6

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SHRIDEVI INSTITUTE OF ENGINEERING AND TECHNOLOGY

(Recognised by Govt. of Karnataka, Affiliated to VTU, Belagavi and Approved by AICTE, New Delhi)

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SHRIDEVI
EDUCATION

BLUE BOOK

USN : | 1 | S | V | 1 | 9 | E | E | 0 | 0 | 7 |

Name of the Student : Manu. S. N.

Course : Digital Signal processing Code : 18EE63

Semester : 6th Branch : EEE

ACY - 2021 - 22

INTERNAL ASSESSMENT MARKS

Date	Test No.	Max. Marks	Marks Obtained	Course Instructor Signature
21/5/22	01	30	21	G. H Rama
28/6/22	02	30	19	G. H Rama
15/7/22	03	30	18	G. H Rama
	Average		19 / 10	G. H Rama

29/40 G. H Rama

CERTIFICATE

This is to certify that Kum / Sri Manu. S. N

with USN 1SV19EE007 has satisfactorily completed the Internal

Assessment tests in the subject Digital Signal processing

with Subject Code 18EE63 as prescribed by the

Visvesvaraya Technological University for the 6th semester

B.E. / M.Tech / MBA degree course in the year 2021-2022

G. H Rama
Course Instructor

G. H Rama
Head of the Department

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SIET., TUMAKURU.

TEST NO. 1

Q.No.	a	b	c	Total
Q1				
Q2				
Q3	6	10		16
Q4	10	2		12
Test - 1 Marks				28

TEST NO. 2

Q.No.	a	b	c	Total
Q1	10	8		18
Q2				
Q3	7			07
Q4				
Test - 2 Marks				25

TEST NO. 3

Q.No.	a	b	c	Total
Q1				
Q2	8			08
Q3	10	6		16
Q4				
Test - 3 Marks				24

REMARKS

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Internal Assessment -1

Digital Signal processing

3a) $x(n) = \{1, 1, 0, 0\}$.

The DFT of the Sequence is,

$x(n) = DFT[X(k)]$

$$X(K) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N}kn}$$

$N = 4$,

$$x(k) = \sum_{n=0}^{4-1=3} x(n) e^{-j\frac{2\pi}{4}kn}$$

$$x(k) = \sum_{n=0}^{3} x(n) e^{-j\frac{\pi}{2}kn}$$

$$x(k) = x(0)e^{-j\frac{\pi}{2}(0)k} + x(1)e^{-j\frac{\pi}{2}(1)k} + x(2)e^{-j\frac{\pi}{2}(2)k} + x(3)e^{-j\frac{\pi}{2}(3)k}$$

$$x(K) = 1 \cdot e^0 + 1 \cdot e^{-j\frac{\pi}{2}k}$$

$$x(K) = 1 + e^{-j\frac{\pi}{2}k} \rightarrow (A)$$

put,

$K = 0$

$$x(0) = 1 + e^{-j\frac{\pi}{2}(0)}$$

$$x(0) = 1 + e^0$$

$$x(0) = 1 + 1$$

$$x(0) = 2$$

put $k=1$

$$x(1) = 1 + e^{-j\frac{\pi}{2}(1)}$$

$$x(1) = 1 + (\overset{0}{\cancel{\cos}\,90} - j\,\overset{1}{\cancel{\sin}\,90})$$

$$x(1) = 1 - j$$

put $k=2$

$$x(2) = 1 + e^{-j\frac{\pi}{2}(2)}$$

$$x(2) = 1 + (\overset{-1}{\cancel{\cos}\,180} - j\,\overset{0}{\cancel{\sin}\,180})$$

$$x(2) = 1 - 1$$

$$x(2) = 0$$

3×180

$\frac{540}{2}$

270

put $k=3$

$$x(3) = 1 + e^{-j\frac{\pi}{2}(3)}$$

$$x(3) = 1 + (\cos 270 - j\sin 270)$$

$$x(3) = 1 + j$$

$$x(k) = \{\underset{x(0)}{2}, \underset{x(1)}{1-j}, \underset{\underset{\uparrow}{x(2)}}{0}, \underset{x(3)}{1+j}\}$$

~~$y(k)$ =~~ Midpoint. $\frac{N}{2} = \frac{4}{2} = 2$

$$Y(k) = \{\underset{Y(0)}{2}, \underset{Y(1)}{1+j}, \underset{Y(2)}{0}, \underset{Y(3)}{1-j}\}$$

IDFT,

$$y(n) = \frac{1}{N}\sum_{k=0}^{N-1} Y(k)\, e^{j\frac{2\pi}{N}kn}$$

$N=4.$

$$y(n) = \frac{1}{4}\sum_{k=0}^{4-1=3} Y(k)\, e^{j\frac{2\pi}{4}kn}$$

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$$y(n) = \frac{1}{4} \sum_{k=0}^{3} Y(k)\, e^{j\frac{\pi}{2}kn}$$

$$y(n) = \frac{1}{4}\left\{ \overset{2}{Y(0)}\, e^{0} + \overset{1+j}{Y(1)}\, e^{j\frac{\pi}{2}(1)n} + \overset{0}{Y(2)}\, e^{j\frac{\pi}{2}(2)n} + \overset{1-j}{Y(3)}\, e^{j\frac{\pi}{2}(3)n} \right\}$$

$$y(n) = \frac{1}{4}\left\{ 2 + (1+j)\, e^{j\frac{\pi}{2}n} + (1-j)\, e^{j\frac{3\pi}{2}n} \right\}$$

put,

$n = 0$

$$y(0) = \frac{1}{4}\left\{ 2 + (1+j)\, e^{j\frac{\pi}{2}(0)} + (1-j)\, e^{0} \right\}$$

$$y(0) = \frac{1}{4}\left\{ 2 + (1+j)(1) + (1-j)(1) \right\}$$

$$y(0) = \frac{1}{4}\left\{ 2 + 1 + \cancel{j} + 1 - \cancel{j} \right\}$$

$$y(0) = \frac{4}{4} = 1$$

$$y(0) = 1$$

put $n = 1$

$$y(1) = \frac{1}{4}\left\{ 2 + (1+j)\, e^{j\frac{\pi}{2}(1)} + (1-j)\, e^{j\frac{3\pi}{2}(1)} \right\}$$

$$y(1) = \frac{1}{4}\left\{ 2 + (1+j)(\cos 90 + j\sin 90) + (1-j)(\cos 270 + j\sin 270) \right\}$$

$$y(1) = \frac{1}{4}\left\{ 2 + (1+j)(0 + j1) + (1-j)(0 + j(-1)) \right\}$$

$$y(1) = \frac{1}{4}\left(2 + 2 + \cancel{j} + j^2 + -\cancel{j} + j^2 \right)$$

$$= \frac{1}{4}\left(3 \; (-1) + (-1) \right.$$

$$= \tfrac{1}{4},0$$

put $n=2$,

$$y(2) = \frac{1}{4}\left\{2 + (1+j)e^{j\frac{\pi}{2}(2)} + (1-j)e^{j\frac{\pi}{2}(2)}\right\}$$

$$y(2) = \frac{1}{4}\left\{2 + (1+j)e^{j\pi} + (1-j)e^{j\pi}\right\}$$

$$y(2) = \frac{1}{4}\left\{2 + (1+j)(\cos 180 + j\sin 180) + (1-j)(\cos 180 + j\sin 180)\right\}$$

$$\cancel{y(2) = \frac{1}{4}\left\{2 + (1+j)(\right.}$$

$$y(2) = 1$$

put, $n=3$,

$$y(3) = \frac{1}{4}\left\{2 + (1+j)e^{j\frac{\pi}{2}(3)} + (1-j)e^{j\frac{\pi}{2}(3)}\right\}$$

$$y(3) = \frac{1}{4}\left\{2 + (1+j)(\cos 270 + j\sin 270) + (1-j)(\cos 270 + j\sin 270)\right\}$$

$$y(3) = \frac{1}{4}\left\{2 + (1+j)(-j) + (1-j)(-j)\right\}$$

$$y(3) = \frac{1}{4}\left\{2 - j - j^2 - j + j^2\right\}$$

$$y(n) = \{1, \quad = \cancel{\tfrac{1}{4}\{2 - 2j + 1 - 1\}}$$

$$= 0$$

$$y(n) = \{1, 0, 1, 0\}$$

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3 b) $x(n) = \left(\frac{1}{2}\right)^n \{u(n) - u(n-4)]$

$N = 4$.

i) If. $G(K) = W_4^{2K} X(K)$

By using the time Shift method

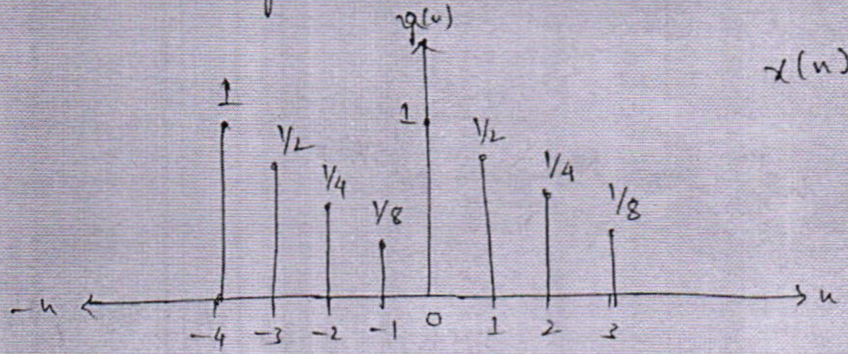
$$x((n-m))_N = W_N^{mK} X(K)$$

$\left(\frac{1}{2}\right)$

$$x((n-2))_N = W_4^{2K} X(K)$$

$$g(n) = x((n-2))_N \quad -(1)$$

$x(n) = \{1, 1/2, 1/4, 1/8\}$



for put $n = 0$,

$$g(0) = x((0-2))_4$$
$$g(0) = x((-2))_4$$
$$g(0) = 1/4.$$

put $n = 1$

$$g(1) = x((1-2))_4$$
$$g(1) = x((-1))_4$$
$$g(1) = 1/8$$

put $n = 2$

$$g(2) = x((2-2))_4$$
$$g(2) = x((0))_4$$
$$g(2) = 1,$$

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put, $n=3$

$$g(3) = x((3-2))_4$$

$$g(3) = x((1))_4$$

$$g(3) = 1/2$$

$$g(n) = \{ 1/4 ,\ 1/8 ,\ 1 ,\ 1/2 \}$$

ii). $\sum_{k=0}^{3} x(k)\, x^*(k)$

By using parseval's Theorem:

$$\sum_{k=0}^{N-1} x(n)\, W_N^{kn} = N \sum_{k=0}^{N-1} |X(k)|^2$$

$x(n) = \{ 1,\ 1/2,\ 1/4,\ 1/8 \}$.

$$= 4 \sum_{k=0}^{4-1=3} |x(k)|^2$$

$$= 4 \left[x(0) + x(1) + x(2) + x(3) \right]$$

$$= 4 \left[1 + (1/2)^2 + (1/4)^2 + (1/8)^2 \right]$$

$$= 4 \left[1 + \frac{1}{4} + \frac{1}{16} + \frac{1}{64} \right]$$

8	64	16	4
2	8	2	4
2	4	1	2
2	2	1	1

$$= 4 \left[\frac{64 + 16 + 4 + 1}{64} \right]$$

$$= \cancel{4} \left[\frac{85}{\cancel{64}_{16}} \right] = \frac{85}{16}$$

iii) $X(0) + X(2)$

By putting Value of $k = 0$, and 2.

$$x(n) = \sum_{n=0}^{N-1} X(k)\, e^{-j\frac{2\pi}{N}kn}$$

$$x(n) = \sum_{n=0}^{4-1=3} X(k)\, e^{-j\frac{2\pi}{4}(0)}$$

$$x(n) = \sum_{n=0}^{3} x(k)\, e^{0}$$

$$x(n) = x(0) + x(1) + x(2) + x(3)$$

$$= 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8}$$

$$X(0) = \frac{8+4+2+1}{8} = \frac{15}{8}$$

$k = 2$

$$x(n) = \sum_{n=0}^{N-1} X(k)\, e^{-j\frac{2\pi(2)n}{4}}$$

$$x(n) = \sum_{n=0}^{N-1} X(k)\, e^{-j\pi n}$$

$$= \left[x(0)\, e^{0} + x(1)\, e^{-j\pi(1)} + x(2)\, e^{-j\pi(2)} + x(3)\, e^{-j\pi(3)}\right]$$

$$= \left[1 + \frac{1}{2}(\cos\pi - j\sin\pi) + \frac{1}{4}(\cos 2\pi - j\sin 2\pi) + \frac{1}{8}(\cos 3\pi - j\sin 3\pi)\right]$$

$$X(2) = \frac{5}{8}$$

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$= X(0) + X(2)$

$= \frac{15}{8} + \frac{5}{8}$

$= \frac{20}{8}$

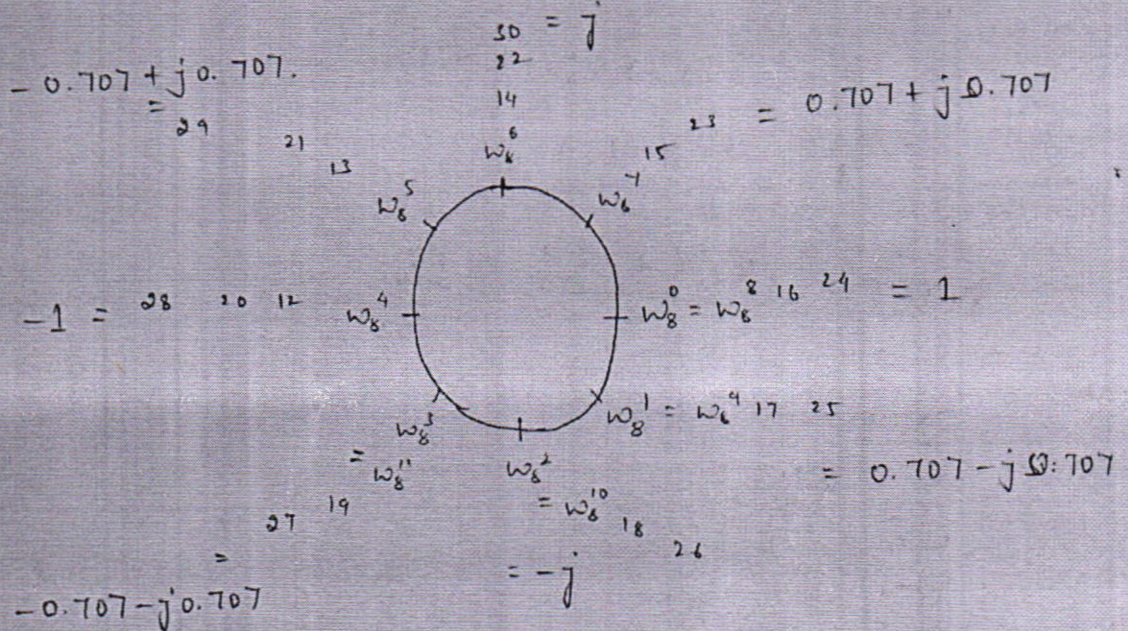
10

4a) $x(n) = \{1, 1, 1, 1\}$

$N = 8.$

for the 8 point DFT i.e.,

$x(n) = \{1, 1, 1, 1, 0, 0, 0, 0\}$



$$
\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \\ X(4) \\ X(5) \\ X(6) \\ X(7) \end{bmatrix}
=
\begin{array}{c|cccccccc}
 & n=0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\
\hline
k=0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 \\
1 & W_8^0 & W_8^1 & W_8^2 & W_8^3 & W_8^4 & W_8^5 & W_8^6 & W_8^7 \\
2 & W_8^0 & W_8^2 & W_8^4 & W_8^6 & W_8^8 & W_8^{10} & W_8^{12} & W_8^{14} \\
3 & W_8^0 & W_8^3 & W_8^6 & W_8^9 & W_8^{12} & W_8^{15} & W_8^{18} & W_8^{21} \\
4 & W_8^0 & W_8^4 & W_8^8 & W_8^{12} & W_8^{16} & W_8^{20} & W_8^{24} & W_8^{28} \\
5 & W_8^0 & W_8^5 & W_8^{10} & W_8^{15} & W_8^{20} & W_8^{25} & W_8^{30} & W_8^{35} \\
6 & W_8^0 & W_8^6 & W_8^{12} & W_8^{18} & W_8^{24} & W_8^{30} & W_8^{36} & W_8^{42} \\
7 & W_8^0 & W_8^7 & W_8^{14} & W_8^{21} & W_8^{28} & W_8^{35} & W_8^{42} & W_8^{49}
\end{array}
\cdot
\begin{bmatrix} x(0) \\ x(1) \\ x(2) \\ x(3) \\ x(4) \\ x(5) \\ x(6) \\ x(7) \end{bmatrix}
$$

$$
\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \\ X(4) \\ X(5) \\ X(6) \\ X(7) \end{bmatrix}
=
\begin{bmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & \left(\frac{1}{\sqrt2} - j\frac{1}{\sqrt2}\right) & (-j) & \left(-\frac{1}{\sqrt2} - j\frac{1}{\sqrt2}\right) & \theta & \theta & \theta & \theta \\
1 & (-j) & (-1) & (j) & \theta & \theta & \theta & \theta \\
1 & \left(-\frac{1}{\sqrt2} - j\frac{1}{\sqrt2}\right) & (j) & \left(\frac{1}{\sqrt2} - j\frac{1}{\sqrt2}\right) & \theta & \theta & \theta & \theta \\
1 & (-1) & (1) & (-1) & \theta & \theta & \theta & \theta \\
1 & \left(-\frac{1}{\sqrt2} + j\frac{1}{\sqrt2}\right) & (-j) & \left(\frac{1}{\sqrt2} + j\frac{1}{\sqrt2}\right) & - & - & - & - \\
1 & (j) & (-1) & (-j) & - & - & - & - \\
1 & \left(\frac{1}{\sqrt2} + j\frac{1}{\sqrt2}\right) & (j) & \left(-\frac{1}{\sqrt2} + j\frac{1}{\sqrt2}\right) & - & - & - & -
\end{bmatrix}
\cdot
\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}
$$

$$
\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \\ X(4) \\ X(5) \\ X(6) \\ X(7) \end{bmatrix}
=
\begin{bmatrix}
1+1+1+1+0+0+0+0 \\
1+\left(\frac{1}{\sqrt{2}}-j\frac{1}{\sqrt{2}}\right)+(-j)+\left(-\frac{1}{\sqrt{2}}-j\frac{1}{\sqrt{2}}\right)+0+0+0+0 \\
1+(-j)+(-1)+(j)+0+0+0+0 \\
1+\left(-\frac{1}{\sqrt{2}}-j\frac{1}{\sqrt{2}}\right)+(j)+\left(\frac{1}{\sqrt{2}}-j\frac{1}{\sqrt{2}}\right)+0+0+0+0 \\
1+(-1)+(1)+(-1)+0+0+0+0 \\
1+\left(-\frac{1}{\sqrt{2}}+j\frac{1}{\sqrt{2}}\right)+(-j)+\left(\frac{1}{\sqrt{2}}+j\frac{1}{\sqrt{2}}\right)+0+0+0+0 \\
1+j+(-1)+(-j)+0+0+0+0 \\
1+\left(\frac{1}{\sqrt{2}}+j\frac{1}{\sqrt{2}}\right)+(j)+\left(-\frac{1}{\sqrt{2}}+j\frac{1}{\sqrt{2}}\right)+0+0+0+0
\end{bmatrix}
$$

$$
\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \\ X(4) \\ X(5) \\ X(6) \\ X(7) \end{bmatrix}
=
\begin{bmatrix}
4 \\
1-j2.4142 \\
0 \\
1-j0.4142 \\
0 \\
1+j0.4142 \\
0 \\
1+j2.4142
\end{bmatrix}
$$

$$X(k)=\{4,\ 1-j2.4142,\ 0,\ 1-j0.4142,\ 0,\ 1+j0.4142,\ 0,\ 1+j2.4142\}.$$

Magnitude

$$|X(K)| = \sqrt{(X(K))_R^2+(X(K))_I^2}$$

$$= \sqrt{(4)^2+(0)^2}$$

$$= 4$$

$$\angle x(k) = \tan^{-1}\left[\frac{x(k)_I}{x(k)_R}\right]$$

$$= \tan^{-1}\left[\frac{0}{4}\right]$$

$$= \tan^{-1}(0) = 0$$

$$|x(k)| = \sqrt{(1)^2 + (-2.4142)^2}$$

$$= \sqrt{1 + 5.828}$$

$$= \sqrt{6.826}$$

$$|x(k)| = 2.6131$$

$$\angle x(k) = \tan^{-1}\left[\frac{-2.4142}{1}\right]$$

$$= -67.49 \times \frac{\pi}{180}$$

$$\angle x(k) = -1.1779$$

$$|x(k)| = \sqrt{(1)^2 + (-0.4142)^2}$$

$$= \sqrt{(1)^2 + 0.1715}$$

$$= \sqrt{1.1715}$$

$$|x(k)| = 1.0823$$

$$\angle x(k) = \tan^{-1}\left[\frac{-0.4142}{1}\right]$$

$$= -22.49 \times \frac{\pi}{180}$$

$$= -0.3926.$$

$$|x(k)| = \sqrt{(1)^2 + (0.4142)}$$

$$|x(k)| = 1.0823$$

$$= \tan^{-1}\left(\frac{0.4142}{1}\right)$$

$$= 0.3926$$

$$|x(k)| = \sqrt{(1)^2 + (2.4142)}$$

$$= \sqrt{1 + 5.828}$$

$$= \sqrt{6.828}$$

$$= 2.6131$$

$$\angle x(k) = -1.1779$$

$$|x(k)| = \{1, 2.6131, 0, 1.0823, 0, 1.0823, 0, 2.6131\}$$

10

4b) $G(k) = \{1+j, -2.1+j3.2, -1.2+j2.4, 0, 0.9+j3.1, -0.3+j3\}$

$h(n) = g((n-4))_6$. $\quad N = 6$

$$h(n) = g((n-4))_6$$

$$h(n) = g(0)\, e^{-j\frac{2\pi}{6}kn}$$

$$h(n) = (1+j)(1)$$

$$h(n) = 1.$$

$h(n) = g(1)\, e^{-j\frac{2\pi}{6}(1)n}$

$h(n) = (-2.1+j3.2)(\cos 60 - j\sin 60) = (-2.1+j3.2)(0.5-j0.866)$

$h(n) = -1.05 + j1.8186 + j1.6 - j^2 2.7712.$

$h(n) = -1.05 + j3.4186 + 2.7712$

$h(n) = 1.7212 + j3.4186$ //

$h(n) = g(2).\, e^{-j\frac{2\pi}{6}(2)}$

$= g(2).\, e^{-j\frac{4\pi}{6}} = g(2).\, e^{-j2\pi/3}$

$= (-1.2+j2.4)(\cos 120 - j\sin 120) = (-1.2+j2.4)(-0.5-j0.866)$

$=$

02

$\frac{28}{40} = \frac{21}{30}$

2nd internal Assessment

1 a) $h(n) = \{1, 1, 1\}$.

$x(n) = \{3, -1, 0, 1 / 3, 2, 0, 1 / 2, 1\}$.

$N = L + M - 1 \qquad M = 3 \qquad 2^3 = 8.$

$8 = L + 3 - 1$

$8 = L + 2$

$L = 6,$

$x_1(n) = \{0, 0, 3, -1, 0, 1\}$

$x_2(n) = \{0, 1, 3, 2, 0, 1\}$

$x_3(n) = \{0, 1, 2, 1, 0, 0\}$

$h(n) = \{1, 1, 1, 0, 0, 0\}$

$$\begin{bmatrix} y_1(0) \\ y_1(1) \\ y_1(2) \\ y_1(3) \\ y_1(4) \\ y_1(5) \\ \cancel{y_1(6)} \\ y_7(n) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & -1 & 3 & 0 \\ 0 & 0 & 1 & 0 & -1 & 3 \\ 3 & 0 & 0 & 1 & 0 & -1 \\ -1 & 3 & 0 & 0 & 1 & 0 \\ 0 & -1 & 3 & 0 & 0 & 1 \\ 1 & 0 & -1 & 3 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1+0 \\ 0+0+1 \\ 3+0+0 \\ -1+3+0 \\ -1+3 \\ \cancel{1-1} \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 3 \\ 2 \\ 2 \\ 0 \end{bmatrix}$$

$y_1(n) = \{1, 1, 3, 2, 2, 0\}$

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$$\begin{bmatrix} y_2(0) \\ y_2(1) \\ y_2(2) \\ y_2(3) \\ y_2(4) \\ y_2(5) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 2 & 3 & 1 \\ 1 & 0 & 1 & 0 & 2 & 3 \\ 3 & 1 & 0 & 1 & 0 & 2 \\ 2 & 3 & 1 & 0 & 1 & 0 \\ 0 & 2 & 3 & 1 & 0 & 1 \\ 1 & 0 & 2 & 3 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0+1+0 \\ 1+1 \\ 3+1 \\ 2+3+1 \\ 2+3 \\ 1+2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 4 \\ 6 \\ 5 \\ 3 \end{bmatrix}$$

$$y_2(n) = \{1, 2, 4, 6, 5, 3\}$$

$$\begin{bmatrix} y_3(0) \\ y_3(1) \\ y_3(2) \\ y_3(3) \\ y_3(4) \\ y_3(5) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 & 2 & 1 \\ 1 & 0 & 0 & 0 & 1 & 2 \\ 2 & 1 & 0 & 0 & 0 & 1 \\ 1 & 2 & 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1+0 \\ 2+1 \\ 1+2+1 \\ 1+2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 3 \\ 4 \\ 3 \\ 1 \end{bmatrix}$$

$$y_3(n) = \{0, 1, 3, 4, 3, 1\}$$

$$y_1(n) = \{\underbrace{\boxed{1}\,\boxed{1}}_{\substack{(D-1) \\ \text{discard}}}\,\boxed{3}\,\boxed{2}\,\boxed{2}\,\boxed{0}\}$$

$$y_2(n)\quad \underbrace{\boxed{1}\,\boxed{2}}_{\substack{(D-1) \\ \text{discard}}}\,\boxed{4}\,\boxed{6}\,\boxed{5}\,\boxed{3}$$

$$y_3(n)\quad \underbrace{\boxed{0}\,\boxed{1}}_{\substack{(D-1) \\ \text{discard}}}\,\boxed{3}\,\boxed{4}\,\boxed{3}\,\boxed{1}$$

$$y(n) = \{3, 2, 2, 0, 4, 6, 5, 3, 3, 4, 3, 1\}$$

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1 b) For the Complex Multiplication and the Complex addition is having an Expressions,

for addition $N(N-1)$ and the $N\log_2 N$

and for Multiplication N^2 and $\frac{N}{2}\log_2 N$

	Complex addition		Complex Multiplication	
Sl.No N	FFT $N(N-1)$	$N\log_2 N$	N^2	$\frac{N}{2}\log_2 N$
16	240	64	256	32
32	992	160	1024	80
128	16,256	896	16,384	448

$N=16$
$=N(N-1)$
$=16(16-1)$
$=16(15)$
240

$N\log_2 N$
$N.\frac{\log N}{\log 2}$
$16\frac{\log 16}{\log 2}$

N^2
$(16)(16)$

$\frac{N}{2}\log_2 N$
$\frac{N}{2}\frac{\log N}{\log 2}$
$\frac{16}{2}\frac{\log 16}{\log 2}$

$N=32$
$=32(32-1)$
$32(31)$
$=992$

$N=32$
$32.\frac{\log 32}{\log 2}$
$32(5)=160$

$(32)(32)$

$\frac{32}{2}\frac{\log 32}{\log 2}$
$(16)(5)$
$=80.$

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$N = 128$

$128(128-1)$

$128(127)$

$= 16{,}256$

$(128) \cdot \frac{\log 128}{\log 2}$

$(128)(7)$

$= 896$

$(128)^2$

$= 16{,}384.$

$\frac{128}{2} \frac{\log 128}{\log 2}$

$(64)(7)$

$\Rightarrow 448$

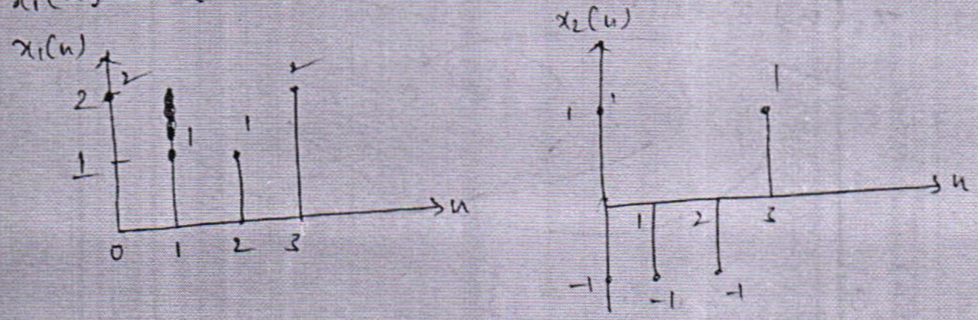
3a) $x(0) = 0$, $X(1) = 2 + j2$, $X(2) = -j4$, $X(3) = 2 - j2$. ~~$X(4) = 0$~~

2b) ~~$x(n) = 2^{n}$~~ ~~$N = 8$.~~

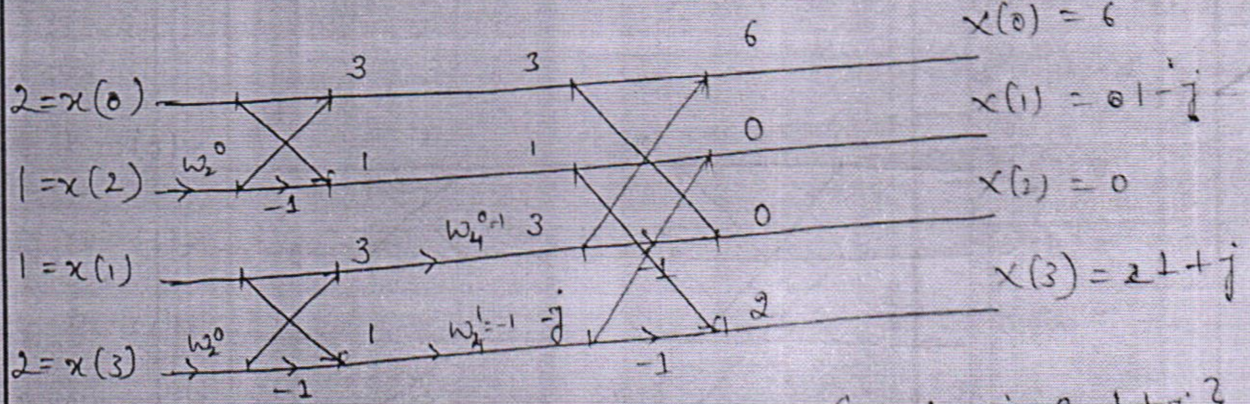
~~$n = 0, 1, 2, 3, 4 \ldots$~~

~~$N = L + M - 1$~~

3b) $x_1(n) = \{2, 1, 1, 2\}$. $x_2(n) = \{1, -1, -1, 1\}$

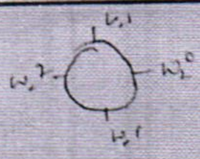


$x_1(n) \circledast x_2(n)$ for $N = 4$.



$X_1(k) = \{6, 1-j, 0, 1+j\} = \{6, 1-j, 0, 1+j\}$

$j = -1$



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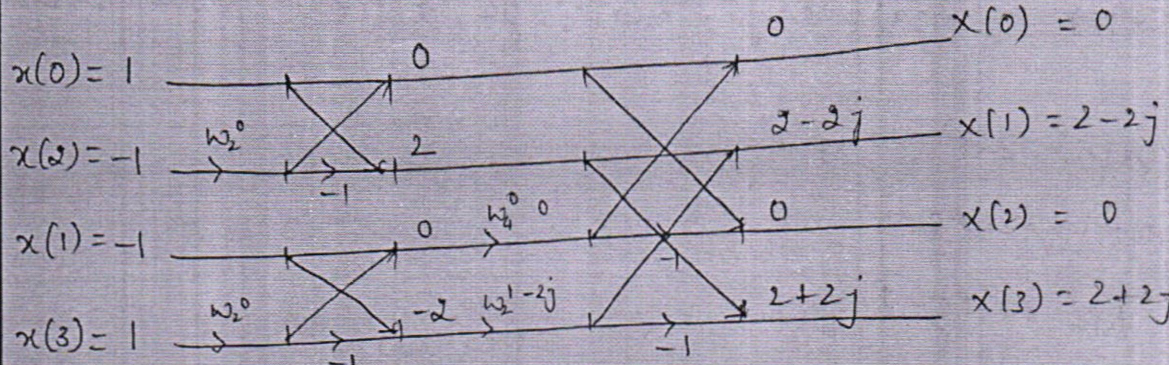
DIT FFT	Bit reversal (2 1)	
00	00	0
01	10	2
10	01	1
11	11	3



$$x_3(n) = x_1(n) \circledast x_2(n)$$

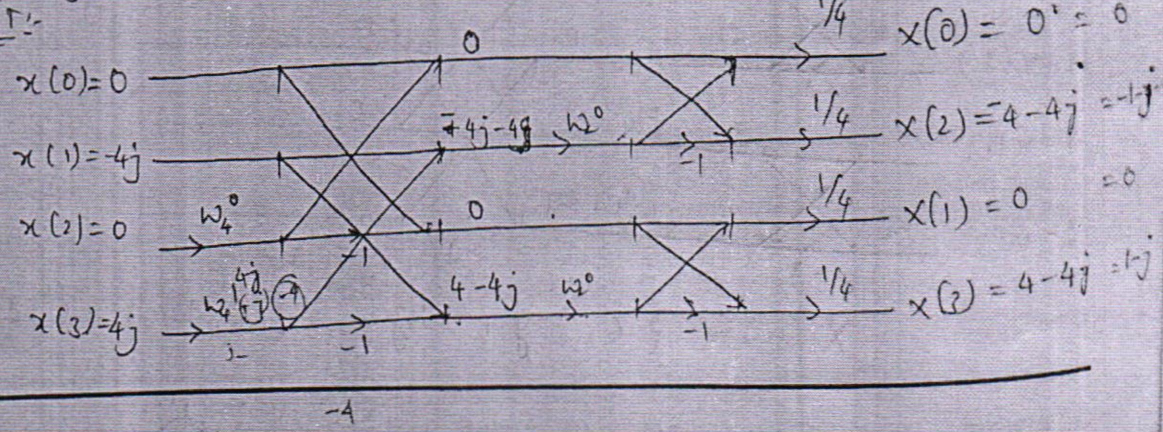
$$= \{6, 1-j, 0, 1+j\} * \{0, 2-2j, 0, 2+2j\}$$

$$x_3(n) = \{0, -4j, 0, 4j\}$$

$(1-j)(2-2j)$ $\qquad$ $(1+j)(2+2j)$

$\cancel{2} - 2j - 2j + \cancel{2}j^2$ $\qquad$ $2+2j+2j+2j^2_{(-1)}$

IDFT:-



$x_3(n) = \{0,\ -1-j,\ 0,\ 1-j\}$

3a) $X(0) = 0,\ X(1) = 2+j2,\ X(2) = -j4,\ X(3) = 2-j2,\ X(4) = 0$

Midpoint $\frac{8}{2} = 4$

$\{x(0),\ x(1),\ x(2),\ x(3),\ x(4),\ x(5),\ x(6),\ x(7)\}$

Midpoint

$x(1) = x^*(7) = +2+j2$

$x(2) = x^*(6) = -j4$

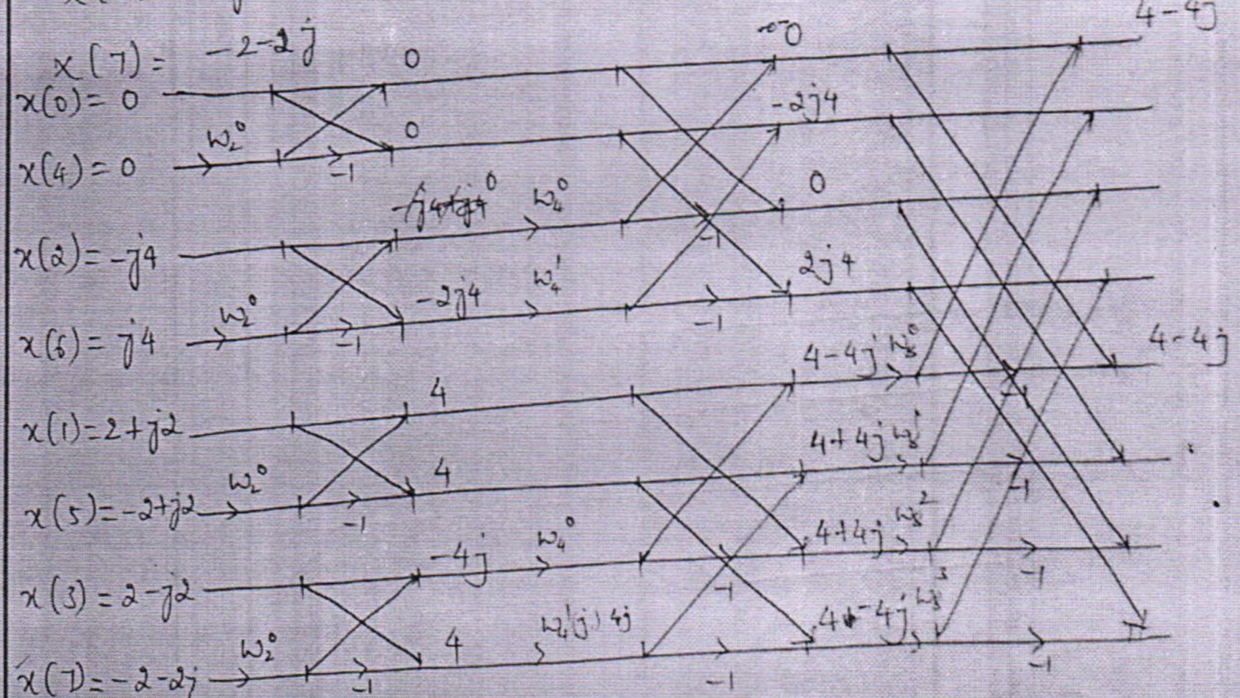
$x(3) = 2-j2$

$x(4) = 0$

$x(5) = -2+j2$

$x(6) = j4$

$x(7) = -2-2j$



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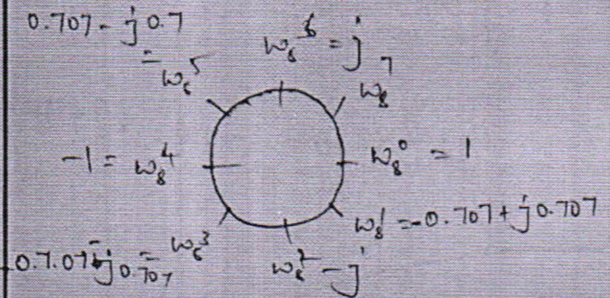

0.707 - j0.7 = W_8^5
$W_8^6 = j$
W_8^7
$-1 = W_8^4$
$W_8^0 = 1$
$W_8^1 = 0.707 + j0.707$
-0.707 - j0.707 = W_8^3
$W_8^2 = -j$

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Internal Assessment - III

3a) The bilinear transformation is used to converting the signals from analog signal to digital signal. and in the bilinear transformation the trapezoidal is linked.

Consider the 1st order differential Equation,

$$\frac{d}{dt}y(t) = x(t) \quad — (1)$$

Now, applying integration on both sides.

$$\int_{(n-1)T}^{nT} \frac{d}{dt} y(t)\,dt = \int_{(n-1)T}^{nT} x(t)\,dt$$

Now the above Equation becomes

$$\Big[y(t)\Big]_{(n-1)T}^{nT} = \int_{(n-1)T}^{nT} x(t)\,dt \quad — (2)$$

$$\Big[y(nT) - y(n-1)T\Big] = \int_{(n-1)T\ (a)}^{nT\ (b)} x(t)\,dt$$

By the trapezoidal Rule.

$$\int_a^b x(t)\,dt = \frac{b-a}{2}\left[\cancel{f}(a) + \cancel{f}(b)\right]$$

$$\left[y(nT) - y(n-1)T\right] = \frac{\left[nT - (n-1)T\right]}{2}\left[x(nT) + x(n-1)T\right]$$

$$\left[y(nT) - y(n-1)T\right] = \frac{n\cancel{T} - \cancel{n}T + T}{2}\left[x(nT) + x(n-1)T\right]$$

$$\left[y(nT) - y(n-1)T\right] = \frac{T}{2}\left[x(nT) + x(n-1)T\right]$$

$$y(n) = y(t)\Big|_{t=nT}$$

$$y(n) = y(nT)$$

By applying z- transform on to the above equation

$$\left[y(n) - y(n-1)\right] = \frac{T}{2}\left[x(n) + x(n-1)\right]$$

$$\left[y(z) - z^{-1}y(z)\right] = \frac{T}{2}\left[\cancel{y}(z) + z^{-1}x(z)\right]$$

$$\left[y(z) - z^{-1}y(z)\right]\frac{2}{T} = \left[\cancel{y}(z) + z^{-1}x(z)\right]$$

$$\cancel{\frac{2}{T}\left[y(z) - z^{-1}y(z) + z^{-1}x(z)\right] = x(z)}$$

$$y(z)\left[1 - z^{-1}\right]\frac{2}{T} = x(z)\left[1 + z^{-1}\right]$$

$$x(z) = \frac{2}{T}\,y(z)\cdot\frac{1-z^{-1}}{1+z^{-1}} \quad — \quad (3)$$

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Now. By applying Laplace transform.

Considering Eq^n 1

$$\frac{d}{dt} y(n) = x(n)$$

$$s Y(s) = X(s)$$

$$s Y(s) = \frac{2}{T} \frac{1-z^{-1}}{1+z^{-1}} y(z) \quad - 4$$

Now. Comparing Equation. 3 & 4

$$s = \frac{2}{T} \frac{1-z^{-1}}{1+z^{-1}} y(z). \quad - 5$$

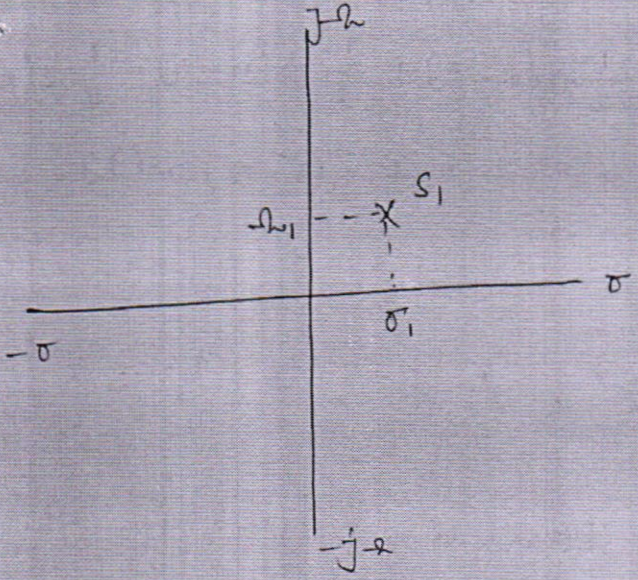
The above Equation is know as the Bilinear Transformation.

Now. Consider Eq^n 5. i.e.,

$$s = \frac{2}{T} \frac{1-z^{-1}}{1+z^{-1}} y(z)$$

$$s\frac{T}{2} (1+z^{-1}) = (1-z^{-1}) y(z)$$

$$s\frac{T}{2} \frac{1+z^{-1}}{1-z^{-1}} = y(z)$$



$$S = e^{S_1(\sigma_1 + \Omega_1)} \qquad e^{p_k t} \quad t = nT$$

$$e^{p_k nT}$$

$$S_1 = e^{(p_1\sigma_1 + p_1\Omega_1)S_1}$$

$$S_1 = e^{p_1\sigma_1 S_1} \cdot e^{p_1\Omega_1 S_1}$$

3) $$S = \frac{2}{T}\,\frac{1-z^{-1}}{1+z^{-1}}\,y(z)$$

$$\frac{ST}{2}(1+z^{-1}) = 1 - z^{-1}\,y(z)$$

$$\frac{ST}{2}\left(1+\frac{1}{z}\right) = \left(1-\frac{1}{z}\right)y(z)$$

$$\frac{ST}{2}\left(\frac{z+1}{\cancel{z}}\right) = \left(\frac{z-1}{\cancel{z}}\right)y(z)$$

$$\boxed{\frac{ST}{2}\,\frac{(z+1)}{(z-1)} = y(z)}$$

3 b) $h(n) = \delta(n) - \frac{1}{4}\delta(n-1) + \frac{1}{2}\delta(n-2) + \frac{1}{2}\delta(n-3) - \frac{1}{4}\delta(n-4)$ 2a)

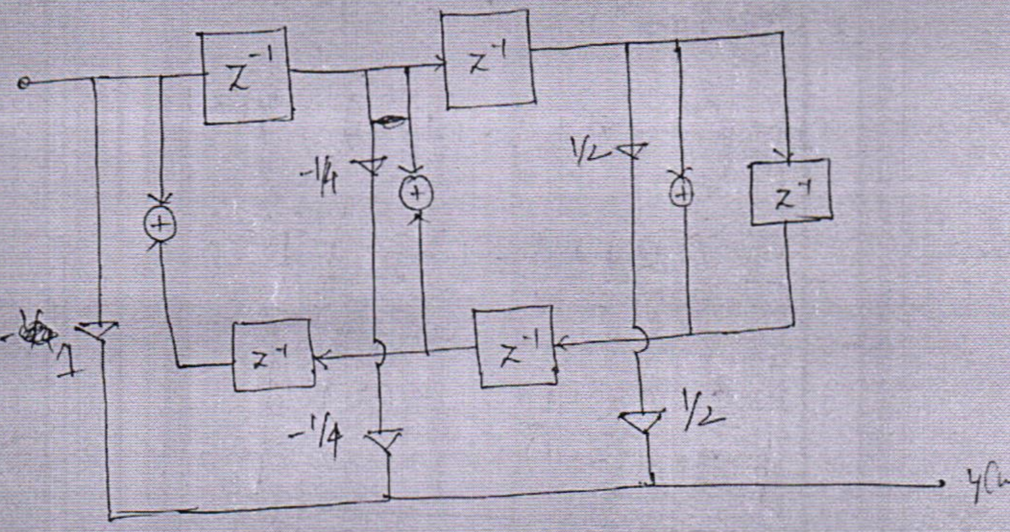
$+ \delta(n-5) \quad -(1)$

Direct form I :-

$$h(z) = 1 - \frac{1}{4}z^{-1} + \frac{1}{2}z^{-2} + \frac{1}{2}z^{-3} - \frac{1}{4}z^{-4} + 1\,z^{-5}$$

The odd No. of z^{-1} term's are present in the above Equation.

Direct form I :



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2a) Impulse invarient Method :-

The impulse invariant Method. it is used to converting the analog signal to the digital signal. and in the transformation from the analog signal to the digital signal.

$$h_a(t)$$

$$h(n) = h_a(t) \quad — ⑧$$

where $t = nT$

$$h(n) = h_a(nT)$$

for the impulse invariant Method. Considering the partial differential Equation by the Consideration of uniform values.

$$h(n) = \sum_{K=1}^{N} \frac{A_K}{S-P_K} \quad — ④$$

$$h(n) = \frac{A_1}{S-P_1} + \frac{A_2}{S-P_2} + \cdots \frac{A_N}{S-P_N}$$

where.-

A_K is the gain. of the. K^{th} pole.

P_K is the. pole of K^{th} pole.

Now by Applying the. Laplace Transform for the above Eq^n we get.

$$h(s) = \sum_{K=1}^{N} A_K e^{P_K t} \quad — ②. \Rightarrow A_K \frac{1}{1-e^{P_K nT}}$$

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Now.

$$\boxed{h(n) = \sum_{K=1}^{M} \frac{A_K}{S - P_K}} \quad -(3)$$

$$h(n) = h(t)\Big|_{t = nT}$$

$$h(n) = h(nT)$$

Consider.

$$h(n) = Z\left[h(t)\right]$$

$$h(t) = \sum_{K=1}^{n} \frac{A_K}{S-P_K} z^{-n}$$

$$h(t) = \sum_{K=-\infty}^{\infty} \sum_{K=1}^{n} \frac{A_K}{S-P_K} z^{-n}$$

$$h(t) = \sum_{K=0}^{\infty} A_K \sum_{K=1}^{n} \frac{1}{S-P_K} z^{-n}$$

$$h(t) = \sum_{K=1}^{n} A_K \sum_{n=0}^{\infty} \frac{1}{S-P_K} z^{-n}$$

By Z-transforming.

$$h(z) = A_K \sum_{n=0}^{\infty} \left(e^{+P_K n} z^{-n} \right)$$

$$h(z) = A_k \sum_{n=0}^{\infty} \left(e^{p_k} z^{-1} \right)^n$$

$$\sum a^n = \frac{1}{1-a}$$

$$h(z) = \frac{A_k}{1 - e^{p_k} z^{-1}} \quad \text{—} \ (4)$$

Now Comparing the above Equation with Laplace transform Equation

$$\frac{A_k}{s - p_k} \longrightarrow \frac{A_k}{1 - e^{p_k} z^{-1}}$$

$$\frac{1}{s - p_k} \longrightarrow \frac{1}{1 - e^{p_k} z^{-1}}$$

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The above Equation the Conversion of the Signal from analog to digital transformation Equation.

2b) $T = 1$ Sec

$$H(z) = \frac{1}{s^2 + \sqrt{2}s + 1}$$

$f = \frac{1}{T} \Rightarrow f = \frac{1}{1} \Rightarrow 1$ HZ. $= 2\pi f$

$= 2\pi(1)$

$=$

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