

C-E-R-T-I-F-I-C-A-T-E

This is to certify that Mr. / Ms. ...*Rayham.K*.....

with USN ..*15VISECO22*..... has satisfactorily completed the

course assignment in the subject of

R
Signature of the Student

AS
Staff in Charge

AS
Head of the Department

5/5

Nanda Sanyal

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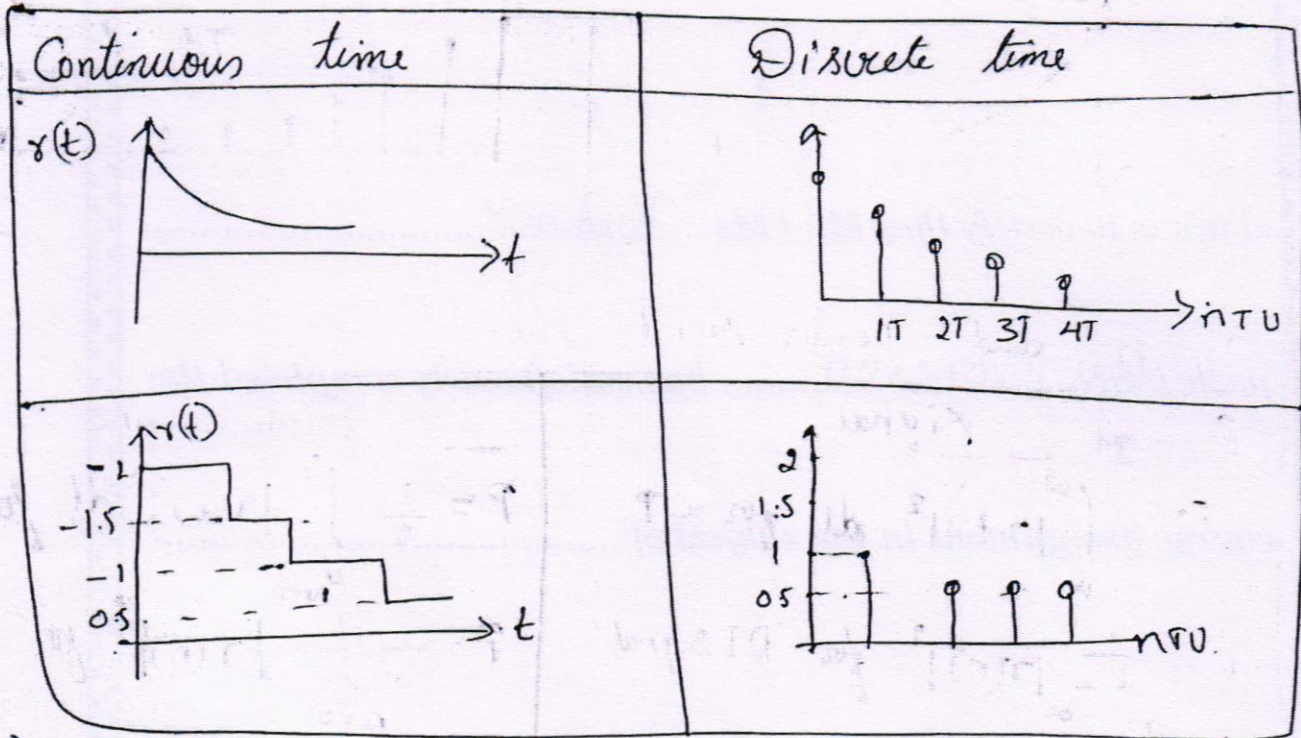
Assignment

Define Signal

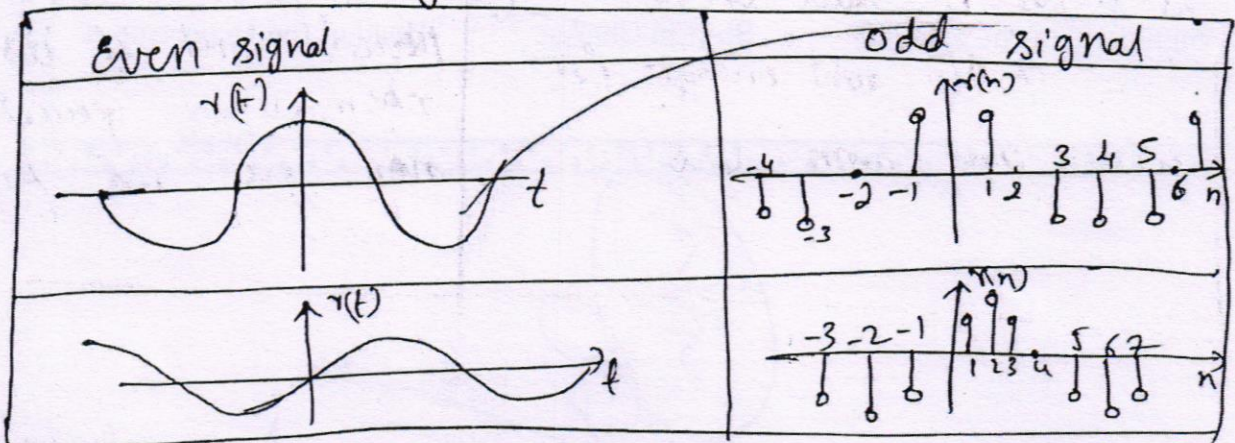
Signal is formally defined as a function of one or more variables that convey information of physical phenomena

with an example distinguish b/w the following

i) Continuous time & discrete time signal

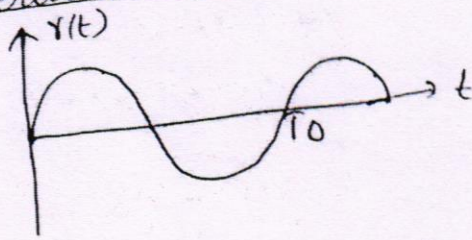


(ii) even and odd signal

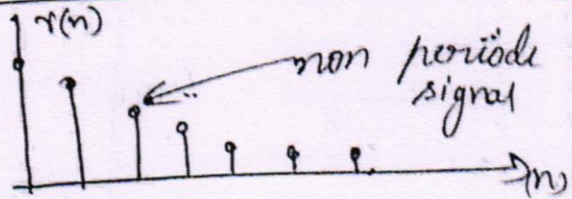
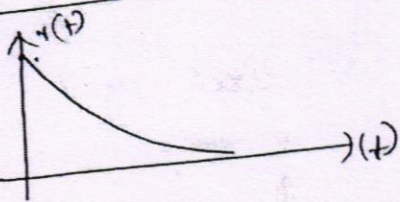
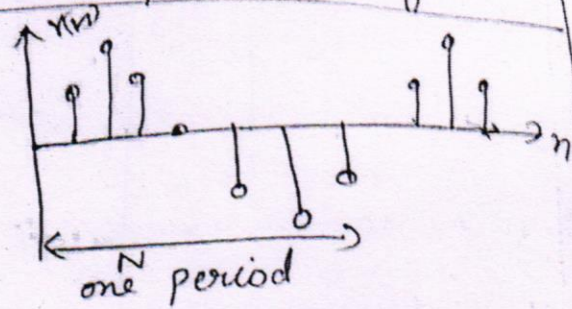


Periodic and non periodic sig

Periodic signal



non periodic signal



Energy and power signal

Energy signal

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt \text{ for CT}$$

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2 \text{ for DT signal}$$

A signal is said to be energy signal if its total energy is finite and non zero.

Power signal

$$P = \frac{1}{T} \int^T |x(t)|^2 dt \text{ for CT}$$

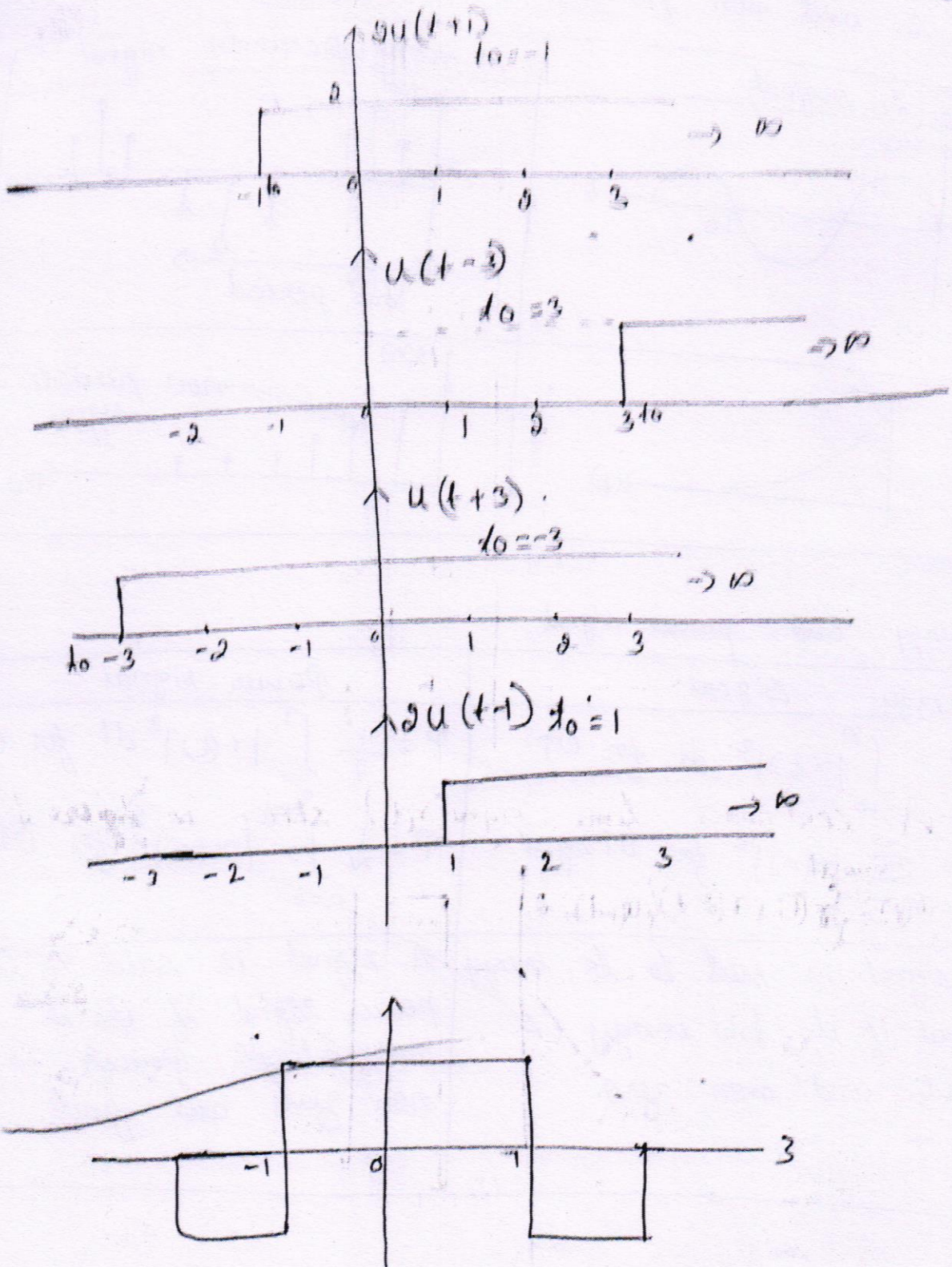
$$P = \frac{1}{N} \int_0^{N-1} |x(n)|^2 \text{ for DT}$$

A signal is said to be power signal if its normalized power is non zero and finite.

Nandhu Srinivasan

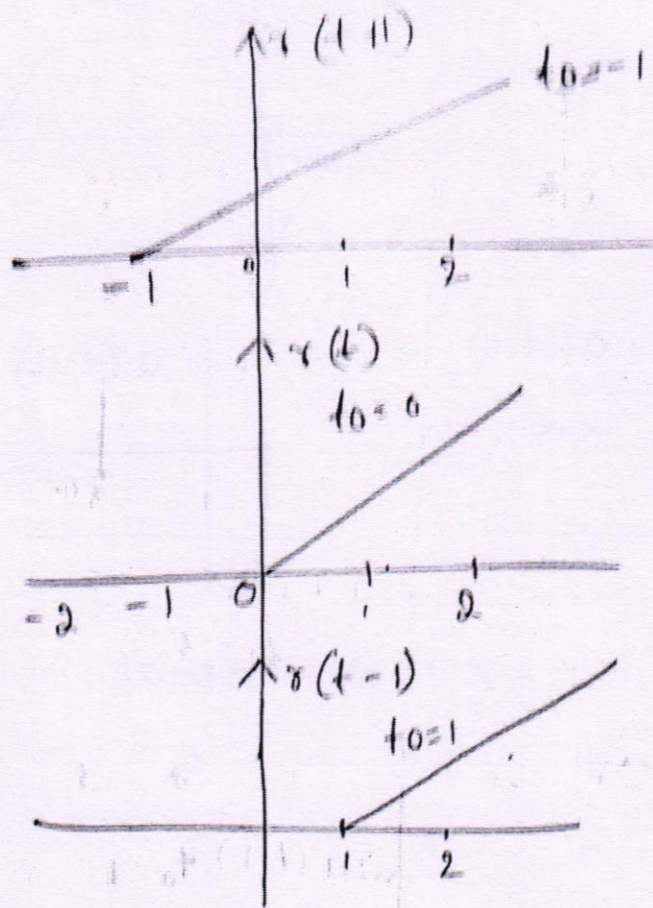
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Sketch the signal $x(t) = -u(t+3) + 2u(t+1) - 2u(t-1) + u(t-3)$



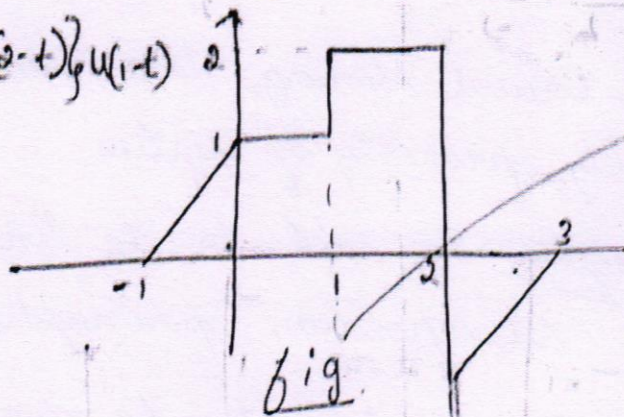
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8) Sketch the signal $y(t) = x(t+1) + x(t-1)$

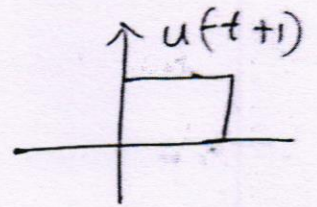
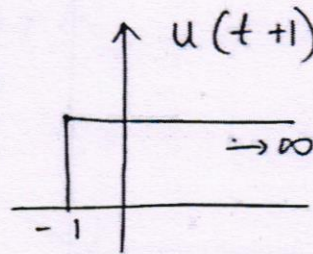
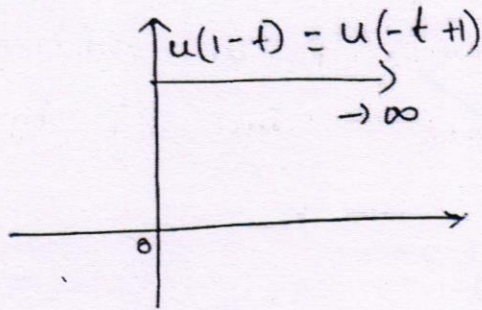
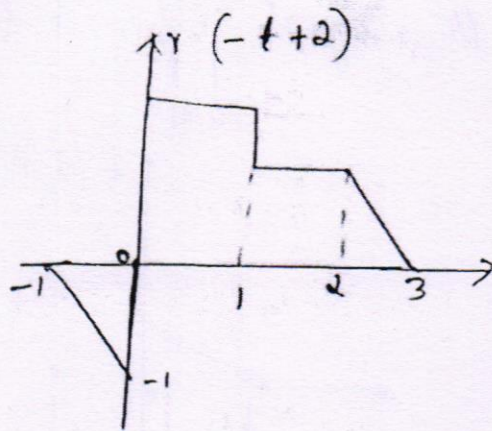
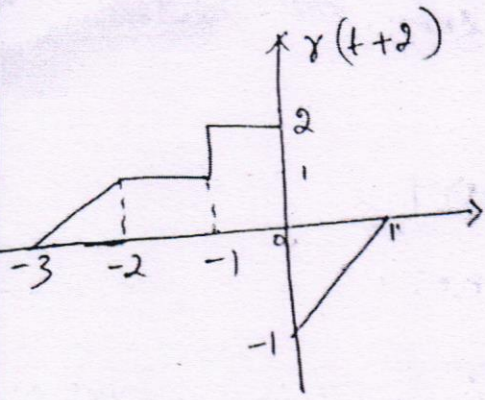


11) a) A continuous time signal $x(t)$ shown in figure 1. Draw the

signal
 $y(t) = \int x(t) + x(2-t) u(1-t)$



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	$x(t)$		$x(-t+2)$	result
-1 to 0	$x(t)$	+	$-x(t)$	0
0 to 1	1	+	2	3
1 to 2	2	+	1	3
2 to 3	$x(t)$	+	$-x(t)$	0

Explain linearity, Causal, Memory, time invariability, invertibility, stability, property of system

Linearity :-> A system is said to be linear if it satisfies the superposition principles

Causal :-> A system is said to be causal which represent value of o/p depends on present or past value of the i/p or q/t depends on both present & Past value of the input signal. If the o/p depends on

when even more future value then it is said to be causal

eg:- $y(n) = \frac{1}{3} (x(n) + x(n-1) + x(n-2)) \rightarrow$ Causal

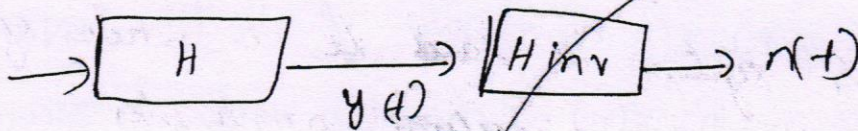
$y(n) = \frac{1}{3} (x(n) + x(n-1) + x(n+2)) \rightarrow$ non causal

Time invariance :- A system is said to be time invariance if a time delay or time advance of the i/p signal leads to the identical time shift in the o/p signal

i.e $x(t) \rightarrow y(t)$
 $x(t-m) \rightarrow y(t-m)$
 $x(t+m) \rightarrow y(t+m)$ } Time invariant system

$x(t-m) \rightarrow y(t)$
 $x(t+m) \rightarrow y(t)$ } Time variant

Invertibility :- The system is said to be invertible if the i/p of the system can be recover from the o/p



$H \cdot H_{inv} = I$

$x(t) = H_{inv} \{y(t)\}$

$x(t) = (H_{inv} H) \{x(t)\}$

Stability Property: An arbitrary system is said to be BIBO stable if and only if the bounded i/p produces a bounded o/p. Let $y(t)$ be the response to the bounded input refers to the finite value of $y(t)$ refers to the finite value of the i/p signal $x(t)$ for any value of t which is given by

$$|x(t)| \leq M_x < \infty$$

where M_x is a constant after computing magnitude of $x(t)$.

Memory: The continuous time system is said to be static or memory less instantaneous, if the o/p depends upon the present i/p only.

Determine whether the system is linear, time invariant, memoryless, causal, stable. $T\{x(n)\} = g(n) \cdot x(n)$

$$T\{x(n)\} = g(n) \cdot x(n)$$

$$y(n) = g(n) \cdot x(n)$$

$$y(n-n_0) = g(n) \cdot x(n-n_0) \rightarrow \text{o/p if it is time invariant}$$

$$T\{x(n)\} = g(n) \cdot x(n) \rightarrow \text{depends only present. The system is said to be memory less}$$

$$T\{x(n)\} = g(n) \cdot x(n) \rightarrow \text{depends only on present hence it is causal.}$$

$$T\{x(n)\} = g(n) \cdot x(n) = |g(n) \cdot x(n)| \leq M_x < \infty = |g(n)| \cdot \underbrace{|x(n)|}_{M_x < \infty}$$

The system is stable if $g(n)$ & $x(n)$ is stable

Determine & sketch the even (odd) signal shown in fig

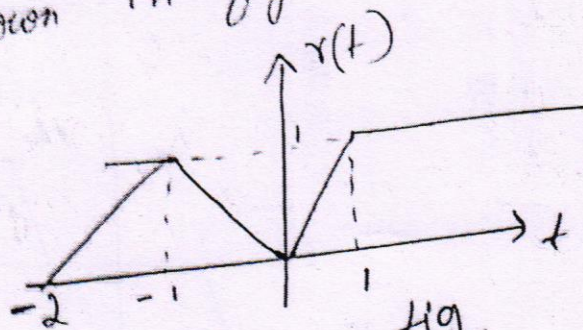
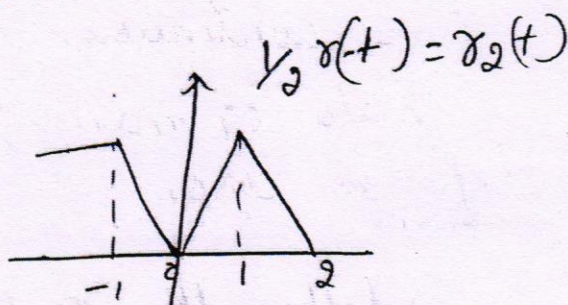
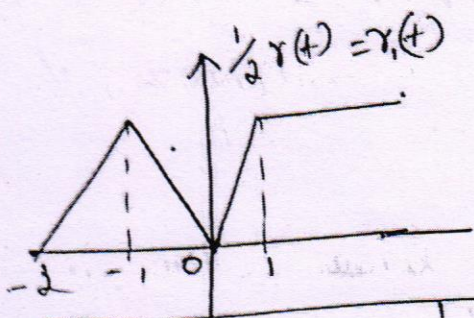


Fig.

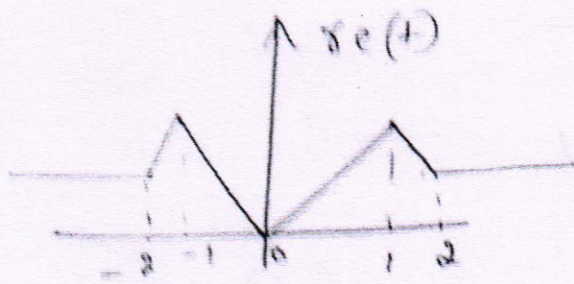
$$r_e(t) = \frac{1}{2} [r(t) + r(-t)] = \frac{1}{2} r(t) + \frac{1}{2} r(-t)$$

$$r_o(t) = \frac{1}{2} [r(t) - r(-t)] = \frac{1}{2} r(t) - \frac{1}{2} r(-t)$$

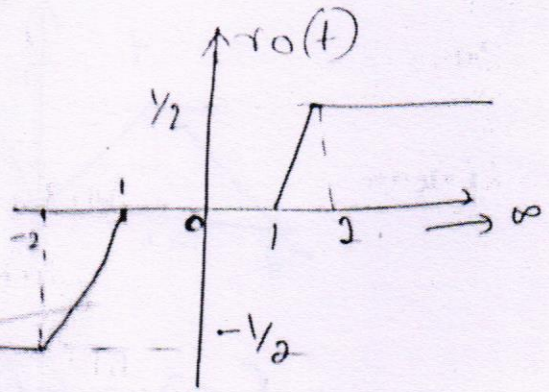


Time duration	$r_1(t)$	$r_2(t)$	$r_e(t)$	$r_o(t)$
$-\infty$ to -2	0	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$
-2 to -1	$r(t)$ ($0 \rightarrow \frac{1}{2}$)	$\frac{1}{2}$	$r(t) (\frac{1}{2} \rightarrow 1)$	$r(t) (-\frac{1}{2} \rightarrow 0)$
-1 to 0	$-r(t)$ ($\frac{1}{2} \rightarrow 0$)	$-r(t)$ ($\frac{1}{2} \rightarrow 0$)	$-r(t) (1 \rightarrow 0)$	0
0 to 1	$r(t)$ ($0 \rightarrow \frac{1}{2}$)	$r(t)$ $0 \rightarrow \frac{1}{2}$	$r(t) (0 \rightarrow 1)$	0
1 to 2	$\frac{1}{2}$	$-r(t)$	$-r(t) (1 \rightarrow \frac{1}{2})$	$r(t) (0 \rightarrow \frac{1}{2})$
2 to ∞	$\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2}$

Even function



odd function



Seen

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Assignment

Properties of discrete time Fourier Series

Linearity

$$x(n) \xrightarrow{\text{DTFS}} X(k)$$

$$y(n) \xrightarrow{\text{DTFS}} Y(k)$$

$$\underbrace{a x(n) + b y(n)}_{Z(n)} \xrightarrow{\text{DTFS}} \underbrace{a X(k) + b Y(k)}_{Z(k)}$$

$$X(k) = \frac{1}{N} \sum_{n=\langle N \rangle} x(n) e^{-jk\Omega_0 n}$$

$$Z(k) = \frac{1}{N} \sum_{n=\langle N \rangle} Z(n) e^{-jk\Omega_0 n}$$

$$Z(k) = \frac{1}{N} \left[a \sum_{n=\langle N \rangle} x(n) e^{-jk\Omega_0 n} + b \sum_{n=\langle N \rangle} y(n) e^{-jk\Omega_0 n} \right]$$

$$\boxed{Z(k) = a X(k) + b Y(k)}$$

LHS = R.H.S. Hence proved

Periodicity

If $x(n) \xrightarrow{\text{DTFS}} X(k)$ with period N then

$$\boxed{X(k+N) = X(k)}$$

Proof $X(k) = \frac{1}{N} \sum_{n=\langle N \rangle} r(n) e^{-jk\Omega_0 n}$

let k be replaced with $k+N$. Then we have

$$r(k+N) = \frac{1}{N} \sum_{n=\langle N \rangle} r(n) e^{-j(k+N)\Omega_0 n}$$

$$= \frac{1}{N} \sum_{n=\langle N \rangle} r(n) e^{-jk\Omega_0 n} \cdot e^{-jN\Omega_0 n}$$

Here $\Omega_0 = \frac{2\pi}{N}$ hence, $e^{-jN\Omega_0 n} = e^{-jN \frac{2\pi}{N} n} = e^{-j2\pi n} = 1$ (always)

$$\therefore X(k+N) = \frac{1}{N} \sum_{n=\langle N \rangle} r(n) e^{-jk\Omega_0 n} \cdot 1 = X(k)$$

$$\therefore \boxed{X(k+N) = X(k)}$$

3) Time shift

of $r(n) \xrightarrow{\text{DTFS}} X(k)$ then

$$\boxed{y(n) = r(n-n_0) \xrightarrow{\text{DTFS}} Y(k) = e^{-jk\Omega_0 n_0} X(k)}$$

Proof:- $Y(k) = \frac{1}{N} \sum_{n=\langle N \rangle} y(n) e^{-jk\Omega_0 n} = \frac{1}{N} \sum_{n=\langle N \rangle} r(n-n_0) e^{-jk\Omega_0 n}$

let $n-n_0=m$, then

$$Y(k) = \frac{1}{N} \sum_{m=\langle N \rangle} r(m) e^{-jk\Omega_0 (m+n_0)}$$

$$= e^{-jk\Omega_0 n_0} \frac{1}{N} \sum_{m=\langle N \rangle} r(m) e^{-jk\Omega_0 m}$$

$$\Rightarrow \boxed{Y(k) = e^{-jk\Omega_0 n_0} X(k)}$$

Proved

Frequency shift

of $x(n)$ (DTFS) $\rightarrow x(k)$ then,

$$y(n) = e^{jk_0 \Omega_0 n} x(n) \xrightarrow{\text{DTFS}} Y(k) = X(k - k_0)$$

Proof:

$$Y(k) = \frac{1}{N} \sum_{n=\langle N \rangle} y(n) e^{-jk \Omega_0 n} = \frac{1}{N} \sum_{n=\langle N \rangle} e^{jk_0 \Omega_0 n} x(n) e^{-jk \Omega_0 n}$$
$$= \frac{1}{N} \sum_{n=\langle N \rangle} x(n) e^{-j(k - k_0) \Omega_0 n}$$

$$\boxed{Y(k) = X(k - k_0)}$$

Convolution :-

of $x(n)$ (DTFS) $\rightarrow x(k)$ and $y(n)$ (DTFS) $\rightarrow y(k)$

then, $\boxed{z(n) = x(n) * y(n) \xrightarrow{\text{DTFS}} N x(k) y(k)}$

Proof:

$$Z(k) = \frac{1}{N} \sum_{n=\langle N \rangle} z(n) e^{-jk \Omega_0 n} = \frac{1}{N} \sum_{n=\langle N \rangle} [x(n) * y(n)] e^{-jk \Omega_0 n}$$

$$= \frac{1}{N} \sum_{n=\langle N \rangle} \left\{ \sum_{l=\langle N \rangle} x(l) y(n-l) \right\} e^{-jk \Omega_0 n}$$

$$= \frac{1}{N} \sum_{l=\langle N \rangle} x(l) \sum_{n=\langle N \rangle} y(n-l) e^{-jk \Omega_0 n}$$

Let $n-l=m$, we get

$$Z(k) = \frac{1}{N} \sum_{l=\langle N \rangle} x(l) \sum_{m=\langle N \rangle} y(m) e^{-jk \Omega_0 (m+l)}$$

$$= \frac{1}{N} \sum_{l=\langle N \rangle} \gamma(l) \sum_{m=\langle N \rangle} y(m) e^{-jk\Omega_0 m} e^{-jk\Omega_0 l}$$

$$z(k) = \frac{1}{N} \sum_{l=\langle N \rangle} \gamma(l) e^{-jk\Omega_0 l} \sum_{m=\langle N \rangle} y(m) e^{-jk\Omega_0 m}$$

$$z(k) = x(k) \cdot N y(k)$$

$$z(k) = N x(k) \cdot y(k)$$

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Assignment

State and prove linearity and Time shifting property of DTFS

Linearity: $x(n) \xrightarrow{\text{DTFS}} X(k)$ and $y(n) \xrightarrow{\text{DTFS}} Y(k)$

then, $Z(n) = ax(n) + by(n) \xrightarrow{\text{DTFS}} Z(k) = aX(k) + bY(k)$

Proof $Z(k) = \frac{1}{N} \sum_{n=\langle N \rangle} z(n) e^{-jk\Omega_0 n}$

$$= \frac{1}{N} \sum_{n=\langle N \rangle} [ax(n) + by(n)] e^{-jk\Omega_0 n}$$

$$= a \frac{1}{N} \sum_{n=\langle N \rangle} x(n) e^{-jk\Omega_0 n} + b \frac{1}{N} \sum_{n=\langle N \rangle} y(n) e^{-jk\Omega_0 n}$$

$$Z(k) = aX(k) + bY(k)$$

Time Shift:

$x(n) \xrightarrow{\text{DTFS}} X(k)$ then

$$y(n) = x(n-n_0) \xrightarrow{\text{DTFS}} Y(k) = e^{-jk\Omega_0 n_0} X(k)$$

Proof: $Y(k) = \frac{1}{N} \sum_{n=\langle N \rangle} y(n) e^{jk\Omega_0 n}$

$$= \frac{1}{N} \sum_{n=\langle N \rangle} x(n-n_0) e^{-jk\Omega_0 n}$$

Let $n-n_0 = m$, then

$$Y(k) = \frac{1}{N} \sum_{m=\langle N \rangle} x(m) e^{-jk\Omega_0 (m+n_0)}$$

$$= e^{jk\Omega_0 n_0} \frac{1}{N} \sum_{m=\langle N \rangle} x(m) e^{-jk\Omega_0 m}$$

$$Y(k) = \frac{e^{jk\Omega_0 n_0} x(k)}{N}$$

Consider the signal $x(n) = 2 + 2 \cos \frac{\pi}{4} n + \cos \frac{\pi}{2} n + \frac{1}{2} \cos \frac{3\pi}{4} n$

Using NTFB i) Determine & sketch its power density spectrum

ii) Evaluate the power of the signal

$$\left. \begin{aligned} \Omega_{01} &= \frac{\pi}{4} \\ \Omega_{02} &= \frac{\pi}{2} \\ \Omega_{03} &= \frac{3\pi}{4} \end{aligned} \right\} \Omega_0 = \text{gcd} \{ \Omega_{01}, \Omega_{02}, \Omega_{03} \}$$

$$= \text{gcd} \{ \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4} \}$$

$$= \text{gcd} \{ \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4} \}$$

$$\Rightarrow \frac{\pi}{4} = \Omega_0$$

$$\text{where } \Omega_0 = \frac{2\pi}{N}$$

$$\therefore \frac{\pi}{4} = \frac{2\pi}{N}$$

$$N = 8$$

$$x(n) = 2e^{j0} + 2 \left[\frac{e^{j(0)\frac{\pi}{4}n} + e^{j(1)\frac{\pi}{4}n}}{2} \right] + \frac{e^{j(2)\frac{\pi}{4}n} + e^{j(-2)\frac{\pi}{4}n}}{2}$$

$$+ \frac{1}{2} \left[\frac{e^{j(3)\frac{\pi}{4}n} + e^{j(-3)\frac{\pi}{4}n}}{2} \right]$$

$$x(n) = 2e^{j0} + e^{j\frac{\pi}{4}n} + e^{-j\frac{\pi}{4}n} + \frac{1}{2} e^{j\frac{\pi}{2}n} + \frac{1}{2} e^{-j\frac{\pi}{2}n} + \frac{1}{4} e^{j\frac{3\pi}{4}n} + \frac{1}{4} e^{-j\frac{3\pi}{4}n}$$

$$n(n) = \sum_{k \in \mathbb{Z}(N)} x(k) e^{j k \omega_0 n}$$

Comparing with the standard equation

$$x(0) = 2$$

$$x(1) = 1$$

$$x(-1) = 1$$

$$x(2) = 1/2$$

$$x(-2) = 1/2$$

$$x(3) = 1/4$$

$$x(-3) = 1/4$$

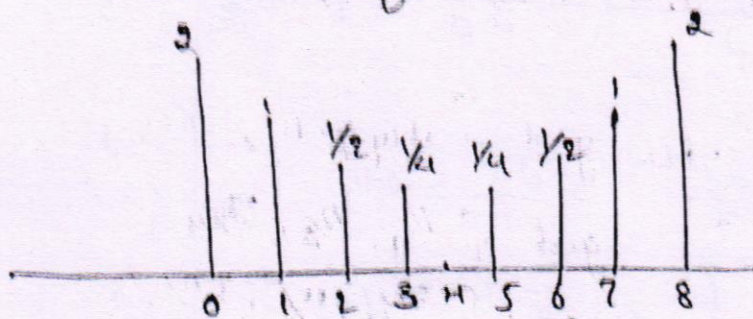
$$x(4) = 1/2, x(5) = 1/4$$

$$x(8) = 2$$

$$x(10) = 1/2, x(11) = 1/4$$

$$x(9) = 1$$

Take an interval from 0 to 7 & draw the magnitude



Total power can be obtained by parallel theorem

$$P = \frac{1}{N} \sum_{n \in \mathbb{Z}(N)} |x(n)|^2$$

$$= \sum_{k \in \mathbb{Z}(N)} |x(k)|^2$$

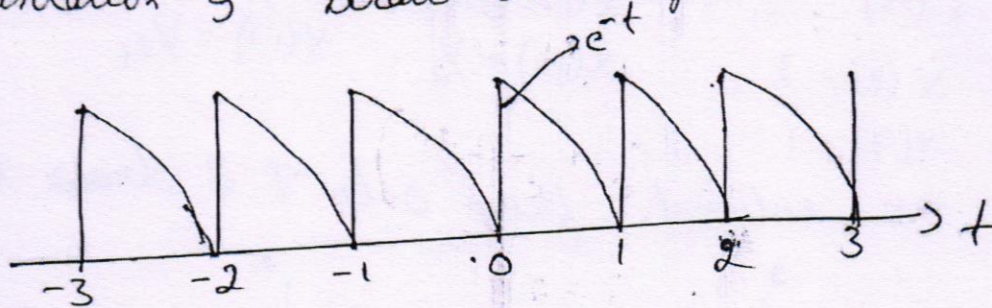
$$P = \sum_{k=0}^7 |x(k)|^2$$

$$P = (x(0))^2 + (x(1))^2 + (x(2))^2 + (x(3))^2 + (x(4))^2 + (x(5))^2 + (x(6))^2 + (x(7))^2$$

$$P = 4 + 1 + \frac{1}{4} + \frac{1}{16} + 0 + \frac{1}{16} + \frac{1}{4} + 1$$

$$P = \underline{\underline{6.625 \text{ Watts}}}$$

For the signal $x(t)$ shown in fig find the fourier series representation & draw its magnitude and phase spectra



$$T = 1, \omega_0 = \frac{2\pi}{T} = \frac{2\pi}{1} = 2\pi$$

$$a(k) = \frac{1}{T} \int_{<T>} x(t) e^{-jk\omega_0 t} dt$$

$$x(k) = \frac{1}{1} \int_0^1 e^{-t} e^{-jk\omega_0 t} dt$$

$$x(k) = \int_0^1 \frac{e^{-(1+jk\omega_0)t}}{-(1+jk\omega_0)} dt$$

$$= \frac{e^{-(1+jk\omega_0)t}}{-(1+jk\omega_0)} \Big|_0^1$$

$$= \frac{e^{-(1+jk\omega_0)} - 1}{-(1+jk\omega_0)} + \frac{1}{(1+jk\omega_0)}$$

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$$x(k) = \frac{1 - e^{-j k \omega_0}}{1 + j k \omega_0}$$

$$= \frac{1 - e^{-j k 2\pi}}{1 + j k 2\pi}$$

$$= \frac{1 - [e^{-1} e^{-j k 2\pi}]}{1 + j k 2\pi}$$

$$x(k) = \frac{1 - e^{-1}}{1 + j k 2\pi}$$

$$k = 0, 1, 2, 3, 4, -1, -2, -3, -4$$

$$x(0) = \frac{1 - e^{-1}}{1 + 0} = \frac{1 - e^{-1}}{1} = 0.6321$$

$$x(k) = a + jb = \frac{(1 - e^{-1})(1 - j k 2\pi)}{1 - j 2k^2 2j\pi^2}$$

$$= \frac{(1 - e^{-1})(1 - j k 2\pi)}{1 + k^2 4\pi^2}$$

$$= \frac{0.6321 (1 - j k 2\pi)}{1 + k^2 4\pi^2}$$

$$x(k) = \frac{0.6321 - j 3.9018}{1 + 4\pi^2}$$

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$$x(1) = \frac{0.6321 - j3.4018}{1 + 4\pi^2}$$

$$x(1) = 0.0156 - j0.0963$$

$$|x(1)| = \sqrt{0.0156^2 + 0.0963^2}$$

$$|x(1)| = 0.1$$

$$\alpha(1) = \tan^{-1} b/a$$

$$= \tan^{-1} \left(\frac{-0.0963}{0.0156} \right)$$

$$= -1.41$$

$$x(2) = \frac{0.6321 - j7.8036}{1 + 16\pi^2} = 3.97 \times 10^{-3} - j0.0491$$

$$|x(2)| = \sqrt{0.0039^2 + 0.0491^2}$$

$$|x(2)| = 0.05$$

$$\alpha(2) = \tan^{-1}(b/a)$$

$$= \tan^{-1} \left(\frac{-0.0491}{0.0039} \right)$$

$$\alpha(2) = -1.49$$

Find the fourier series coefficient of the signal $x(t)$ in the figure and draw the spectra

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When k is even

$$k = \pm 2, \pm 4, \pm 6$$

$$X(k) = \frac{1 - \cos k\pi}{k^2 \pi^2}$$

$$X(k) = 0$$

When k is odd

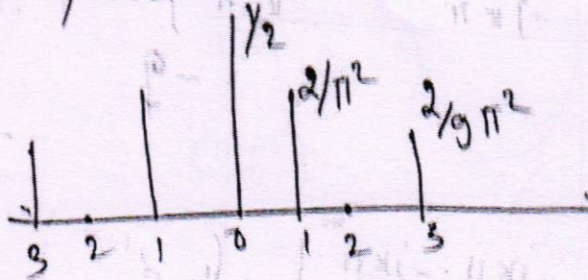
$$k = \pm 1, \pm 3, \pm 5$$

$$X(k) = \left(\frac{1 - \cos k\pi}{k^2 \pi^2} \right) \Rightarrow X(1) = \left(\frac{1 - \cos \pi}{\pi^2} \right)$$

$$X(1) = \frac{2}{\pi^2}$$

$$X(3) = \frac{2}{9\pi^2}$$

The spectra is as shown in the below figure



1) Find the DTFT of the signal, $x(n) = \alpha^n u(n)$, $|\alpha| < 1$ Draw its magnitude spectrum

2) Proof: we have the DTFT

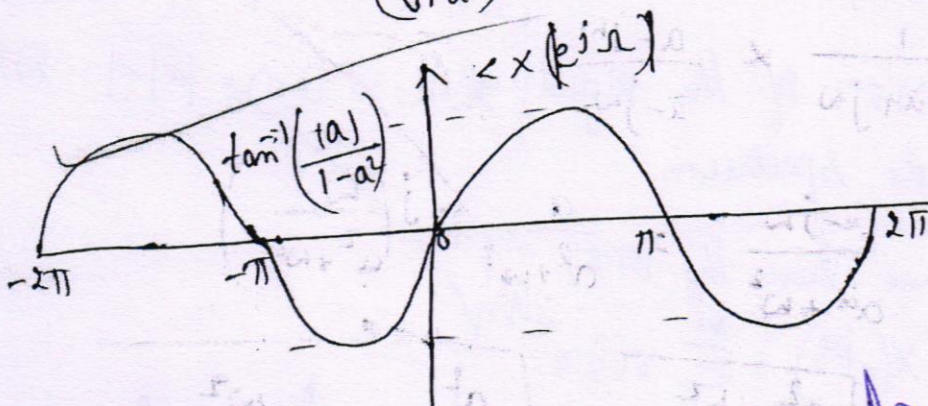
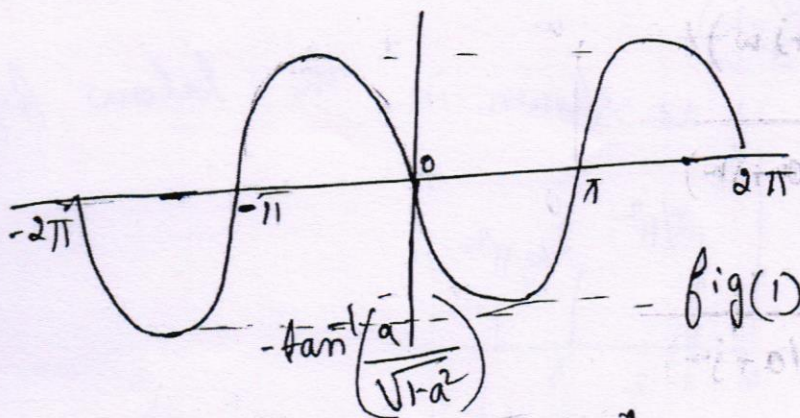
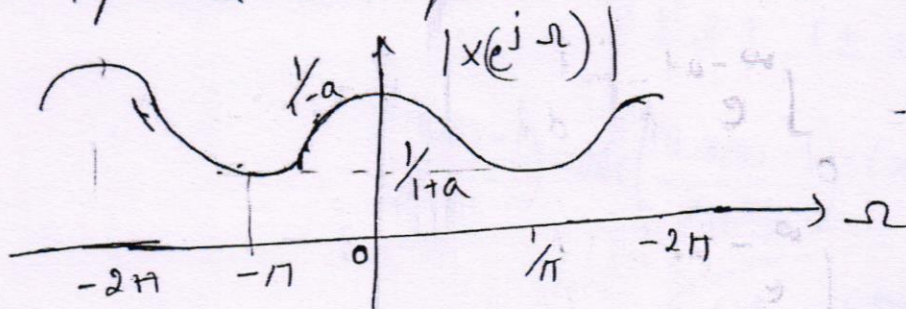
$$X(e^{j\omega}) = \sum_{n=0}^{\infty} \alpha^n e^{-j\omega n}$$
$$= \sum_{n=0}^{\infty} \alpha^n e^{-j\omega n}$$

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$$= \sum_{n=0}^{\infty} (\alpha e^{-j\Omega} e^{jn})^n$$

$$x(e^{j\Omega}) = \frac{1}{1 - \alpha e^{j\Omega}}$$

The magnitude and phase of $x(e^{j\Omega})$ is shown in the figure 1 and fig 2 $a > 0$ and $a < 0$ respectively. These spectra are periodic with period 2π



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6) Obtain the Fourier transform of $x(t) = e^{-at} u(t)$; also draw its magnitude and phase spectrum

$$X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{j\omega t} dt$$

$$= \int_{-\infty}^{\infty} e^{-at} u(t) e^{j\omega t} dt$$

$$= \int_0^{\infty} e^{-at} e^{j\omega t} dt$$

$$= \int_0^{\infty} e^{-(a+j\omega)t} dt$$

$$= \frac{e^{-(a+j\omega)t}}{-(a+j\omega)} \Big|_0^{\infty}$$

$$= 0 + \frac{1}{(a+j\omega)}$$

$$= \frac{1}{a+j\omega} \times \frac{a-j\omega}{a-j\omega}$$

$$= \frac{a-j\omega}{a^2+\omega^2} = \frac{a}{a^2+\omega^2} - j \frac{\omega}{a^2+\omega^2}$$

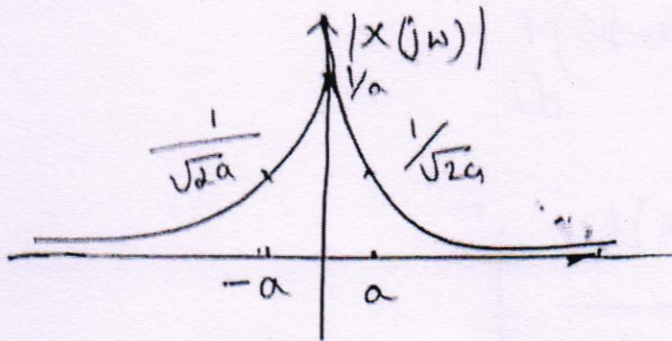
$$|X(j\omega)| = \sqrt{a^2 + \omega^2} = \sqrt{\frac{a^2}{(a^2+\omega^2)^2} + \frac{\omega^2}{(a^2+\omega^2)^2}}$$

$$\sqrt{\frac{a^2 + \omega^2}{(a^2 + \omega^2)^2}}$$

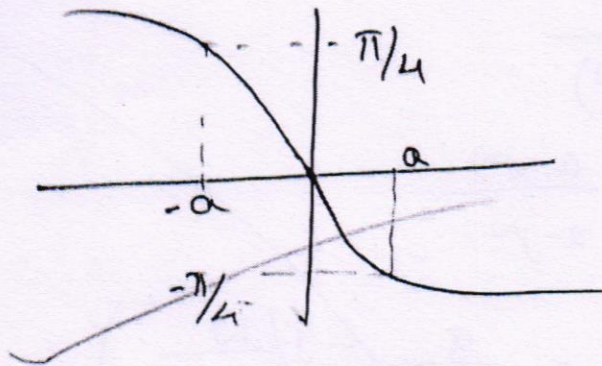
$$|X(j\omega)| = \frac{1}{\sqrt{a^2 + \omega^2}}$$

$$\angle X(j\omega) = \tan^{-1}(b/a) = \tan^{-1}\left(\frac{-\omega}{a^2 + b^2}\right)$$

$$\angle X(j\omega) = \tan^{-1} \frac{-\omega}{a}$$



→ magnitude



→ phase

[Signature]

N. Srinivasan

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