sin^2(60) = 0.75$

$n = 1$, $x(1) = \sin^2\left(\frac{\pi}{2} + \frac{\pi}{3}\right) \Rightarrow x(1) = \sin^2(150) \Rightarrow 0.25$

$n = 2$, $x(2) = \sin^2\left(\frac{2\pi}{2} + \frac{\pi}{3}\right) \Rightarrow \sin^2\left(\frac{4\pi}{3}\right) \Rightarrow \sin^2(240) \Rightarrow 0.75$

$n = 3$, $x(3) = \sin^2\left(\frac{3\pi}{2} + \frac{\pi}{3}\right) \Rightarrow \sin^2\left(\frac{4\pi}{6}\right) \Rightarrow \sin^2(120) \Rightarrow 0.75$

$$\therefore x(n) = \{\underset{x(0)}{0.75}, \underset{x(1)}{0.25}, \underset{x(2)}{0.75}, \underset{x(3)}{0.75}\}$$

we know,

$$X(K) = \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi}{N}kn}$$

$$X(K) = \sum_{n=0}^{4-1} x(n)\, e^{-j\frac{2\pi}{4}kn} \Rightarrow \sum_{n=0}^{3} x(n)\, e^{-j\frac{\pi}{2}kn}$$

$$X(K) = x(0)\, e^{0} + x(1)\, e^{-j\frac{\pi}{2}k(1)} + x(2)\, e^{-j\frac{\pi}{2}k(2)} + x(3)\, e^{-j\frac{\pi}{2}k(3)}$$

$$x(k) = 0.75(1) + 0.25\,e^{-j\pi k/2} + 0.75\,e^{-j\pi k} + 0.75\,e^{-j\frac{3\pi k}{2}}$$

put $K=1$.

$$X(1) = 0.75 + 0.25\,e^{-j\pi/2(1)} + 0.75\,e^{-j\pi(1)} + 0.75\,e^{-j\frac{3\pi}{2}(1)}$$

$$X(1) = \cancel{0.75}\,[\cancel{1} + \cancel{e^{-j}}$$

$$= 0.75 + 0.25\left[\cos\overset{0}{\frac{\pi}{2}} - j\sin\overset{1}{\frac{\pi}{2}}\right] + 0.75\left[\cos\overset{1}{\pi} - j\sin\overset{0}{\pi}\right]$$

$$+ 0.75\left[\cos\overset{0}{\left(\frac{3\pi}{2}\right)} - j\sin\overset{-1}{\left(\frac{3\pi}{2}\right)}\right]$$

$$= 0.75 + 0.25[-j] + 0.75[1] + 0.75[j]$$

$$= 0.75 - 0.25j + 0.75 + j0.75$$

$$X(1) = 1.5 + j0.5$$

put $K=0$

$$X(0) = 0.75 + 0.25\,\overset{1}{e^{0}} + 0.75\,\overset{1}{e^{0}} + 0.75\,\overset{1}{e^{0}}$$

$$= 0.75 + 0.25 + 0.75 + 0.75$$

$$X(0) = 2.5$$

when $K=2$.

$$X(2) = 0.75 + 0.25\,e^{-j\frac{\pi 2}{2}} + 0.75\,e^{-j\pi(2)} + 0.75\,e^{-j\frac{3\pi(2)}{2}}$$

$$X(2) = 0.75 + 0.25\,e^{-j\pi} + 0.75\,e^{-j2\pi} + 0.75\,e^{-j3\pi}$$

$$X(2) = 0.75 + 0.25\left[\cos\overset{-1}{\pi} - j\sin\overset{0}{\pi}\right] + 0.75\left[\cos\overset{1}{2\pi} - j\sin\overset{0}{2\pi}\right]$$

$$+ 0.75\left[\cos\overset{-1}{\pi} - j\sin\overset{0}{3\pi}\right]$$

$$x(2) = 0.75 - 0.25 + \cancel{0.75} - \cancel{0.75}$$

$$x(2) = 0.5$$

when $K = 3$,

$$X(3) = 0.75 + 0.25\, e^{-j\pi(3)/2} + 0.75\, e^{-j\pi(3)} + 0.75\, e^{-j\frac{3\pi(3)}{2}}$$

$$X(3) = 0.75 + 0.25\, e^{-j3\pi/2} + 0.75\, e^{-j3\pi} + 0.75\, e^{-j9\pi/2}$$

$$X(3) = 0.75 + 0.25\left[\cos 270 - j\sin 270\right] + 0.75\left[\cos 540 - j\sin 540\right] + 0.75\left[\cos(810) - j\sin 810\right]$$

$$X(3) = \cancel{0.75} + j0.25 + -\cancel{0.75} - j0.75$$

$$X(3) = -j0.5$$

$$X(k) = \{2.5,\ 1.5 + j0.5,\ 0.5,\ -j0.5\}$$

4. Compute the 4-point DFT of the sequence $x(n) = \{1, 0, 1, 0\}$. also find $y(n)$ = if $Y(k) = X((k-2))_4$

$$\Rightarrow X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi}{N}kn}$$

$$= \sum_{n=0}^{4-1=3} x(n)\, e^{-j\frac{2\pi}{4}kn}$$

$$= \sum_{n=0}^{3} x(n)\, e^{-j\frac{\pi}{2}kn}$$

$$X(k) = x(0)\, e^{0} + x(1)\, e^{-j\frac{\pi}{2}k(1)} + x(2)\, e^{-j\frac{\pi}{2}k(2)} + x(3)\, e^{-j\frac{\pi}{2}k(3)}$$

$$X(k) = 1 + e^{-j\pi k} \quad \text{— (A)}$$

when.

$K=0, \quad x(0) = 1 + e^{0} = 1 + 1 = 2$

$K=1, \quad x(1) = 1 + e^{-j\pi} = 1 - 1 = 0$

$K=2, \quad x(2) = 1 + e^{-j\pi 2} = 1 + 1 = 2$

$K=3, \quad x(3) = 1 + e^{-j\pi 3} = 1 - 1 = 0$

$x(k) = \{2, 0, 2, 0\}$

we know. frequency Shift property.

$$x((k-l))_N \longleftrightarrow \omega_N^{-ln} x(n)$$

(or)

$$x((k-l))_N \longleftrightarrow e^{\frac{2\pi}{N} ln} x(n)$$

Given

$$y(k) = x((k-2))_4$$

$$y(n) = e^{j\frac{2\pi}{N} ln} x(n)$$

$$y(n) = e^{j\frac{2\pi}{4} 2n} x(n)$$

$$y(n) = e^{j\pi n} x(n)$$

$n=0, \; y(0) = e^{0} x(0) = 1.$

$n=1, \; y(1) = e^{j\pi} x(1) = 0$

$n=2, \; y(2) = e^{j2\pi} x(2) = 1$

$n=3, \; y(3) = e^{j\pi 3} x(3) = 0 \qquad y(n) = \{1, 0, 1, 0\}$

5. Find DFT for finite duration DT Signal of length L.

$$x(n) = \begin{cases} 1, & 0 \le n \le L \\ 0 & \text{else where assume } N > L. \end{cases}$$

$\Rightarrow$ $X(K) = DFT[x(n)]$

$$= \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi}{N}Kn}$$

But Length is 'L'. & $x(n) = 1$

$$= \sum_{n=0}^{L-1} 1 \cdot e^{-j\frac{2\pi}{N}Kn}$$

WKT, $\sum_{k=0}^{N} a^k = \frac{1 - a^{N+1}}{1 - a}$

$$X(K) = \sum_{K=0}^{L-1} \left(e^{-j\frac{2\pi}{N}K}\right)^n \Rightarrow \frac{\left(1 - e^{-j\frac{2\pi}{N}K}\right)^{L-1+1}}{1 - e^{-j\frac{2\pi}{N}K}}$$

$$X(K) = \frac{1 - e^{-j\frac{2\pi}{N}KL}}{1 - e^{-j\frac{2\pi}{N}K}} = \frac{e^{-j\frac{\pi}{N}KL}\left(e^{j\frac{\pi}{N}KL} - e^{-j\frac{\pi}{N}KL}\right)}{e^{-j\frac{\pi}{N}K}\left(e^{j\frac{\pi}{N}K} - e^{-j\frac{\pi}{N}K}\right)} \div 2j$$

$$= e^{-j\frac{\pi}{N}KL}\, e^{\frac{\pi}{N}K} \frac{\sin\frac{\pi}{N}KL}{\sin\frac{\pi}{N}K}$$

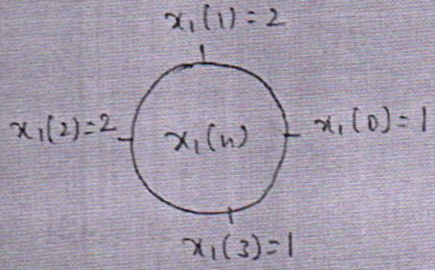
$$X(K) = e^{\frac{\pi}{N}K(L-1)} \frac{\sin\frac{\pi}{N}KL}{\sin\frac{\pi}{N}K} \quad (\text{or}) \quad e^{-\frac{\pi}{N}K(L-1)} \frac{\sin\frac{\pi}{N}KL}{\sin\frac{\pi}{N}K}.$$

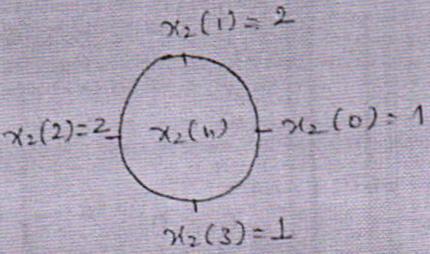
2. Determine 4-point Circular Convolution using tabular arrays method for $x_1(n) = x_2(n) = \{1, 2, 2, 1\}$. Verify your answer by Stockham's method.

Soln.

$x_1(n) = \{1, 2, 2, 1\} \quad x_2(n) = \{1, 2, 2, 1\}$

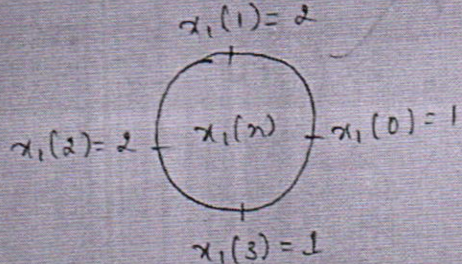
$$x_3(m) = \sum_{n=0}^{N-1} x_1(n)\, x_2((m-n))_N$$

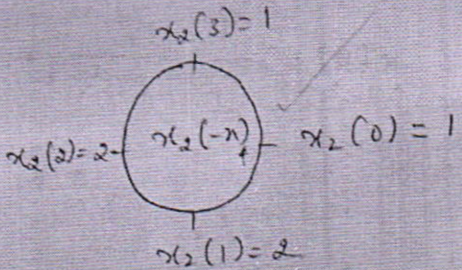




when, $m=0$.

$$x_3(0) = \sum_{n=0}^{4-1=3} x_1(n)\, x_2((-n))_4$$





$$x_3(0) = (1\times1)+(2\times1)+(2\times2)+(1\times2)$$
$$= 1+2+4+2$$
$$x_3(0) = 9$$

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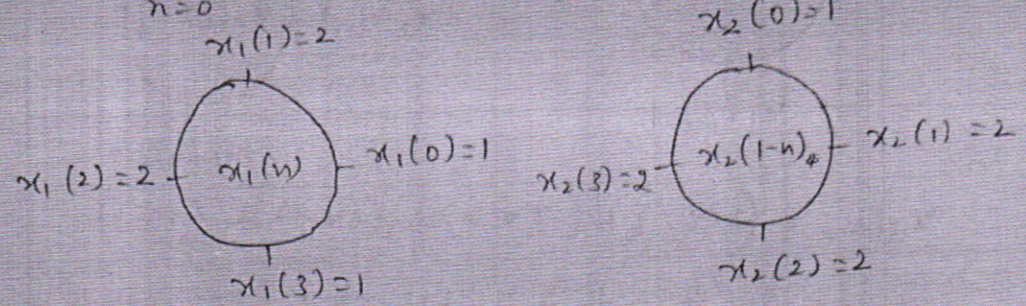
when, $m = 1$

$$x_3(1) = \sum_{n=0}^{3} x_1(n)\, x_2((1-n))_4$$



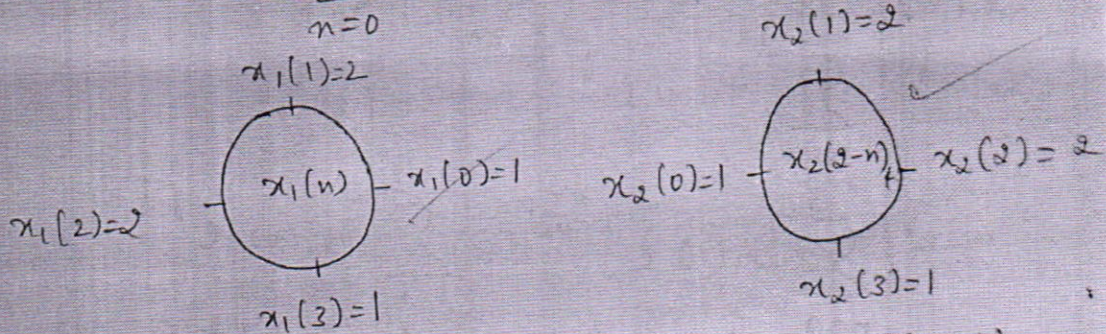
$$= (1\times 2) + (2\times 1) + (2\times 1) + (1\times 2)$$

$$= 2 + 2 + 2 + 2$$

$$x_3(1) = 8$$

when, $m = 2$.

$$x_3(2) = \sum_{n=0}^{3} x_1(n)\, x_2((2-n))_4$$



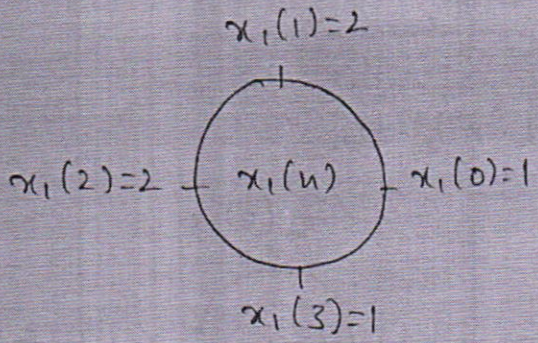
$$x_3(2) = (1\times 2) + (2\times 2) + (2\times 1) + (1\times 1)$$

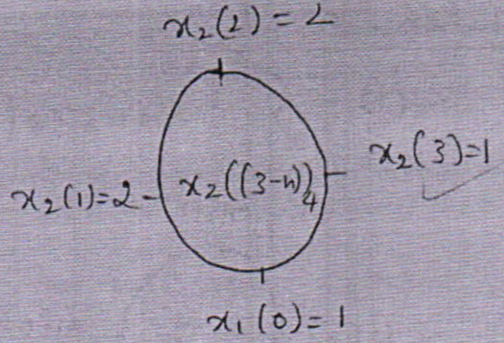
$$= 2 + 4 + 2 + 1$$

$$x_3(2) = 9$$

when, $m = 3$

$$x_3(3) = \sum_{n=0}^{3} x_1(n)\, x_2((3-n))_4$$





$$x_3(n) = (2\times 2) + (2\times 2) + (1\times 1) + (1\times 1)$$
$$= 4 + 4 + 1 + 1$$
$$x_3(3) = 10,$$
$$x_3(n) = \{9, 8, 9, 10\}.$$
$$x_3(m) = x_1(n) \; (N) \; x_2(n) = \{9, 8, 9, 10\}.$$

By Stackham's Method.

$$X_1(K) = DFT[x_1(n)]$$
$$X_1(K) = \sum_{n=0}^{4-1} x_1(n) e^{-j\frac{2\pi}{N}Kn}$$
$$= \sum_{n=0}^{3} x_1(n) e^{-j\frac{2\pi}{4}Kn} = \sum_{n=0}^{3} x_1(n) e^{-j\frac{\pi}{2}Kn}$$
$$X_1(K) = x_1(0) e^{0} + x_1(1) e^{-j\frac{\pi}{2}K(1)} + x_1(2) e^{-j\frac{\pi}{2}K(2)} + x_1(3) e^{-j\frac{\pi}{2}K(3)}$$
$$X_1(K) = 1 + 2e^{-j\frac{\pi K}{2}} + 2e^{-j\pi K} + 1e^{-j\frac{3\pi K}{2}}$$

when, $K=0$

$$X_1(0) = 1 + 2e^{0} + 2e^{0} + e^{0}$$
$$= 1 + 2 + 2 + 1$$
$$X_1(0) = 6$$

$k=1$

$$X_1(1) = 1 + 2e^{-j\frac{\pi}{2}(1)} + 2e^{-j\pi(1)} + e^{-j\frac{3\pi}{2}(1)}$$

$$X_1(1) = 1 + 2[\cos 90 - j\sin 90] + 2[\cos 180 - j\sin 180] + [\cos 270 - j\sin 270].$$

$$X_1(1) = 1 + 2(0 - j) + 2(-1) + -j$$

$$= 1 - 2j - 2 - j \Rightarrow -1-3j$$

$$X_1(1) = -1-3j$$

$k=2$

$$X_1(2) = 1 + 2e^{-j\frac{\pi(2)}{2}} + 2e^{-j\pi(2)} + e^{-j\frac{3\pi(2)}{2}}$$

$$X_1(2) = 1 + 2e^{-j\pi} + 2e^{-j2\pi} + e^{-j3\pi}$$

$$X_1(2) = 1 + 2[\cos 180 - j\sin 180] + 2[\cos 360 - j\sin 360] + [\cos 540 - j\sin 540].$$

$$X_1(2) = 1 + 2(-1-0) + 2[1-0] + (-1)$$

$$= \cancel{1} - \cancel{2} + \cancel{2} - \cancel{1}$$

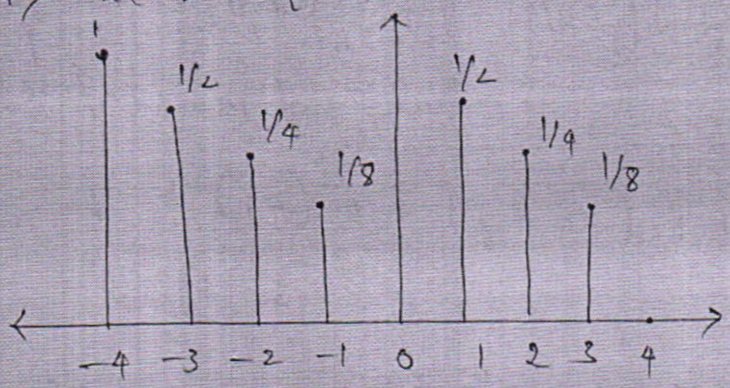
$$X_1(2) = 0$$

6). Given $x(n) = \left[\frac{1}{2}\right]^n [u(n) - u(n-4)]$ Determine the following without computing 4-point DFT $X(K)$.

i). $\sum_{K=0}^{3} X(K)\, X^*(K)$. ii) If $G(K) = W_4^{2K} X(K)$ find $g(n)$

iii) $X(0) + X(2)$

$\Rightarrow$ i) $x(n) = \{1, 1/2, 1/4, 1/8\}$.



when, $n=0$, $g(0) = x(-2)$

$g(0) = 1/4$.

$n=1$, $g(1) = x(-1) = 1/8$

$n=2$, $g(2) = x(0) = 1$

$n=3$, $g(3) = 1/2$

$g(n) = \{1/4, 1/8, 1, 1/2\}$

ii). A/c to parsevals Theorem:

$$\frac{1}{N}\sum_{K=0}^{N-1} X(K)\, X^*(K) = \sum_{n=0}^{N-1} x(n)\, x^*(n)$$

Given $x(n) = \{1, 1/2, 1/4, 1/8\}$

$$\sum_{n=0}^{4-1} |X(K)|^2 = 4\sum_{n=0}^{3} |x(n)|^2$$

$$\sum_{k=0}^{3} |X(k)|^2 = 4\left\{ |x(0)|^2 + |x(1)|^2 + |x(2)|^2 + |x(3)|^2 \right\}$$

$$= 4\left\{ 1 + \frac{1}{4} + \frac{1}{16} + \frac{1}{64} \right\}$$

$$= 4\left\{ \frac{85}{64} \right\}$$

$$= \frac{85}{16}$$

iii) $X(0) + X(2)$

$$X(K) = \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi}{N}kn}$$

$$X(K) = \sum_{n=0}^{4-1} x(n) \cdot e^{-j\frac{2\pi}{4}kn}$$

$$= \sum_{n=0}^{3} x(n)\, e^{-j\frac{\pi}{2}kn} \quad \text{—} (A)$$

when $k=0$, $X(0) = \sum_{n=0}^{3} x(n) e^{0}$

$$= x(0) + x(1) + x(2) + x(3)$$

$$= 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8}$$

$= \frac{15}{8}$

when $K=2$, $X(2) = \sum_{n=0}^{3} x(n)\, e^{-j\pi/2 (2) n}$

$$X(2) = \sum_{n=0}^{3} x(n).\, e^{-j\pi n}$$

$$X(2) = x(0).\, e^{0} + x(1).\, e^{-j\pi} + x(2).\, e^{-j\pi 2} + x(3)\, e^{-j\pi 3}$$

$$= 1 + \frac{1}{2}(-1) + \frac{1}{4} e^{-j2\pi} + \frac{1}{8}(-1)$$

$$= 1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8}$$

$$= \frac{5}{8}$$

$$X(0) + X(2) = \frac{15}{8} + \frac{5}{8} = \frac{20}{8}$$

3. Determine 4-point DFT of $x(n) = \{0, 1, 2, 3\}$ then verify the result by taking IDFT using Linear Transformation.

$\Rightarrow$ $N = 4$.

$x(n) = \{0, 1, 2, 3\}$

$$X(K) = \text{DFT}\left[x(n)\right]$$

$$= \sum_{n=0}^{N-1} x(n).\, e^{-j\frac{2\pi}{N} Kn}$$

But $N=4$

$$x(k) = \sum_{n=0}^{4-1=3} x(n) e^{-j\frac{2\pi}{4} kn}$$

$$x(k) = \sum_{n=0}^{3} x(n) e^{-j\frac{\pi}{2} kn}$$

$$x(k) = x(0)e^{0} + x(1)e^{-j\frac{\pi}{2}k(1)} + x(2)e^{-j\frac{\pi}{2}k(2)} + x(3)e^{-j\frac{\pi}{2}k(3)}.$$

$$x(k) = (0)(1) + 1e^{-j\frac{\pi}{2}k} + 2e^{-j\pi k} + 3e^{-j\frac{3\pi k}{2}}$$

put $k=0$

$$x(0) = 1e^{-j\frac{\pi}{2}(0)} + 2e^{-j\pi(0)} + 3e^{-j\frac{3\pi}{2}(0)}$$

$$x(0) = 1 + 2(1) + 3(1) \Rightarrow 6. \Rightarrow \boxed{X(0)=6}$$

put $k=1$.

$$x(1) = 1e^{-j\frac{\pi}{2}(1)} + 2e^{-j\pi(1)} + 3e^{-j\frac{3\pi}{2}(1)}$$

$$x(1) = (\cos 90 - j\sin 90) + 2(\cos 180 - j\sin 180) + 3(\cos 270 - j\sin 270)$$

$$x(1) = -j - 2 + 3j \Rightarrow \boxed{X(1) = -2+2j}$$

put $k=2$.

$$x(2) = 1e^{-j\frac{\pi}{2}(2)} + 2e^{-j\pi(2)} + 3e^{-j\frac{3\pi}{2}(2)}$$

$$x(2) = (\cos 180 - j\sin 180) + 2(\cos 360 - j\sin 360) + 3(\cos 540 - j\sin 540)$$

$$x(2) = -1 + 2 - 3 \Rightarrow \boxed{X(2) = -2}$$

put $k=3$.

$$x(3) = 1e^{-j\frac{\pi}{2}(3)} + 2e^{-j\pi(3)} + 3e^{-j\frac{3\pi}{2}(3)}$$

$$= (\cos 270 - j\sin 270) + 2(\cos 540 - j\sin 540) + 3(\cos 810 - j\sin 810)$$

$$= +j - 2 - 3j \Rightarrow -2 - 2j \Rightarrow -2 - 2j$$

$$x(k) = \{6, -2+2j, \underset{\uparrow}{-2}, -2-2j\}.$$

Midpoint, $\frac{N}{2} = \frac{4}{2} = 2$

8) Compute 4-point DFT of the Sequence $x(n) = \{1, 0, 1, 0\}$ also find $y(n)$ if $X(K) = X((K-2))_4$

$\Rightarrow$ $$X(K) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N}Kn}$$

$$= \sum_{n=0}^{4-1=3} x(n) e^{-j\frac{2\pi}{4}Kn}$$

$$= \sum_{n=0}^{3} x(n) e^{-j\frac{\pi}{2}Kn}$$

$$X(K) = x(0).e^0 + x(1).e^{-j\frac{\pi}{2}K(1)} + x(2).e^{-j\frac{\pi}{2}K(2)} + x(3)e^{-j\frac{\pi}{2}K(3)}$$

$$X(K) = 1 + e^{-j\pi K} \quad \text{— (1)}$$

when $K = 0$, $X(0) = 1 + e^0 = 1 + 1 = 2$

$K = 1$, $X(1) = 1 + e^{-j\pi} = 1 - 1 = 0$

$K = 2$, $X(2) = 1 + e^{-j\pi 2} = 1 + 1 = 2$

$K = 3$, $X(3) = 1 + e^{-j\pi 3} = 1 - 1 = 0$

$$X(K) = \{2, 0, 2, 0\}$$

we know frequency Shift property

$$X((K-l)) \longleftrightarrow W_N^{-ln} x(n).$$

(or)

$$X((K-l))_N \longleftrightarrow e^{j\frac{2\pi}{N}ln} x(n).$$

Given :-

$$Y(K) = X((K-2))$$

$$y(n) = e^{j\frac{2\pi}{N}ln} x(n)$$

$$y(n) = e^{j\frac{2\pi}{4}(2)n} x(n)$$

$$y(n) = e^{j\pi n} x(n)$$

$n=0, \quad y(0) = e^{0} x(0) = 1$

$n=1, \quad y(1) = e^{j\pi} x(1) = 0$

$n=2, \quad y(2) = e^{j\pi 2} x(2) = 1$

$n=3, \quad y(3) = e^{j\pi 3} x(3) = 0$

$$y(n) = \{1, 0, 1, 0\}.$$

Assignment - 2

1. Find the o/p. $y(n)$ of a filter whose impulse response is $h(n) = \{1, 1, 1\}$ & i/p Signal.
$x(n) = \{3, -1, 0, 1, 3, 2, 0, 1, 2, 1\}$ using
i) over-lap Save method.
ii) over-lap add method use Circular Convolution

$\Rightarrow$

$N = 2^M$ $\qquad$ $N = L + M - 1$

$N = 2^3$ $\qquad$ $8 = L + 3 - 1$

$N = 8$ $\qquad$ $8 = L + 2$

$L = 8 - 2$

$\boxed{L = 6}$

i). Save Method :-

$x_1(n) = \{3, -1, 0, 1, 3, 2\}$ But $N = 8$

$x_1(n) = \{0, 0, 3, -1, 0, 1, 3, 2\}$

$x_2(n) = \{3, 2, 0, 1, 2, 1, 0, 0\}$.

$h(n) = \{1, 1, 1, 0, 0, 0, 0, 0\}$

$$\begin{bmatrix} y_1(0) \\ y_1(1) \\ y_1(2) \\ y_1(3) \\ y_1(4) \\ y_1(5) \\ y_1(6) \\ y_1(7) \end{bmatrix} = \begin{bmatrix} 0 & 2 & 3 & 1 & 0 & -1 & 3 & 0 \\ 0 & 0 & 2 & 3 & 1 & 0 & -1 & 3 \\ 3 & 0 & 0 & 2 & 3 & 1 & 0 & -1 \\ -1 & 3 & 0 & 0 & 2 & 3 & 1 & 0 \\ 0 & -1 & 3 & 0 & 0 & 2 & 3 & 1 \\ 1 & 0 & -1 & 3 & 0 & 0 & 2 & 3 \\ 3 & 1 & 0 & -1 & 3 & 0 & 0 & 2 \\ 2 & 3 & 1 & 0 & -1 & 3 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \\ 3 \\ 2 \\ 2 \\ 0 \\ 4 \\ 6 \end{bmatrix}$$

$$y_1(n) = \{5, 2, 3, 2, 2, 0, 4, 6\}$$

$$\begin{bmatrix} y_2(0) \\ y_2(1) \\ y_2(2) \\ y_2(3) \\ y_2(4) \\ y_2(5) \\ y_2(6) \\ y_2(7) \end{bmatrix} = \begin{bmatrix} 3 & 0 & 0 & 1 & 2 & 1 & 0 & 2 \\ 2 & 3 & 0 & 0 & 1 & 2 & 1 & 0 \\ 0 & 2 & 3 & 0 & 0 & 1 & 2 & 1 \\ 1 & 0 & 2 & 3 & 0 & 0 & 1 & 2 \\ 2 & 1 & 0 & 2 & 3 & 0 & 0 & 1 \\ 1 & 2 & 1 & 0 & 2 & 3 & 0 & 0 \\ 0 & 1 & 2 & 1 & 0 & 2 & 3 & 0 \\ 0 & 0 & 1 & 2 & 1 & 0 & 2 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \\ 5 \\ 3 \\ 3 \\ 4 \\ 3 \\ 1 \end{bmatrix}$$

$$y_2(n) = \{3, 5, 5, 3, 3, 4, 3, 1\}$$

$$y_1(n) = \{\cancel{5}, \cancel{2}, 3, 2, 2, 0, 4, 6\}$$

$$y_2(n) = \{\cancel{3}, \cancel{5}, 5, 3, 3, 4, 3, 1\}$$

$$y(n) = \{3, 2, 2, 0, 4, 6, 5, 3, 3, 4, 3, 1\}$$

Add Method :- $x_1(n) = \{3, -1, 0, 1, 3, 2, 0, 0\}$

$x_2(n) = \{0, 1, 2, 1, 0, 0, 0, 0\}$

$h(n) = \{1, 1, 1, 0, 0, 0, 0, 0\}$

$$\begin{bmatrix} y_1(0) \\ y_1(1) \\ y_1(2) \\ y_1(3) \\ y_1(4) \\ y_1(5) \\ y_1(6) \\ y_1(7) \end{bmatrix} = \begin{bmatrix} 3 & 0 & 0 & 2 & 3 & 1 & 0 & 0 \\ -1 & 3 & 0 & 0 & 2 & 3 & 1 & 0 \\ 0 & -1 & 3 & 0 & 0 & 2 & 3 & 1 \\ 1 & 0 & -1 & 3 & 0 & 0 & 2 & 3 \\ 3 & -1 & 0 & -1 & 3 & 0 & 0 & 2 \\ 2 & 3 & 1 & 0 & -1 & 3 & 0 & 0 \\ 0 & 2 & 3 & 1 & 0 & -1 & 3 & 0 \\ 0 & 0 & 2 & 3 & 1 & 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 2 \\ 0 \\ 4 \\ 6 \\ 5 \\ 2 \end{bmatrix}$$

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$$\begin{bmatrix} y_2(0) \\ y_2(1) \\ y_2(2) \\ y_2(3) \\ y_2(4) \\ y_2(5) \\ y_2(6) \\ y_2(7) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 2 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 2 \\ 2 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 2 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 2 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 3 \\ 4 \\ 3 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$y_2(n) = \{0, 1, 3, 4, 3, 1, 0, 0\}$

$y_1(n)$

3	2	2	0	4	6	5	2

+ +

$y_2(n)$

0	1	3	4	3	1	0	0

$y(n) = \{3, 2, 2, 0, 4, 6, 5, 3, 3, 4, 3, 1, \ldots\}$

2) Derive an Expression for decimation in time (DIT) FFT algorithm.

A) $x(n) = \{x(0), x(1), x(2) \ldots x(N-1)\}$

Even indexed $\{x(0), x(2), x(4) \ldots x(N-2)\}$

odd indexed $\{x(1), x(3), x(5) \ldots x(N-1)\}$

W.K.T. DFT eqn.

$$X(K) = \sum_{n=0}^{N-1} x(n) W_N^{nK} \quad \text{—} ①$$

Separate. $x(n)$ into even & odd values of $x(n)$

$$X(K) = \sum_{n=\text{even}} x(n) W_N^{nK} + \sum_{n=\text{odd}} x(n) W_N^{nK}.$$

$$X(K) = \sum_{n=0}^{\frac{N}{2}-1} x_e(2n) W_N^{2nK} + \sum_{n=0}^{\frac{N}{2}-1} x_o(2n+1) W_N^{(2n+1)K}$$

$$W_N^{2nK} = \frac{W_N^{nK}}{2} \Rightarrow W_N = e^{-j\frac{2\pi}{N}}$$

$$W_N^{2nK} = e^{-j\frac{2\pi}{N}2nK} = e^{\frac{-j2\pi}{N/2}nK} = W_{N/2}^{nK}$$

$$X(K) = \underbrace{\sum_{n=0}^{\frac{N}{2}-1} x_e(2n) W_{N/2}^{nK}}_{\frac{N}{2}\text{ point of DFT of even Seq.}} + W_N^K \underbrace{\sum_{n=0}^{\frac{N}{2}-1} x_o(2n+1) W_{N/2}^{nK}}_{\frac{N}{2}\text{ DFT of odd Seq.}}$$

$$X(K) = G(K) + W_N^K H(K) \rightarrow Ⓐ$$

where. $$G(K) = \sum_{n=0}^{\frac{N}{2}-1} x_e(2n) W_{N/2}^{nK}$$

$$H(K) = \sum_{n=0}^{N/2-1} x_o(2n+1) W_{\frac{N}{2}}^{2K}$$

Since. $G(K)$ & $H(K)$ are periodic with period $\frac{N}{2}$

$\therefore G\left(k+\frac{N}{2}\right) = G(k)$ & $H\left(k+\frac{N}{2}\right) = H(k)$

So. eqⁿ (A) Becomes.

$$x\left(k+\frac{N}{2}\right) = G\left(k+\frac{N}{2}\right) W_N^{k+N/2} H\left(k+\frac{N}{2}\right)$$

$$x\left(k+\frac{N}{2}\right) = G(k) + W_N^k W_N^{N/2} H(k)$$

$$W_N^{N/2} = e^{-j2\pi/N (N/2)} = e^{-j\pi} = (-1)$$

$$\therefore x\left(k+\frac{N}{2}\right) = G(k) - W_N^k H(k) \rightarrow \text{(B)} \quad 0 \le k \le \frac{N}{2}-1$$

Consider $N=8$. then eqⁿ (A)

when $k=0$, $x(0) = G(0) + W_N^0 H(0)$

$k=1$, $x(1) = G(1) + W_N^1 H(1)$

$k=2$, $x(2) = G(2) + W_N^2 H(2)$

$k=3$, $x(3) = G(3) + W_N^3 H(3)$

from eqⁿ (B) i.e.,

$$x\left(k+\frac{N}{2}\right) = G(k) - W_N^k H(k)$$

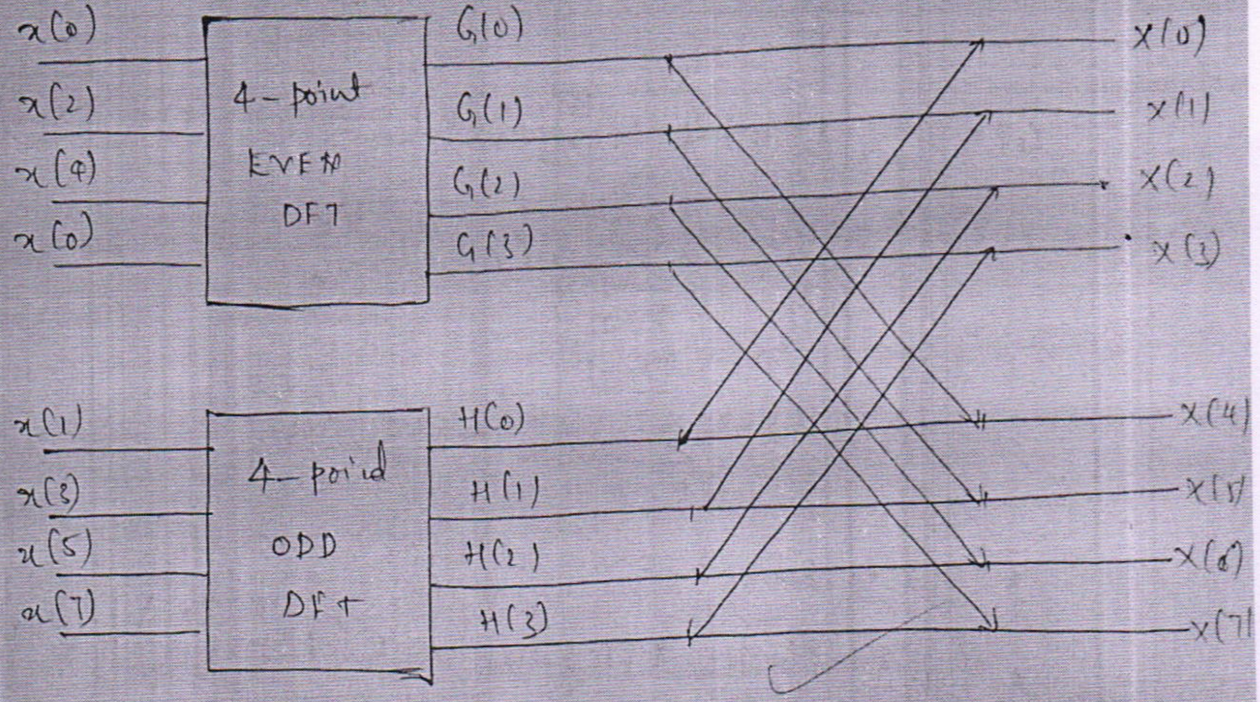
when. $k=0$. $x(N/2) = G(0) - W_N^0 H(k)$

$x(4) = G(0) - W_8^0 H(0)$

$k=1$, $x(5) = G(1) - W_8^1 H(1)$

$k=2$, $x(6) = G(2) - W_8^2 H(2)$

$k=3$, $x(7) = G(3) - W_8^3 H(3)$.



Ist Stage decomposition

II nd Stage deComposition :-

$$\left\{ G(K) = \sum_{n=0}^{N/2-1} x_e(2n)\, W_{N/2}^{nK} \right\}$$

$$G(K) = \sum_{n=0}^{N/4-1} x_{ee}(2n)\, W_{\frac{N}{2}}^{nK} + \sum_{n=0}^{N/4-1} x_{eo}(2n)\, W_{N/2}^{nK}$$

Substituting $n = 2n$ for 1st term & $n = 2n+1$ for IInd term.

$$G(K) = \sum_{n=0}^{N/4-1} x_{ee}\, 2(2n)\, W_{N/2}^{2nK} + \sum_{n=0}^{N/4-1} x_{eo}\left[2(2n+1)\right] W_{N/2}^{(2n+1)K}$$

$$G(K) = \sum_{n=0}^{N/4-1} x_{ee}(4n)\, W_{N/4}^{nK} + \sum_{n=0}^{N/4-1} x_{eo}\left[4n+2\right] W_{N/2}^{2nK}\, W_{N/2}^{K}$$

$$G(k) = \sum_{n=0}^{N/4-1} x(4n) W_{N/4}^{nk} + W_{N/2}^{k} \sum_{n=0}^{N/4-1} x(4n+2) W_{N/4}^{nk}$$

$$G(k) = A(k) + W_{N/2}^{nk} B(k) \rightarrow © \quad 0 \le k \le \frac{N}{4}-1$$

where $A(k) = \sum_{k=0}^{N/4-1} x(4n) W_{N/4}^{nk}$

$$B(k) = \sum_{k=0}^{N/4-1} x(4n+2) W_{N/4}^{nk}$$

$A(k)$ & $B(k)$ are periodic with the period $N/4$

$A\left(k+\frac{N}{4}\right) = A(k)$. $\quad B\left(k+\frac{N}{4}\right) = B(k)$

Eqn ©.

$$G\left(k+\frac{N}{4}\right) = A\left(k+\frac{N}{4}\right) + W_{\frac{N}{2}}^{k+N/4} B\left(k+\frac{N}{4}\right) \quad 0 \le k \le \frac{N}{4}-1$$

$$\therefore G\left(k+\frac{N}{4}\right) = A(k) + W_{N/2}^{k} W_{N/2}^{N/4} B(k)$$

$$W_{N/2}^{N/4} = e^{-j\frac{2\pi}{N/2}\left[N/4\right]} = e^{-j\frac{4\pi}{N}\left[\frac{N}{4}\right]} = e^{-j\pi} = -1$$

$$G\left(k+\frac{N}{4}\right) = A(k) - W_{\frac{N}{2}}^{k} B(k) \rightarrow ① \quad 0 \le k \le \frac{N}{4}-1$$

from eqn ©.

when $k=0$, $G(2) = A(0) + W_4^0 B(0)$

$k=1$, $G(3) = A(1) + W_4^1 B(1)$

$k=2$, $G(4) = A(2) + W_4^2 B(2)$

$k=3$, $G(5) = A(3) + W_4^3 B(3)$

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$$H(K) = \sum_{n=0}^{N/2-1} x_0(2n+1) W_{N/2}^{nK} \qquad 0 \le K \le N/2 - 1$$

Decimation $H(K)$ into even & odd sequence

$$H(K) = \sum_{n=0}^{N/4-1} x\left[2(2n+1)\right] W_{N/2}^{2nK} + \sum_{n=0}^{N/4-1} \left[2\left[2n+1\right]+1\right]\} W_{N/2}^{(2n+1)K}$$

$$= \sum_{n=0}^{N/4-1} x(4n+1) W_{N/4}^{nK} + \sum_{n=0}^{N/4-1} x\left(4n+2+1\right] W_{N/2}^{2nK} W_{N/2}^{K}$$

$$H(K) = \sum_{n=0}^{N/4-1} x(4n+1) W_{N/4}^{nK} + W_{N/8}^{K} \sum_{n=0}^{N/4-1} x(4n+3) W_{N/4}^{nK}$$

$$H(K) = C(K) + W_{N/2}^{K} D(K)$$

where, $K=0$, $H(0) = C(0) + W_4^0 D(0)$

$K=1$, $H(1) = C(1) + W_4^1 D(1)$

$K=2$, $H(2) = C(2) + W_4^2 D(2)$

$K=3$, $H(3) = C(3) + W_4^3 D(3)$

$C(K)$ & $D(K)$ are period with $N/4$.

$C(K+N/4) = C(K)$. $\quad D(K+N/4) = D(K)$

$$\therefore H\left(K+\frac{N}{4}\right) = C\left(K+\frac{N}{4}\right) + W_{N/2}^{K+N/4} D\left(K+N/4\right)$$

$$H\left(K+\frac{N}{4}\right) = C(K) + W_{N/2}^{K} W_{N/2}^{N/4} D(K)$$

$$W_{N/2}^{N/4} = e^{-j\,2\pi/N/2 \cdot N/4} = e^{-j4\pi/N(N/4)} = e^{-j\pi} = -1$$

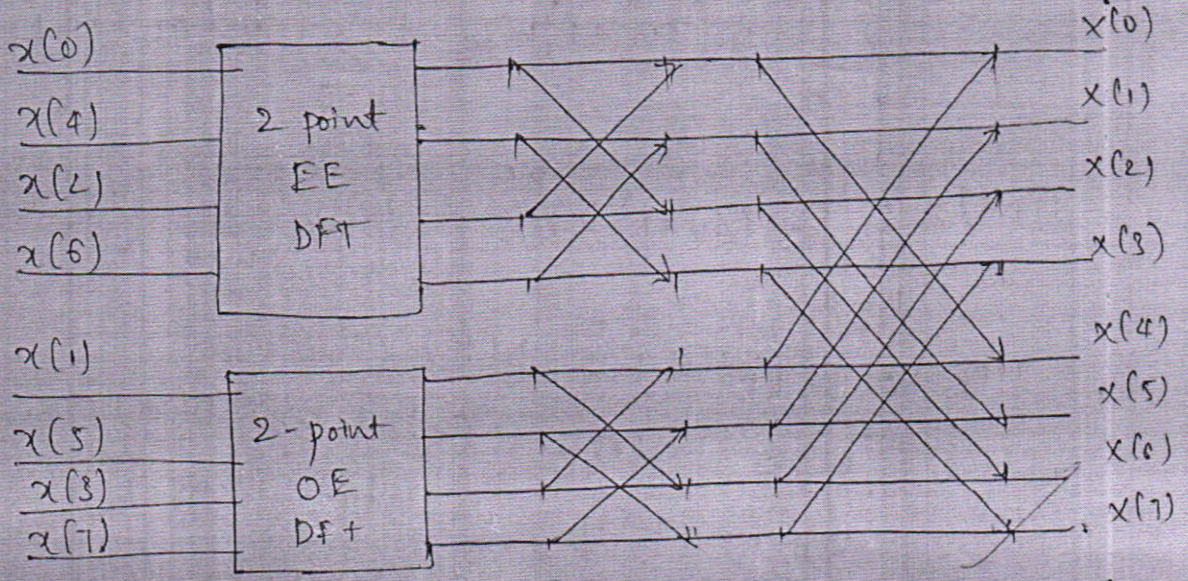
$$H\left(K+\frac{N}{4}\right) = C(K) - W_{N/2}^{K} D(K).$$

$K=0, \; H(2) = C(0) - W_4^0 D(0)$

$K=1, \; H(3) = C(1) - W_4^1 D(1)$

$K=2, \; H(4) = C(2) - W_4^2 D(2)$

$K=3, \; H(5) = C(3) - W_4^3 D(3)$



II-Stage deComposition

<u>Stage 03</u> :- WK, $A(K) = \sum_{n=0}^{N/4-1} x(4n)\, W_{N/4}^{nK}$ $\left\{ \frac{N}{4} = \frac{8}{4} = 2 \right\}$

$$A(K) = \sum_{n=0}^{1} x(4n) W_2^{nK}$$

$$A(K) = x(0) \cdot W_2^0 + x(4) W_2^K \quad \Big| \; W_2^0 = e^{-j\frac{2\pi}{2}(0)} \quad e^0 = 1$$

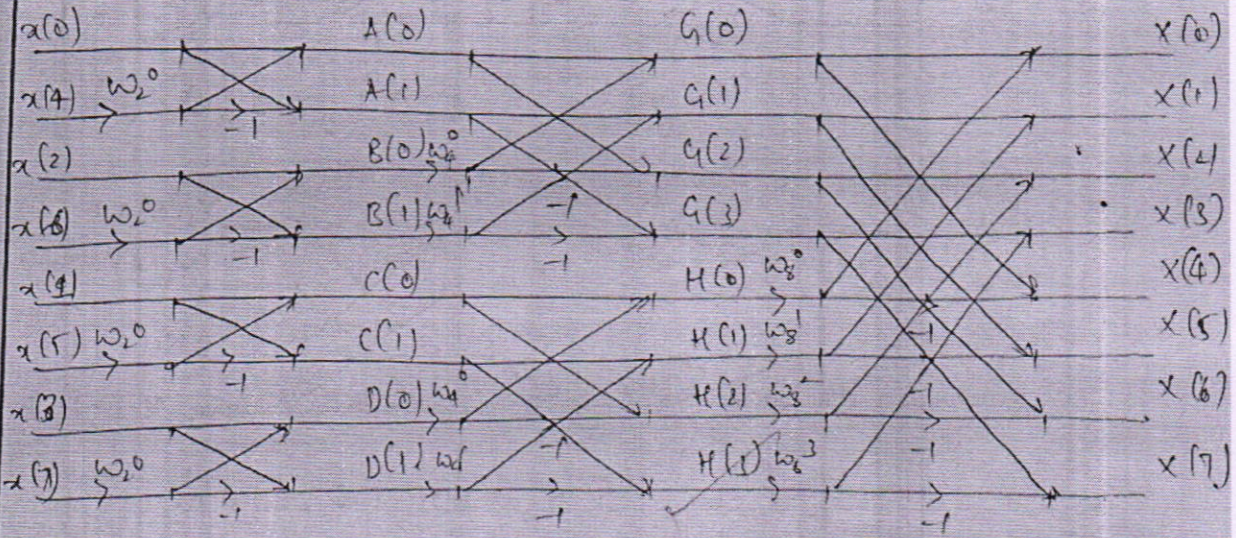
when. $K = 0$. $A(0) = x(0) + W_2^0 \, x(4)$

$A(0) = x(0) + x(4)$

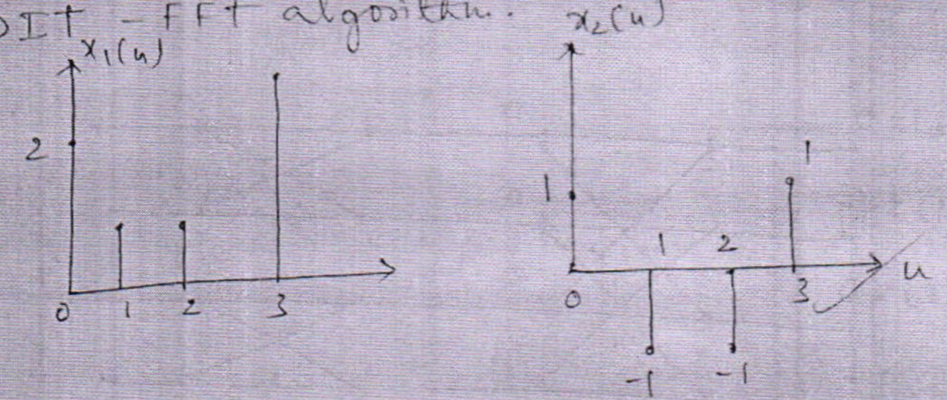
$K = 1$, $A(1) = x(0) + W_2^1 x(4)$

$A(1) = x(0) - W_2^0 x(4)$

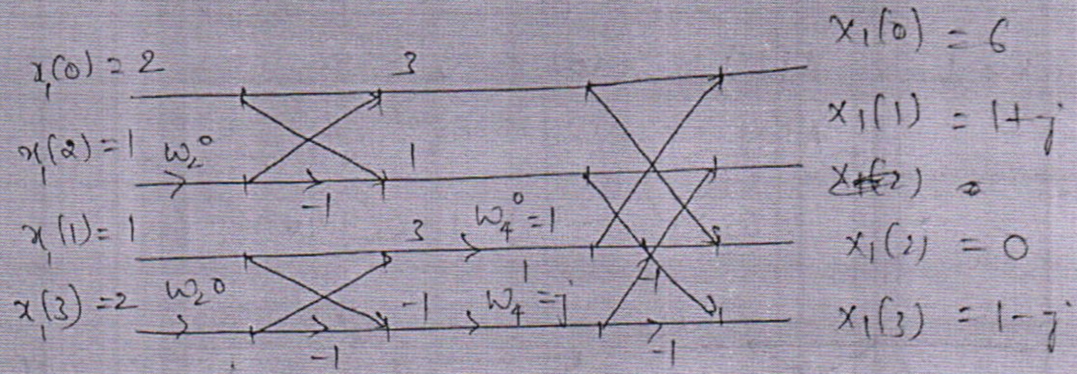
$K = 2$, $A(2) = x(0) + W_2^2 x(4)$



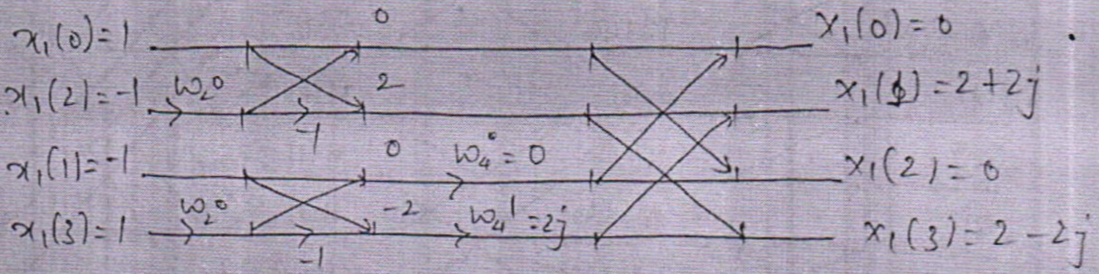
3. Given the Sequences $x_1(n)$ & $x_2(n)$ below. Compute the Circular Convolution $x_1(n) \circledast x_2(n)$ for $N=4$ use DIT - FFT algorithm.



Solⁿ:- $x_1(n) = \{2, 1, 1, 2\}$ $\quad x_2(n) = \{1, -1, -1, 1\}$



$$X_1(k) = \{6, 1+j, 0, 1-j\}$$

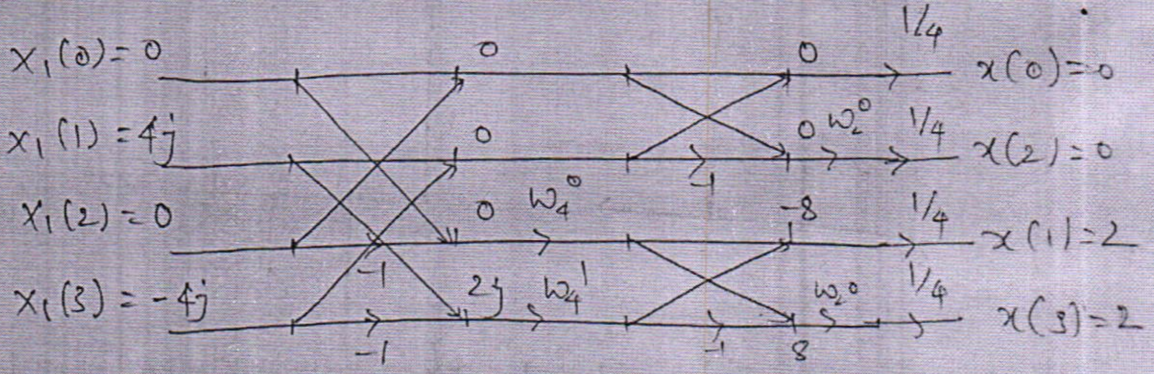


$$X_2(K) = \{0, 2+2j, 0, 2-2j\}$$

$$X_3(K) = X_1(K)\, X_2(K)$$

$$= \{6, 1+j, 0, 1-j\}\{0, 2+2j, 0, 2-2j\}$$

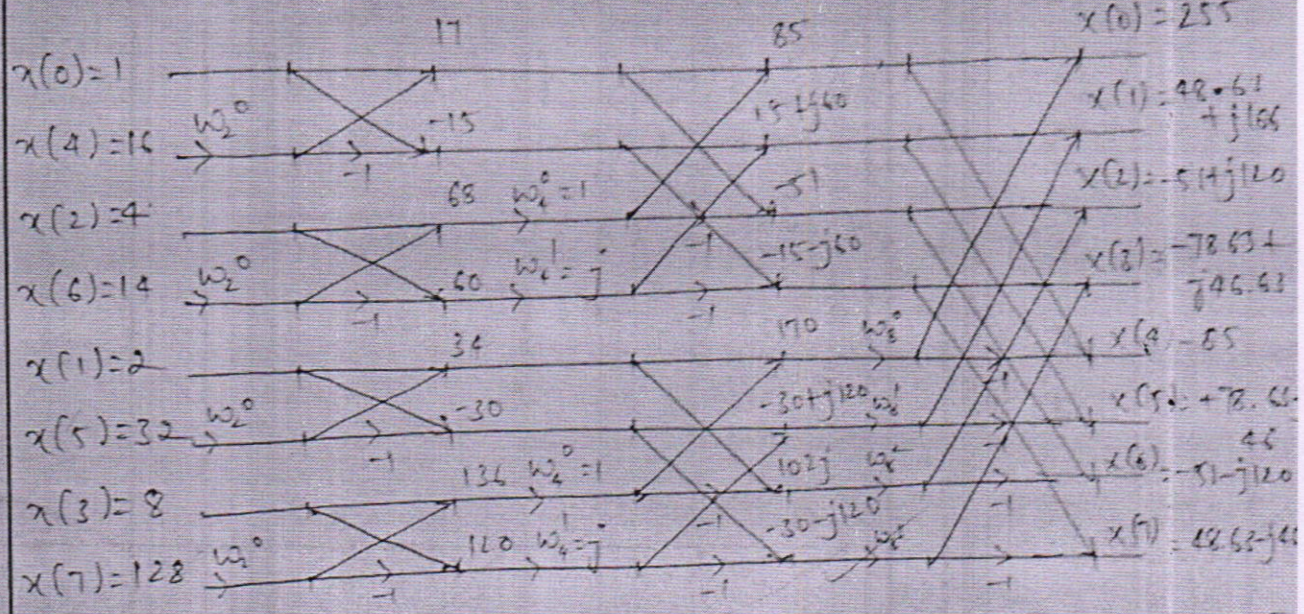
IDFT



$$x_1(n) \circledast x_2(n) = \{0, -2, 0, 2\}$$

4) Given $x(n) = 2^n$ & $N=8$. Find $X(k)$ using DIF-FFT algorithm.

=> $x(n) = \{1, 2, 4, 8, 16, 32, 64, 128\}$.



$X(K) = \{255, 48.63 + j166.05, -51 + j120, -78.63 + j46.05,$
$-85, -78.63 - j46.05, -51 - j120, 48.63 - j166.05\}$

5.) Tabulate the Comparision of Complex multiplication and addition for direct Computation of DFT v/s the FFT algorithm for $N = 16, 32, 128$.

No.	No. of addition		No. of Multiplication	
	Direct DFT $(N(N-1))$	FFT $N\log_2 N$	Direct DFT N^2	$\frac{N}{2}\log_2 N$
16	240	64	256	32
32	992	166	1024	80
128	16256	896	16384	448

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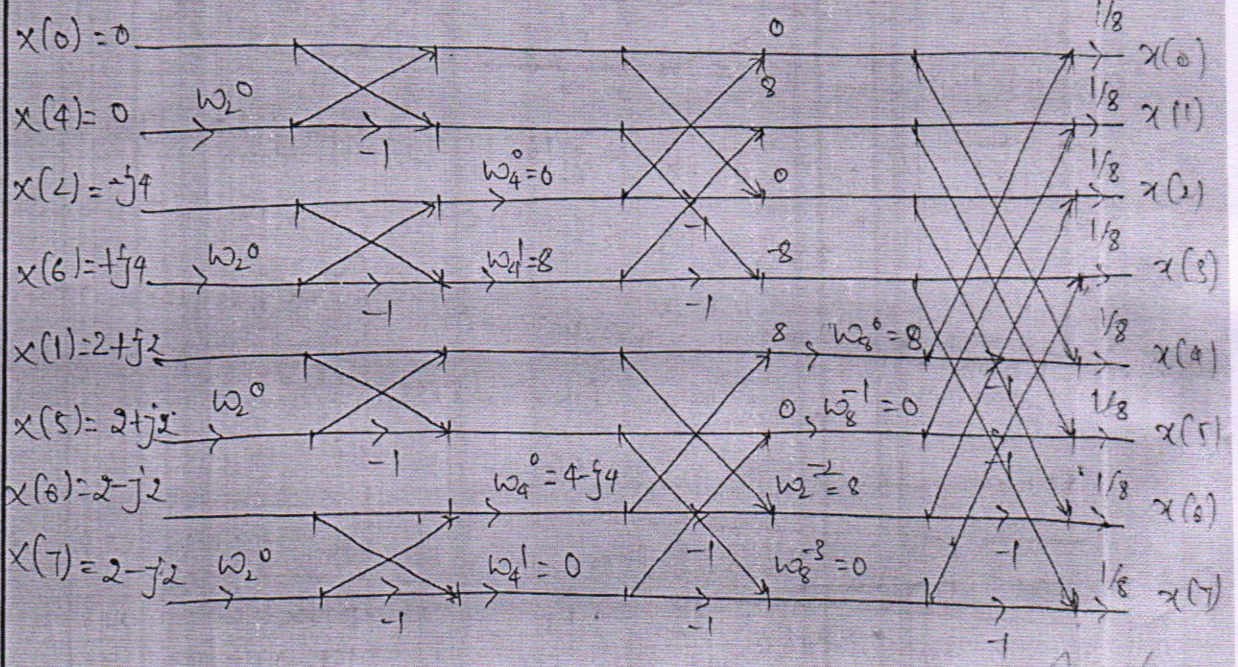
6) First five samples of the 8-point DFT of a real valued sequence is given by $X(0) = 0$, $X(1) = 2+j2$, $X(2) = -j4$, $X(3) = 2-j2$, $X(4) = 0$. Determine the remaining point hence find the original sequence $x(n)$ using DIF-FFT.

$\Rightarrow$ $X(5) = X^*(3) = 2+j2$

$X(6) = X^*(0) = j4$

$X(7) = X^*(1) = 2-j2$



$x(n) = \{1, 1, -1, -1, -1, 1, +1, -1\}$

Assignment - 03

1) Direct form. I, II. Cascode & parallel form for the given. T.F. $H(z) = \dfrac{8z^3 - 4z^2 + 4z - 2}{(z - 1/2)(z^2 - z + 1/4)}$

Sol^n: Numerator. $8z^3 - 4z^2 + 4z - 2 = 0$ Can be expressed as

when $z = 0$, $\Rightarrow 2$

when $z = 1/2 \Rightarrow 8(1/2)^3 - 4(1/2)^2 + 4(1/2) - 2 = 0$

$8(1/8) - 4(1/4) + 2 - 2 = 0$

$1 - 1 + 2 - 2 = 0.$

$\frac{1}{2} = 0.5$

8	-4	4	-2
0	4	0	2

8 0 4 0 $\Rightarrow 8z^2$

$z = \frac{1}{2}$ $8z^2 + 4$

$\left(z - \frac{1}{2}\right)$

$(z - 1/2)(8z^2 + 4)$

$(z - 1/2)4(2z^2 + 1)$

$4(z - 1/2)(2z^2 + 1)$

$2(2z - 1)(2z^2 + 1)$

$(z - 1/2)(8z^4 + 4) = 0$

$8z^3 + 4z - 4z^2 - 2 = 0$

$8z^3 - 4z^2 + 4z - 2 = 0.$

$$H(z) = \frac{2(2z-1)}{(z-1/4)} \cdot \frac{2z^2+1}{(z^2-z+1/2)} \div \frac{z}{z} \; \frac{z^2}{z^2}$$

$$H(z) = \frac{2(2-z^{-1})}{(1-1/4 z^{-1})} \cdot \frac{2+z^{-2}}{(1-z^{-1}+1/2 z^{-2})}$$

$$H_1(z) \qquad H_2(z)$$

$$\frac{W(z)}{X(z)} \cdot \frac{Y(z)}{W(z)} = \frac{2}{(1-1/4 z^{-1})} \cdot \frac{(2-z^{-1})}{1}$$

$$\frac{W(z)}{X(z)} = \frac{2}{1-1/4 z^{-1}}$$

$$W(z) - \frac{1}{4} z^{-1} W(z) = 2X(z)$$

$$w(n) = 2x(n) + \frac{1}{4} w(n-1)$$

$$\frac{Y(z)}{W(z)} = \frac{2-z^{-1}}{1}$$

$$Y(z) = 2W(z) - z^{-1}W(z)$$

$$y(n) = 2w(n) - w(n-1)$$

$$\frac{W(z)}{Y(z)} \cdot \frac{Y(z)}{W(z)} = \frac{1}{(1-z^{-1}+1/2 z^{-2})}$$

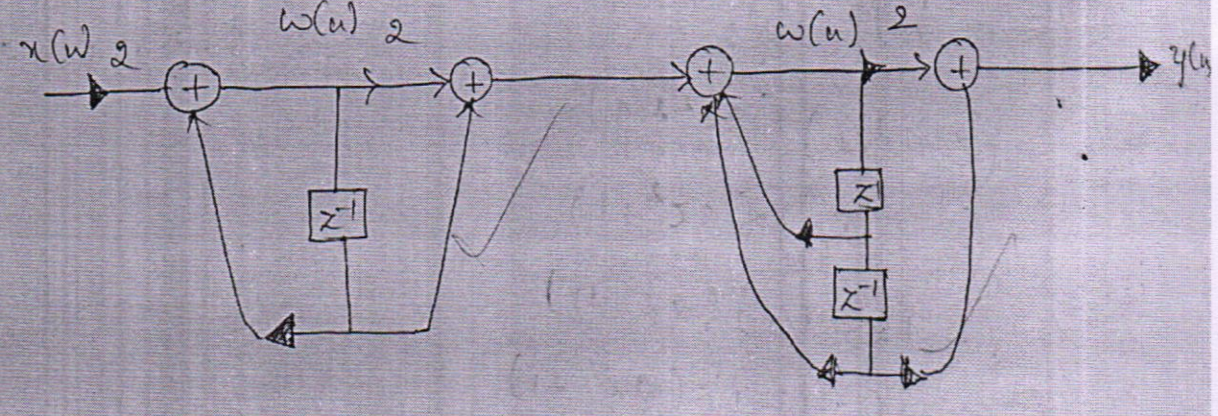
$$\frac{W(z)}{X(z)} = \frac{1}{1-z^{-1}+\frac{1}{2}z^{-2}}$$

$$W(z) - z^{-1}W(z) + \frac{1}{2}z^{-2}W(z) = X(z)$$

$$w(n) = x(n) + w(n-1) - \frac{1}{3} w(n-2)$$

$$Y(z) = 2W(z) + z^{-2}W(z)$$

$$y(n) = 2w(n) + w(n-2)$$



PRINCIPAL
SIET., TUMAKURU.

parallel form:

$$H(z) = \frac{8z^3 - 4z^2 + 4z - 2}{(z - 1/4)(z^2 - z + 1/2)}$$

$$(z - 1/4)(z^2 - z + 1/2) = z^3 - z^2 + \frac{1}{2}z - \frac{1}{4}z^2 + \frac{1}{4}z - \frac{1}{8}$$

$$= z^3 - \left(1 + \frac{1}{4}\right)z^2 + \left(\frac{1}{2} + \frac{1}{4}\right)z - \frac{1}{8}$$

$$= z^3 - \frac{5}{4}z^2 + \frac{3}{4}z - \frac{1}{8}$$

$$H(z) = \frac{8z^3 - 4z^2 + 4z - 2}{z^3 - \frac{5}{4}z^2 + \frac{3}{4}z - \frac{1}{8}}$$

Direct form I :

$$H(z) = \frac{8z^3 - 4z^2 + 4z - 2}{(z - 1/4)(z^2 - z + 1/2)} \quad \begin{array}{l} \div z^3 \\ \div z z^2 \end{array}$$

$$H(z) = \frac{8 - 4z^{-1} + 4z^{-2} - 2z^{-3}}{\left(1 - \frac{1}{4}z^{-1}\right)\left(1 - z^{-1} + \frac{1}{2}z^{-2}\right)}$$

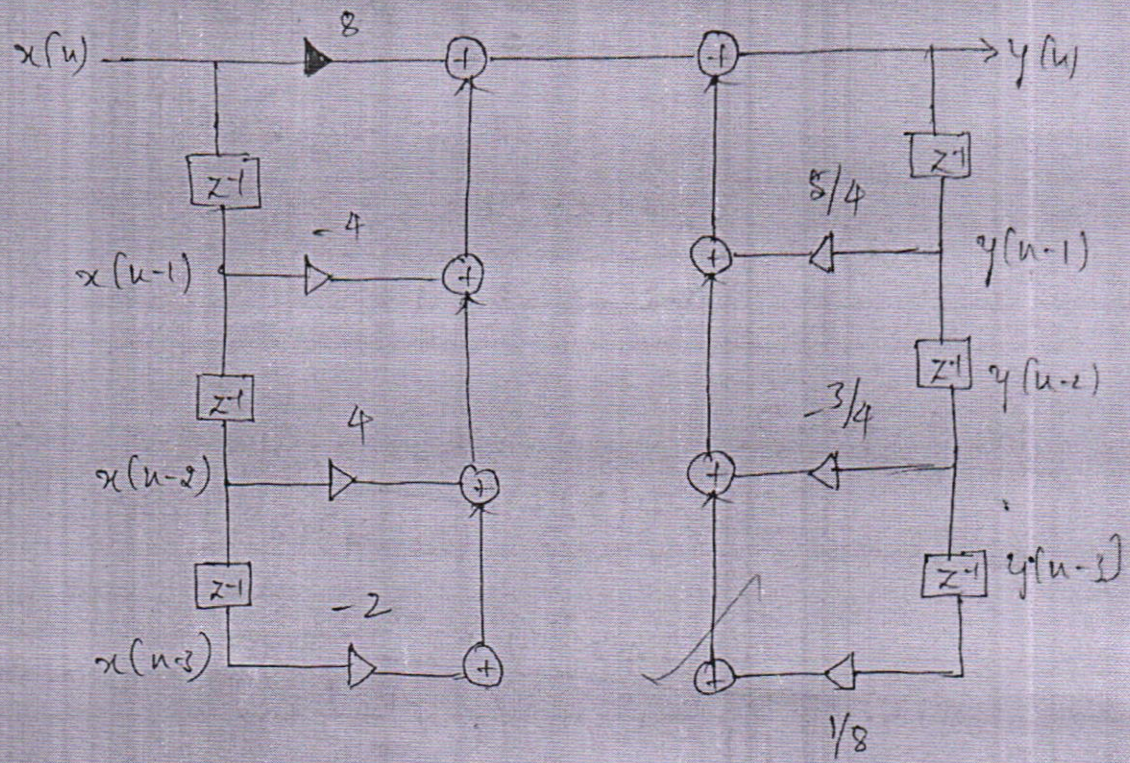
$$H(z) = \frac{8 - 4z^{-1} + 4z^{-2} - 2z^{-3}}{1 - z^{-1} + \frac{1}{2}z^{-2} - \frac{1}{4}z^{-1} + \frac{1}{4}z^{-2} - \frac{1}{8}z^{-3}}$$

$$\frac{Y(z)}{X(z)} = \frac{8 - 4z^{-1} + 4z^{-2} - 2z^{-3}}{1 - \frac{5}{4}z^{-1} + \frac{3}{4}z^{-2} - \frac{1}{8}z^{-3}}$$

$$Y(z)\left[1 - \frac{5}{4}z^{-1} + \frac{3}{4}z^{-2} - \frac{1}{8}z^{-3}\right] = 8x(z) - 4z^{-1}x(z) + 4z^{-2}x(z) - 2z^{-3}x(z).$$

Take inverse z-transform.

$$y(n) = 8x(n) - 4x(n-1) + 4x(n-2) - 2x(n-3) + \frac{5}{4}y(n-1)$$
$$- \frac{3}{4}y(n-2) + \frac{1}{8}y(n-3).$$



Direct form II :

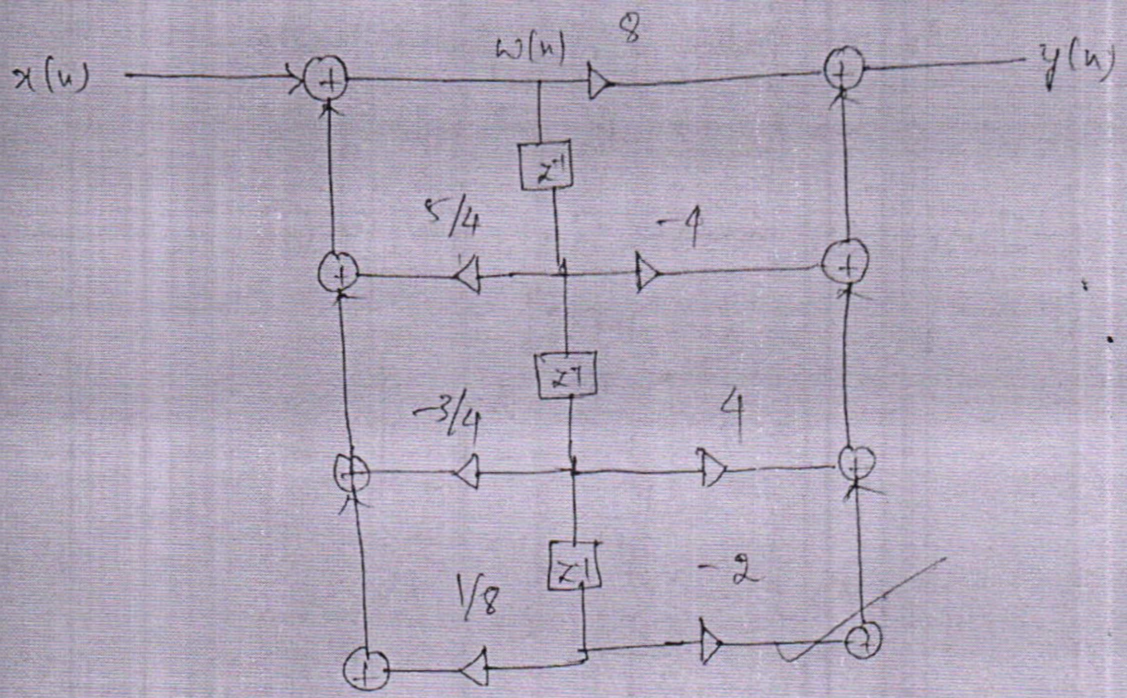
$$\frac{W(z)}{X(z)} \cdot \frac{Y(z)}{W(z)} = \frac{1}{1 - \frac{5}{4}z^{-1} + \frac{3}{4}z^{-2} - \frac{1}{8}z^{-3}} \cdot \frac{8 - 4z^{-1} + 4z^{-2} - 2z^{-3}}{1}$$

$$w(z) - \frac{5}{4}z^{-1}w(z) + \frac{3}{4}z^{-2}w(z) - \frac{1}{8}z^{-3}w(z) = x(z)$$

$$w(n) = x(n) + \frac{5}{4}w(n-1) - \frac{3}{4}w(n-2) + \frac{1}{8}w(n-1)$$

$$Y(z) = 8w(z) - 4z^{-1}w(z) + 4z^{-2}w(z) - 2z^{-3}w(z)$$

$$y(n) = 8w(n) - 4w(n-1) + 4w(n-2) - 2w(n-3).$$



2) Impulse Invariant method.

Sol^n: In this technique, the derived impulse response of the digital filter is obtained by uniformly sampling the impulse response of the equivalent analog filter, putting the $t = nT$.

$$h(n) = h_a(nT) \quad n = 0, 1, 2, 3 \ldots$$

take Laplace transform for $h(t)$

$$H(S) = \mathcal{L}[h(t)]$$

$$H(S) = \sum_{K=1}^{N} \frac{A_K}{S - P_K} \rightarrow (1)$$

$$H(S) = \frac{A_1}{S - P_1} + \frac{A_2}{S - P_2} + \ldots \frac{A_N}{S - P_N}$$

$A_K \rightarrow$ Gain factor for K^{th} pole

$S \rightarrow P_K$ is the K^{th} pole

Take the inverse L.T. for eqn (1)

$$h(t) = L^{-1}[H(S)]$$

$$= L\left[\sum_{K=1}^{N} \frac{A_K}{S-P_K}\right]$$

$$h(t) = \sum_{K=1}^{N} L^{-1} \frac{A_K}{S-P_K}$$

$$h(t) = \sum_{K=1}^{N} A_K e^{P_K t} \rightarrow (2)$$

The impulse response of the digital filter is obtained by uniformly sampling the impulse response of the analog filter.

i.e., $h(n) = h(t)\Big|_{t=nT}$

$$= \sum_{K=1}^{N} A_K e^{P_K t}$$

$$h(n) = \sum_{K=1}^{N} A_K P_K nT \quad -(3).$$

eq^n (3) can be obtained by taking the z-transform

i.e., $H(z) = Z[h(n)]$

$$= \sum_{n=-\infty}^{\infty} h(n) z^{-n}$$

$$= \sum_{n=0}^{\infty} \sum_{K=1}^{N} A_K e^{P_K n T} z^{-n}$$

$$= \sum_{K=1}^{N} \sum_{n=0}^{\infty} \left[A_K \left[e^{P_K T} z^{-1} \right]^n \right]$$

$$= \sum_{K=1}^{N} A_K \sum_{n=0}^{\infty} \left[e^{P_K T} z^{-1} \right]^n$$

A/C. to STD. Summation i.e.,

$$\sum_{n=0}^{\infty} a^n = \frac{1}{1-a}$$

$$H(z) = \sum_{K=1}^{N} A_K \frac{1}{1 - e^{P_K T} z^{-1}} \rightarrow (4)$$

Comparing eq^n (1) & (3). i.e. $H(s)$ & $H(z)$

$$H(s) = \sum_{K=1}^{N} \frac{A_K}{s - P_K} \quad \text{and} \quad H(z) = \sum_{K=1}^{N} \frac{A_K}{1 - e^{P_K T} z^{-1}}.$$

we Say that $\frac{1}{S-P_K} \rightarrow \frac{1}{1-e^{P_K T} z^{-1}} \rightarrow$ (5)

where $T \rightarrow$ Sampling time period

3) problem by using impulse invariant

T.F $H(s) = \frac{1}{s^2+\sqrt{2}s+1}$. To find assume that 3dB. Cutt off frequency of 150KHZ and a Sampling frequency of 1.28KHZ.

$\Rightarrow$ Given. $f_c = 150KHZ$ $\Omega_c = 2\pi f_c = 300\pi$ rad/sec

Sampling period $T = \frac{1}{f_s} = 7.8125 \times 10^{-4}$ sec.

$$H(s) = H_2(s)\Big|_{s \rightarrow \frac{s}{\Omega_c}} = H_2(s)\Big|_{s \rightarrow \frac{s}{300\pi}}.$$

$$Ha(s) = \frac{1}{\left(\frac{s}{300\pi}\right)^2 + \sqrt{2}\left(\frac{s}{300\pi}\right)+1}$$

$$= \frac{888.26 \times 10^3}{s^2 + 1332.86 + 888.26 \times 10^3}$$

$$= \frac{888.26 \times 10^3}{(s+666.43)^2 + 666.43^2}$$

$$Ha(s) = \frac{888.26 \times 10^3}{666.43} \cdot \frac{666.43}{(s+666.43)^2 + (666.43)^2}.$$

$$= 133286 \frac{666.43}{(s+666.43)^2 + (666.42)^2}$$

$$H(z) = 1332.86 \frac{e^{-666.93T} \sin(666.43T) z^{-1}}{1 - 2e^{-666.43T} \cos(666.43T) z^{-1} + e^{-2(666.43)T} z^{-2}}$$

put $T = 7.8125 \times 10^{-4}$

$$H(z) = \frac{393.92z^{-1}}{1.10308z^{-1} + 0.353z^{-2}}$$

4. Explain Bilinear transformation derivation.

The Bilinear Transformation is used for transforming an analog filter to a digital filter. Bilinear transformation can be linked to the trapezoidal formula for numerical integration.

Consider the I^{st} order differential Eqn of an analog system.

$$\frac{dy(t)}{dt} = x(t) \rightarrow (1)$$

Integration on B.S.

$$\int_{(n-1)T}^{nT} \frac{dy(t)}{dt} = \int_{(n-1)T}^{nT} x(t)\,dt$$

$$\left[y(t)\right]_{(n-1)T}^{nT} = \int_{(n-1)T}^{nT} x(t)\,dt$$

$$\left[y(nT) - y(n-1)T\right] = \int_{(n-1)T}^{nT} x(t)\,dt$$

A/c. to trapezoidal rule

$$\int_a^b f(x)\,dx = \frac{b-a}{2}\left[f(a) + f(b)\right].$$

$$y(nT) - y(n-1)T = \frac{nT-(n-1)T}{2}\left[x(n-1) + x(nT)\right]$$

$$= \frac{nT - nT + T}{2}\left[x(nT) + x(n-1)T\right]$$

$$y(nT) - y(n-1)T = \frac{T}{2}\left[x(nT) + x(n-1)T\right]$$

But we know. $y(n) = y(t)\Big|_{t=nT}$

i.e $y(n) = y(nT)$

$$y(n) - y(n-1) = \frac{T}{2}\left[x(n) + x(n-1)\right]$$

z- Transform on both sides

$$Y(z) - z^{-1}Y(z) = \frac{T}{2}\left[X(z) + z^{-1}X(z)\right].$$

$$Y(z)\left[1-z^{-1}\right]=\frac{T}{2}\left[1+z^{-1}\right]X(z)$$

$$Y(z)\left[1-z^{-1}\right]\frac{2}{T}\cdot(1+z^{-1})=X(z)$$

$$\frac{2}{T}\,\frac{1-z^{-1}}{1+z^{-1}}\,Y(z)=X(z) \rightarrow (2)$$

$$\frac{d\,y(t)}{dt}=x(t)$$

take LT

$$s\,Y(s)=X(s) - (3)$$

Comparing (2) & (3)

$$s\,Y(s)=\frac{2}{T}\,\frac{1-z^{-1}}{1+z^{-1}} \rightarrow (4)$$

T – Sampling time period

The Relation b/w s & z is known as Bilinear Transformation.

$$s=\frac{2}{T}\,\frac{(1-z^{-1})}{(1+z^{-1})}$$

$$\frac{sT}{2}=\frac{1-z^{-1}}{1+z^{-1}}$$

$$\frac{ST}{2}(1+z^{-1}) = 1 - z^{-1}$$

$$\frac{ST}{2}\left(1 + \frac{1}{z}\right) = \frac{z-1}{z}$$

$$\frac{ST}{2}z + \frac{ST}{2} = z - 1$$

$$1 + \frac{ST}{2} = z - \frac{ST}{2}z$$

$$= z\left(1 - \frac{ST}{2}\right)$$

$$z = \frac{1 + \frac{ST}{2}}{1 - \frac{ST}{2}}$$

Relation b/w. analog. frequency and digital frequency.

$$S = \frac{2}{T}\,\frac{1-z^{-1}}{1+z^{-1}}$$

but $S = j\Omega$ and $z = e^{j\omega}$

$$j\Omega = \frac{2}{T}\,\frac{1-e^{-j\omega}}{1+e^{-j\omega}}$$

$$j\Omega = \frac{2}{T} \frac{e^{-j\frac{\omega}{2}}\left[e^{j\omega/2} - e^{-j\omega/2}\right]}{e^{-j\frac{\omega}{2}}\left[e^{j\omega/2} + e^{-j\omega/2}\right]}$$

$$j\Omega = \frac{2}{T} \frac{\frac{2j}{2}\left[\frac{e^{j\omega/2} - e^{-j\omega/2}}{2j}\right]}{\left[\frac{e^{j\omega/2} + e^{-j\omega/2}}{2}\right]}$$

$$\Omega = \frac{2}{T} \frac{\cancel{2} \sin\frac{\omega}{2}}{\cancel{2} \cos \omega/2}$$

$$\boxed{\Omega = \frac{2}{T} \tan \omega/2}$$

Analog frequency warping digital frequency.

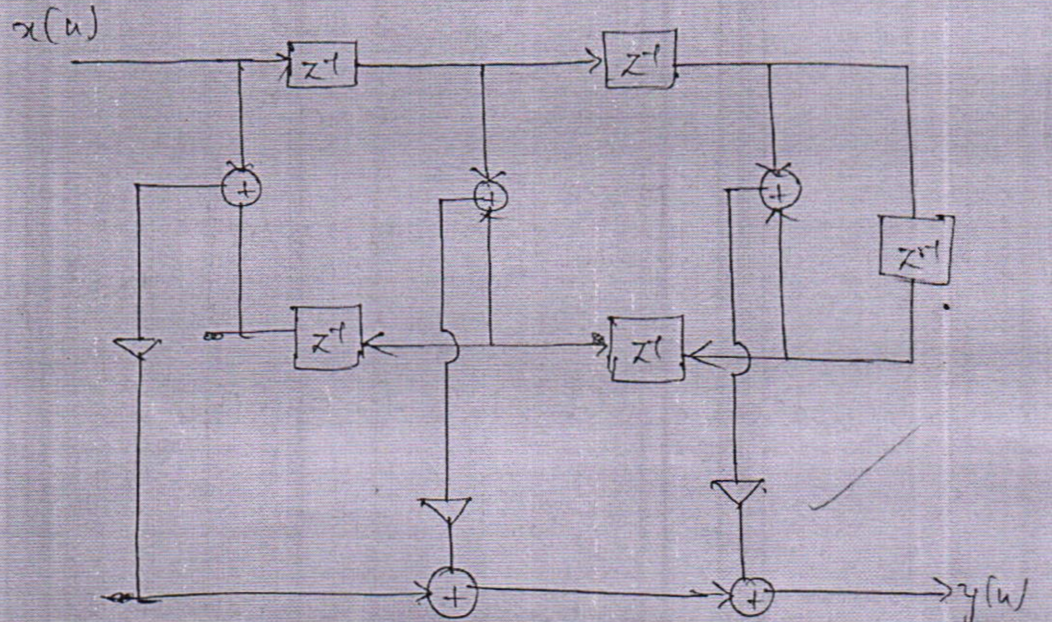
$$\omega = 2 \tan^{-1}\left(\frac{\Omega T}{2}\right)$$

for Small value of $\omega = \Omega T$.

5) Realize the Linear FIR filter having the following impulse response $h(n) = \delta(n) - \frac{1}{4}\delta(n-1) + \frac{1}{2}\delta(n-2) + \frac{1}{2}\delta(n-3) - \frac{1}{4}\delta(n-4) + \delta(n-5)$.

$$\Rightarrow \quad H(z) = 1 + \frac{1}{4}z^{-1} + \frac{1}{2}z^{-2} + \frac{1}{2}z^{-3} - \frac{1}{4}z^{-4} + 1.z^{-5}$$

$$\frac{Y(z)}{X(z)} = 1\left[1 + z^{-5}\right] - \frac{1}{4}\left[z^{-1} + z^{-4}\right] + \frac{1}{2}\left[z^{-2} + z^{-3}\right]$$



FIR filter

$$H(e^{j\omega}) = e^{j\omega\left(\frac{N-1}{2}\right)} \sum_{n=0}^{\frac{N}{2}-1} 2h(n)\cos\omega\left[n - \left(\frac{N-1}{2}\right)\right]$$

for even.

$$H(e^{j\omega}) = e^{-j\omega\left(\frac{N-1}{2}\right)} \left\{ h\left(\frac{N-1}{2}\right) + \sum_{n=0}^{\left(\frac{N-3}{2}\right)} h(n)\cos\left[\omega\left(n - \left(\frac{N-1}{2}\right)\right)\right]\right\}$$

for odd.