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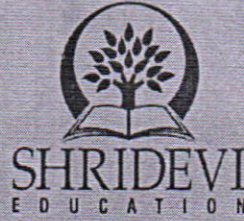
ಶ್ರೀದೇವಿ ಇಂಜಿನಿಯರಿಂಗ್ ಮತ್ತು ತಾಂತ್ರಿಕ ಮಹಾವಿದ್ಯಾಲಯ

# SHRIDEVI INSTITUTE OF ENGINEERING AND TECHNOLOGY

(Recognised by Govt. of Karnataka, Affiliated to VTU, Belagavi and Approved by AICTE, New Delhi. An ISO 9001:2015 Certified Institution)

Sira Road, Tumakuru - 572 106, Karnataka.

Phone: 0816-2212629 / 27 Fax: 0816-2212628 Email: info@shrideviengineering.org Web: http://www.shrideviengineering.org



## ASSIGNMENT BOOK

USN :

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Name of the Student : ..... Manu. S. N

Course : ..... Digital Signal processing ..... Code : ..... 18EE63

Semester : ..... 6<sup>th</sup> ..... Branch : ..... EEE



*Manu S. N*  
PRINCIPAL  
SIET., TUMAKURU.

## ASSIGNMENT MARKS

Date	Assignment No.	Max. Marks	Marks Obtained	Course Instructor Signature
20/5/22	1	10	10	G. H Ramesh
27/6/22	2	10	10	G. H Ramesh
14/7/22	3	10	10	G. H Ramesh
	4			
	5			
	Average	10	10	G. H Ramesh

## CERTIFICATE

This is to certify that Kum/ Sri ..... Manu. S. N. .....

with USN ..... 1SV19EE007 ..... has satisfactorily completed the

Assignments in the subject Digital Signal processing .....

with Subject Code ..... 18EE63 ..... as prescribed by the

Visvesvaraya Technological University for the ..... 6<sup>th</sup> ..... semester.

✓  
B.E. / M.Tech / MBA degree course in the year 2021 -2022

G. H Ramesh  
Course Instructor

G. H Ramesh  
Head of the Department

Manu S. N.  
PRINCIPAL  
SIET., TUMAKURU.

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Assignment - 01  
Digital Signal Processing

3a) Find the DFT of a Sequence  $x(n) = \{1, 1, 0, 0\}$  and find the IDFT of  $Y(k) = \{2, 1+j, 0, 1-j\}$ .

Ans:

$$x(n) = \{1, 1, 0, 0\}$$

$$N=4.$$

$$X(k) = \text{DFT} [x(n)]$$

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j \frac{2\pi}{N} kn}$$

$$X(k) = \sum_{n=0}^{4-1=3} x(n) e^{-j \frac{2\pi}{4} kn}$$

$$X(k) = \sum_{n=0}^3 x(n) e^{-j \frac{\pi}{2} kn}$$

$$X(k) = x(0) e^{j0} + x(1) e^{-j \frac{\pi}{2} k(1)} + x(2) e^{-j \frac{\pi}{2} k(2)} + x(3) e^{-j \frac{\pi}{2} k(3)}$$

$$X(k) = 1 + 1 \cdot e^{-j \frac{\pi}{2} k}$$

put,  $k=0$ ,  $X(0) = 1 + 1e^0$

$$X(0) = 2$$

put,  $k=1$ ,  $X(1) = 1 + 1 \cdot e^{-j \frac{\pi}{2} (1)}$

$$= 1 + \left( \cos \frac{\pi}{2} - j \sin \frac{\pi}{2} \right)$$

$$X(1) = 1 - j$$

put  $k=2$ ,

$$\begin{aligned}x(2) &= 1 + 1e^{-j\frac{\pi}{2}(2)} \\&= 1 + e^{-j\pi} \\&= 1 + \cos\pi - j\sin\pi \\&= 1 - 1 = 0\end{aligned}$$

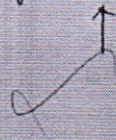
$k=3$ ,

$$\begin{aligned}x(3) &= 1 + 1e^{-j\frac{\pi}{2}(3)} \\&= 1 + e^{-j\frac{3\pi}{2}} \\&= 1 + \cos\frac{3\pi}{2} - j\sin\frac{3\pi}{2} \\&= 1 - j(-1) \\x(3) &= 1 + j\end{aligned}$$

$$x(k) = \{2, 1-j, 0, 1+j\}$$

Midpoint

$$= \frac{N}{2} = \frac{4}{2} = 2$$



1. Compute 4-point DFT of  $x(n) = \sin^2\left(\frac{n\pi}{2} + \frac{\pi}{3}\right)$  from  $x(k)$  obtain the DFT of  $\cos^2\left(\frac{n\pi}{2} + \frac{\pi}{3}\right)$ .

$\Rightarrow N=4$

$$x(n) = \sin^2\left(\frac{n\pi}{2} + \frac{\pi}{3}\right) \quad \begin{array}{l} 0 \leq n \leq N-1 \\ 0 \leq n \leq 4-1 \\ 0 \leq n \leq 3 \end{array}$$

Let us,

$$n=0, x(0) = \sin^2\left(\frac{0\pi}{2} + \frac{\pi}{3}\right)$$

$$x(0) = \sin^2(60) = 0.75$$

$$n=1, x(1) = \sin^2\left(\frac{\pi}{2} + \frac{\pi}{3}\right) \Rightarrow x(1) = \sin^2(150) \Rightarrow 0.25$$

$$n=2, x(2) = \sin^2\left(\frac{2\pi}{2} + \frac{\pi}{3}\right) \Rightarrow \sin^2\left(\frac{4\pi}{3}\right) \Rightarrow \sin^2(240) \Rightarrow 0.75$$

$$n=3, x(3) = \sin^2\left(\frac{3\pi}{2} + \frac{\pi}{3}\right) \Rightarrow \sin^2\left(\frac{10\pi}{3}\right) \Rightarrow \sin^2(120) \Rightarrow 0.75$$

$$\therefore x(n) = \{0.75, 0.25, 0.75, 0.75\}$$

$x(0) \quad x(1) \quad x(2) \quad x(3)$

We know,

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N}kn}$$

$$X(k) = \sum_{n=0}^{4-1} x(n) e^{-j\frac{\pi}{2}kn} \Rightarrow \sum_{n=0}^3 x(n) e^{-j\frac{\pi}{2}kn}$$

$$X(k) = x(0) e^{-j\frac{\pi}{2}k(0)} + x(1) e^{-j\frac{\pi}{2}k(1)} + x(2) e^{-j\frac{\pi}{2}k(2)} + x(3) e^{-j\frac{\pi}{2}k(3)}$$

$$x(k) = 0.75(1) + 0.25e^{-j\pi k/2} + 0.75e^{-j\pi k} + 0.75e^{-j\frac{3\pi k}{2}}$$

put  $k=1$

$$x(1) = 0.75 + 0.25e^{-j\pi/2} + 0.75e^{-j\pi(1)} + 0.75e^{-j\frac{3\pi}{2}(1)}$$

$$\begin{aligned} x(1) &= 0.75 \left[ 1 + e^{-j\pi} \right] \\ &= 0.75 + 0.25 \left[ \cos\frac{\pi}{2} - j\sin\frac{\pi}{2} \right] + 0.75 \left[ \cos\pi - j\sin\pi \right] \\ &\quad + 0.75 \left[ \cos\left(\frac{3\pi}{2}\right) - j\sin\left(\frac{3\pi}{2}\right) \right] \\ &= 0.75 + 0.25[-j] + 0.75[-1] + 0.75[j] \\ &= 0.75 - 0.25j + 0.75 + j0.75 \\ x(1) &= 1.5 + j0.5 \end{aligned}$$

put  $k=0$

$$\begin{aligned} x(0) &= 0.75 + 0.25 + 0.75 + 0.75 \\ &= 0.75 + 0.25 + 0.75 + 0.75 \\ x(0) &= 2.5 \end{aligned}$$

when  $k=2$

$$\begin{aligned} x(2) &= 0.75 + 0.25e^{-j\frac{2\pi}{2}} + 0.75e^{-j2\pi} + 0.75e^{-j\frac{3\pi(2)}{2}} \\ x(2) &= 0.75 + 0.25e^{-j\pi} + 0.75e^{-j2\pi} + 0.75e^{-j3\pi} \\ x(2) &= 0.75 + 0.25 \left[ \cos\pi - j\sin\pi \right] + 0.75 \left[ \cos 2\pi - j\sin 2\pi \right] \\ &\quad + 0.75 \left[ \cos 3\pi - j\sin 3\pi \right] \end{aligned}$$

$$x(2) = 0.75 - 0.25 + 0.75 - 0.75$$

$$x(2) = 0.5 //$$

when  $k=3$ ,

$$x(3) = 0.75 + 0.25 e^{-j\pi(3)/2} + 0.75 e^{-j\pi(3)} + 0.75 e^{-j\frac{3\pi(3)}{2}}$$

$$x(3) = 0.75 + 0.25 e^{-j3\pi/2} + 0.75 e^{-j3\pi} + 0.75 e^{-j9\pi/2}$$

$$x(3) = 0.75 + 0.25 [\cos \uparrow 70 - j \sin \uparrow 70] + 0.75 [\cos \uparrow 40 - j \sin \uparrow 40] + 0.75 [\cos \uparrow 10 - j \sin \uparrow 10]$$

$$x(3) = 0.75 + j0.25 + -0.75 - j0.75$$

$$x(3) = -j0.5 //$$

$$x(k) = \{ 2.5, 1.5 + j0.5, 0.5, -j0.5 \}$$

4. Compute the 4-point DFT of the sequence

$x(n) = \{ 1, 0, 1, 0 \}$ . also find  $y(n)$  if  $Y(k) = X((k-2))_4$

$$\Rightarrow X(k) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N}kn}$$

$$= \sum_{n=0}^{4-1=3} x(n) e^{-j\frac{2\pi}{4}kn}$$

$$= \sum_{n=0}^3 x(n) e^{-j\frac{\pi}{2}kn}$$

$$x(k) = x(0) e^0 + x(1) e^{-j\frac{\pi}{2}k(1)} + x(2) e^{-j\frac{\pi}{2}k(2)} + x(3) e^{-j\frac{\pi}{2}k(3)}$$

$$x(k) = 1 + e^{-j\pi k} \quad \text{--- (A)}$$

Let's see.

$$k=0, \quad x(0) = 1 + e^0 = 1 + 1 = 2$$

$$k=1, \quad x(1) = 1 + e^{-j\pi} = 1 - 1 = 0$$

$$k=2, \quad x(2) = 1 + e^{-j\pi 2} = 1 + 1 = 2$$

$$k=3, \quad x(3) = 1 + e^{-j\pi 3} = 1 - 1 = 0$$

$$x(k) = \{2, 0, 2, 0\}$$

We know frequency shift property.

$$x((k-1))_N \longleftrightarrow \omega_N^{-1} x(n)$$

(\*)

$$x((k-1))_N \longleftrightarrow e^{j\frac{2\pi}{N}kn} x(n)$$

Given

$$y(k) = x((k-2))_4$$

$$y(n) = e^{j\frac{2\pi}{4}kn} x(n)$$

$$y(n) = e^{j\frac{\pi}{2}kn} x(n)$$

$$y(n) = e^{j\pi n} x(n)$$

$$n=0, \quad y(0) = e^0 x(0) = 1$$

$$n=1, \quad y(1) = e^{j\pi} x(1) = 0$$

$$n=2, \quad y(2) = e^{j2\pi} x(2) = 1$$

$$n=3, \quad y(3) = e^{j3\pi} x(3) = 0$$

$$y(n) = \{1, 0, 1, 0\}$$

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5. Find DFT for finite duration DT Signal of length L.

$$x(n) = \begin{cases} 1, & 0 \leq n \leq L-1 \\ 0 & \text{else where assume } N \gg L \end{cases}$$

$$\Rightarrow X(k) = \text{DFT} [x(n)] \\ = \sum_{n=0}^{L-1} x(n) e^{-j \frac{2\pi}{N} kn}$$

But length is L.  $\therefore x(n) = 1$

$$= \sum_{n=0}^{L-1} 1 \cdot e^{-j \frac{2\pi}{N} kn}$$

$$\text{WKT, } \sum_{k=0}^N a^k = \frac{1-a^{N+1}}{1-a}$$

$$X(k) = \sum_{k=0}^{L-1} \left( e^{-j \frac{2\pi}{N} k} \right)^n \Rightarrow \frac{\left( 1 - e^{-j \frac{2\pi}{N} k L} \right)^{L-1+1}}{1 - e^{-j \frac{2\pi}{N} k}}$$

$$X(k) = \frac{1 - e^{-j \frac{2\pi}{N} k L}}{1 - e^{-j \frac{2\pi}{N} k}} = \frac{e^{-j \frac{\pi}{N} k L} \left( e^{j \frac{\pi}{N} k L} - e^{-j \frac{\pi}{N} k L} \right)}{e^{-j \frac{\pi}{N} k} \left( e^{j \frac{\pi}{N} k} - e^{-j \frac{\pi}{N} k} \right)} \quad \div 2j$$

$$= e^{-j \frac{\pi}{N} k L} \frac{e^{\frac{\pi}{N} k L} \sin \frac{\pi}{N} k L}{\sin \frac{\pi}{N} k}$$

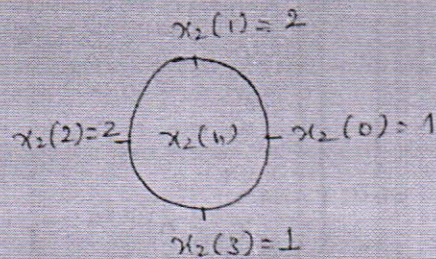
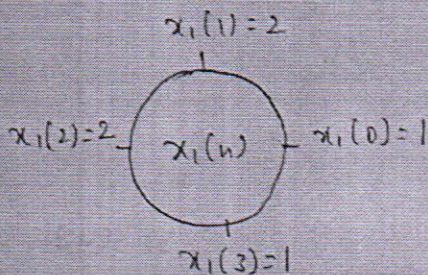
$$X(k) = e^{\frac{\pi}{N} k(L-1)} \frac{\sin \frac{\pi}{N} k L}{\sin \frac{\pi}{N} k} \quad (\propto) \quad e^{-\frac{\pi}{N} k(L-1)} \frac{\sin \frac{\pi}{N} k L}{\sin \frac{\pi}{N} k}$$

2. Determine 4-point Circular Convolution using tabular arrays method for  $x_1(n) = x_2(n) = \{1, 2, 2, 1\}$ .  
Verify your answer by Stackham's method.

Sol<sup>n</sup>:

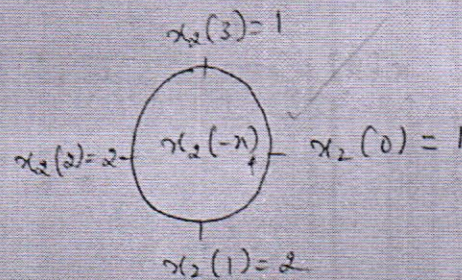
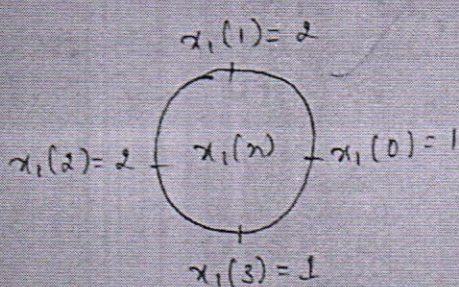
$$x_1(n) = \{1, 2, 2, 1\} \quad x_2(n) = \{1, 2, 2, 1\}$$

$$x_3(m) = \sum_{n=0}^{N-1} x_1(n) x_2((m-n))_N$$



Let  $m=0$ .

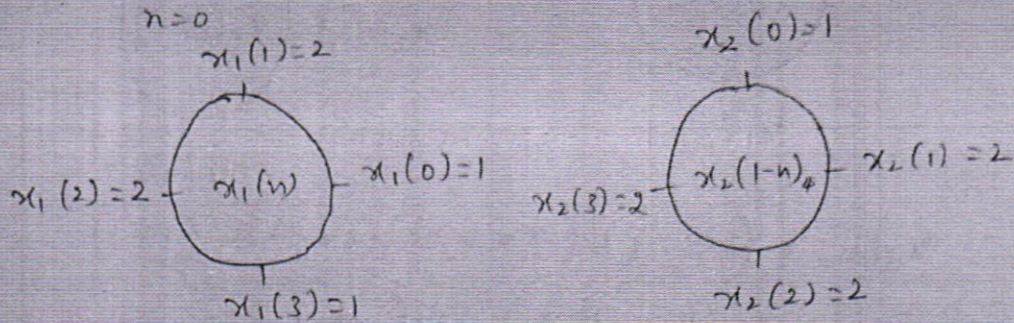
$$x_3(0) = \sum_{n=0}^{4-1=3} x_1(n) x_2((-n))_4$$



$$\begin{aligned} x_3(0) &= (1 \times 1) + (2 \times 1) + (2 \times 2) + (1 \times 2) \\ &= 1 + 2 + 4 + 2 \\ x_3(0) &= 9 \end{aligned}$$

hehen,  $m=1$

$$x_3(1) = \sum_{n=0}^3 x_1(n) x_2((1-n))_4$$



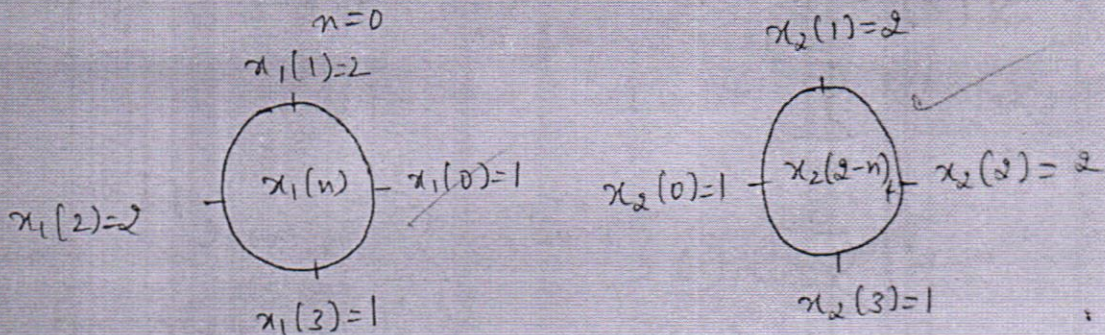
$$= (1 \times 2) + (2 \times 1) + (2 \times 1) + (1 \times 2)$$

$$= 2 + 2 + 2 + 2$$

$$x_3(1) = 8$$

hehen,  $m=2$ .

$$x_3(2) = \sum_{n=0}^3 x_1(n) x_2((2-n))_4$$



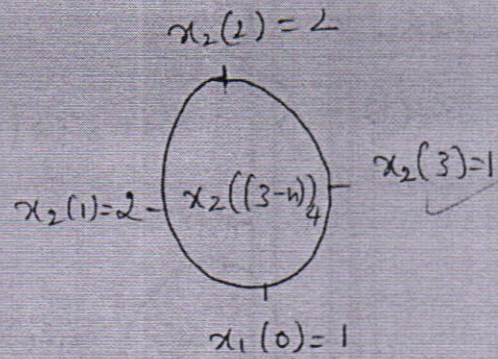
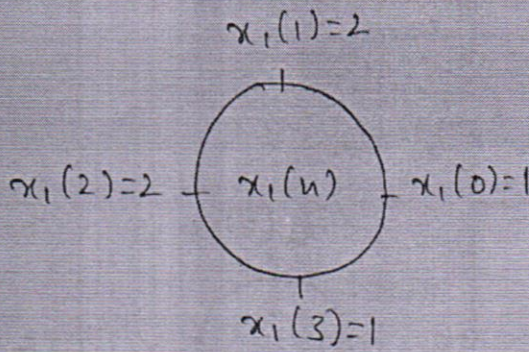
$$x_3(2) = (1 \times 2) + (2 \times 2) + (2 \times 1) + (1 \times 1)$$

$$= 2 + 4 + 2 + 1$$

$$x_3(2) = 9$$

hehen,  $m=3$

$$x_3(3) = \sum_{n=0}^3 x_1(n) x_2((3-n))_4$$



$$x_3(n) = (2 \times 2) + (2 \times 2) + (1 \times 1) + (1 \times 1)$$

$$= 4 + 4 + 1 + 1$$

$$x_3(3) = 10,$$

$$x_3(n) = \{9, 8, 9, 10\}$$

$$x_3(m) = x_1(n) \otimes x_2(n) = \{9, 8, 9, 10\}$$

By Stackham's Method.

$$X_1(K) = \text{DFT} [x_1(n)]$$

$$X_1(K) = \sum_{n=0}^{4-1} x_1(n) e^{-j \frac{2\pi}{N} kn}$$

$$= \sum_{n=0}^3 x_1(n) e^{-j \frac{2\pi}{4} kn} = \sum_{n=0}^3 x_1(n) e^{-j \frac{\pi}{2} kn}$$

$$x_1(k) = x_1(0) e^{-j \frac{\pi}{2} k(0)} + x_1(1) e^{-j \frac{\pi}{2} k(1)} + x_1(2) e^{-j \frac{\pi}{2} k(2)} + x_1(3) e^{-j \frac{\pi}{2} k(3)}$$

$$x_1(k) = 1 + 2e^{-j \frac{\pi k}{2}} + 2e^{-j \pi k} + 1e^{-j \frac{3\pi k}{2}}$$

when,  $k=0$

$$x_1(0) = 1 + 2e^{j0} + 2e^{j0} + e^{j0}$$

$$= 1 + 2 + 2 + 1$$

$$x_1(0) = 6$$

$$k=1$$

$$x_1(1) = 1 + 2e^{-j\frac{\pi}{2}(1)} + 2e^{-j\pi(1)} + e^{-j\frac{3\pi}{2}(1)}$$

$$x_1(1) = 1 + 2[\cos 90^\circ - j\sin 90^\circ] + 2[\cos 180^\circ - j\sin 180^\circ] + [\cos 270^\circ - j\sin 270^\circ]$$

$$x_1(1) = 1 + 2(0 - j) + 2(-1) + -j$$

$$= 1 - 2j - 2 - j \Rightarrow -1 - 3j$$

$$k=2$$

$$x_1(1) = -1 - 3j$$

$$x_1(2) = 1 + 2e^{-j\frac{\pi}{2}(2)} + 2e^{-j\pi(2)} + e^{-j\frac{3\pi}{2}(2)}$$

$$x_1(2) = 1 + 2e^{-j2\pi} + 2e^{-j2\pi} + e^{-j3\pi}$$

$$x_1(2) = 1 + 2[\cos 360^\circ - j\sin 360^\circ] + 2[\cos 720^\circ - j\sin 720^\circ] + [\cos 1080^\circ - j\sin 1080^\circ]$$

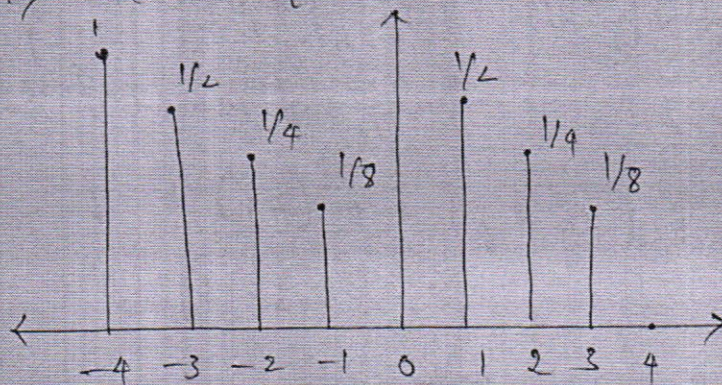
$$x_1(2) = 1 + 2(-1 - 0) + 2[1 - 0] + (-1)$$

$$= 1 - 2 + 2 - 1$$

$$x_1(2) = 0$$

- 6). Given  $x(n) = \left[\frac{1}{2}\right]^n [u(n) - u(n-4)]$  Determine the following without computing 4-point DFT  $X(k)$ .
- i)  $\sum_{k=0}^3 x(k) x^*(k)$  ii) If  $G(k) = W_4^{2k} x(k)$  find  $g(n)$
- iii)  $x(0) + x(2)$

$$\Rightarrow i) x(n) = \{1, 1/2, 1/4, 1/8\}$$



hehen,  $n=0$ ,  $g(0) = x(-2)$

$$g(0) = 1/4$$

$$n=1, g(1) = x(-1) = 1/8$$

$$n=2, g(2) = x(0) = 1$$

$$n=3, g(3) = 1/2$$

$$g(n) = \{1/4, 1/8, 1, 1/2\}$$

ii) A/c to Parseval's Theorem:

$$\frac{1}{N} \sum_{k=0}^{N-1} x(k) x^*(k) = \sum_{n=0}^{N-1} x(n) x^*(n)$$

Given

$$x(n) = \{1, 1/2, 1/4, 1/8\}$$

$$\sum_{n=0}^{4-1} |x(k)|^2 = 4 \sum_{n=0}^3 |x(n)|^2$$

$$\sum_{k=0}^3 |x(k)|^2 = 4 \left\{ |x(0)|^2 + |x(1)|^2 + |x(2)|^2 + |x(3)|^2 \right\}$$

$$= 4 \left\{ 1 + \frac{1}{4} + \frac{1}{16} + \frac{1}{64} \right\}$$

$$= 4 \left\{ \frac{85}{64} \right\}$$

$$= \frac{85}{16}$$

iii)  $x(0) + x(2)$

$$x(k) = \sum_{n=0}^{N-1} x(n) e^{-j \frac{2\pi}{N} kn}$$

$$x(k) = \sum_{n=0}^{4-1} x(n) \cdot e^{-j \frac{2\pi}{4} kn}$$

$$= \sum_{n=0}^3 x(n) e^{-j \frac{\pi}{2} kn} \quad \text{--- (A)}$$

when  $k=0$ ,  $x(0) = \sum_{n=0}^3 x(n) e^0$

$$= x(0) + x(1) + x(2) + x(3)$$

$$= 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8}$$

$$= \frac{15}{8}$$

when  $k=2$ ,  $x(2) = \sum_{n=0}^3 x(n) e^{-j\frac{\pi}{2}(2)n}$

$$x(2) = \sum_{n=0}^3 x(n) \cdot e^{-j\pi n}$$

$$x(2) = x(0) \cdot e^0 + x(1) \cdot e^{-j\pi} + x(2) \cdot e^{-j2\pi} + x(3) \cdot e^{-j3\pi}$$

$$= 1 + \frac{1}{2}(-1) + \frac{1}{4}e^{-j2\pi} + \frac{1}{8}(-1)$$

$$= 1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8}$$

$$= \frac{5}{8}$$

$$x(0) + x(2) = \frac{15}{8} + \frac{5}{8} = \frac{20}{8}$$

3. Determine 4-point DFT of  $x(n) = \{0, 1, 2, 3\}$  hence verify the result by taking IDFT using Linear Transformation.

$\Rightarrow N=4$

$$x(n) = \{0, 1, 2, 3\}$$

$$X(k) = \text{DFT}[x(n)]$$

$$= \sum_{n=0}^{N-1} x(n) \cdot e^{-j\frac{2\pi}{N}kn}$$



But  $N=4$

$$x(k) = \sum_{n=0}^{4-1=3} x(n) e^{-j \frac{2\pi}{4} kn}$$

$$x(k) = \sum_{n=0}^3 x(n) e^{-j \frac{\pi}{2} kn}$$

$$x(k) = x(0)e^0 + x(1)e^{-j \frac{\pi}{2} k(1)} + x(2)e^{-j \frac{\pi}{2} k(2)} + x(3)e^{-j \frac{\pi}{2} k(3)}$$

$$x(k) = (0)(1) + 1e^{-j \frac{\pi}{2} k} + 2e^{-j \pi k} + 3e^{-j \frac{3\pi}{2} k}$$

put  $k=0$

$$x(0) = 1e^{-j \frac{\pi}{2}(0)} + 2e^{-j \pi(0)} + 3e^{-j \frac{3\pi}{2}(0)}$$

$$x(0) = 1 + 2(1) + 3(1) \Rightarrow 6 \Rightarrow \boxed{x(0) = 6}$$

put  $k=1$

$$x(1) = 1e^{-j \frac{\pi}{2}(1)} + 2e^{-j \pi(1)} + 3e^{-j \frac{3\pi}{2}(1)}$$

$$x(1) = (\cos 90^\circ - j \sin 90^\circ) + 2(\cos 180^\circ - j \sin 180^\circ) + 3(\cos 270^\circ - j \sin 270^\circ)$$

$$x(1) = -j + 2 + 3j \Rightarrow \boxed{x(1) = -2 + 2j}$$

put  $k=2$

$$x(2) = 1e^{-j \frac{\pi}{2}(2)} + 2e^{-j \pi(2)} + 3e^{-j \frac{3\pi}{2}(2)}$$

$$x(2) = (\cos 180^\circ - j \sin 180^\circ) + 2(\cos 360^\circ - j \sin 360^\circ) + 3(\cos 540^\circ - j \sin 540^\circ)$$

$$x(2) = -1 + 2 - 3 \Rightarrow \boxed{x(2) = -2}$$

put  $k=3$

$$x(3) = 1e^{-j \frac{\pi}{2}(3)} + 2e^{-j \pi(3)} + 3e^{-j \frac{3\pi}{2}(3)}$$

$$= (\cos 270^\circ - j \sin 270^\circ) + 2(\cos 540^\circ - j \sin 540^\circ) + 3(\cos 810^\circ - j \sin 810^\circ)$$
$$= +j - 2 - 3j \Rightarrow -2 - 2j \Rightarrow -2 - 2j$$

$$x(k) = \{ 6, -2 + 2j, -2, -2 - 2j \}$$

Midpoint, ~~2~~ ~~4~~ ~~2~~

8) Compute 4-point DFT of the Sequence  $x(n) = \{1, 0, 1, 0\}$   
 also find  $y(n)$  if  $x(k) = x((k-2))_4$

$$\Rightarrow X(k) = \sum_{n=0}^{N-1} x(n) e^{-j \frac{2\pi}{N} kn}$$

$$= \sum_{n=0}^{4-1=3} x(n) e^{-j \frac{2\pi}{4} kn}$$

$$= \sum_{n=0}^3 x(n) e^{-j \frac{\pi}{2} kn}$$

$$X(k) = x(0) \cdot e^0 + x(1) \cdot e^{-j \frac{\pi}{2} k(1)} + x(2) \cdot e^{-j \frac{\pi}{2} k(2)} + x(3) \cdot e^{-j \frac{\pi}{2} k(3)}$$

$$X(k) = 1 + e^{-j\pi k} \quad \text{--- (1)}$$

when  $k=0$ ,  $X(0) = 1 + e^0 = 1 + 1 = 2$

$k=1$ ,  $X(1) = 1 + e^{-j\pi} = 1 - 1 = 0$

$k=2$ ,  $X(2) = 1 + e^{-j\pi 2} = 1 + 1 = 2$

$k=3$ ,  $X(3) = 1 + e^{-j\pi 3} = 1 - 1 = 0$

$$X(k) = \{2, 0, 2, 0\}$$

we know frequency shift property.

$$X((k-l)) \longleftrightarrow W_N^{-ln} x(n)$$

(or)

$$X((k-l))_N \longleftrightarrow e^{j \frac{2\pi}{N} ln} x(n)$$

Given:-

$$y(k) = x((k-2))$$

$$y(n) = e^{j\frac{2\pi}{4}nu} x(n)$$

$$y(n) = e^{j\frac{2\pi}{4}(2)n} x(n)$$

$$y(n) = e^{j\pi n} x(n)$$

$$n=0, \quad y(0) = e^0 x(0) = 1$$

$$n=1, \quad y(1) = e^{j\pi} x(1) = 0$$

$$n=2, \quad y(2) = e^{j\pi 2} x(2) = 1$$

$$n=3, \quad y(3) = e^{j\pi 3} x(3) = 0$$

$$y(n) = \{1, 0, 1, 0\}$$

## Assignment - 2

1. Find the o/p.  $y(n)$  of a filter whose impulse response is  $h(n) = \{1, 1, 1\}$  & i/p signal  $x(n) = \{3, -1, 0, 1, 3, 2, 0, 1, 2, 1\}$  using
- i) over-lap-save method
  - ii) over-lap-add method use Circular Convolution

⇒

$$N = 2^M$$

$$N = 2^3$$

$$N = 8$$

$$N = L + M - 1$$

$$8 = L + 3 - 1$$

$$8 = L + 2$$

$$L = 8 - 2$$

$$\boxed{L = 6}$$

⇒

Save Method :-

$$x_1(n) = \{3, -1, 0, 1, 3, 2\} \text{ But } N=8$$

$$x_1(n) = \{0, 0, 3, -1, 0, 1, 3, 2\}$$

$$x_2(n) = \{3, 2, 0, 1, 2, 1, 0, 0\}$$

$$h(n) = \{1, 1, 1, 0, 0, 0, 0, 0\}$$

$$\begin{bmatrix} y_1(0) \\ y_1(1) \\ y_1(2) \\ y_1(3) \\ y_1(4) \\ y_1(5) \\ y_1(6) \\ y_1(7) \end{bmatrix} = \begin{bmatrix} 0 & 2 & 3 & 1 & 0 & -1 & 3 & 0 \\ 0 & 0 & 2 & 3 & 1 & 0 & -1 & 3 \\ 3 & 0 & 0 & 2 & 3 & 1 & 0 & -1 \\ -1 & 3 & 0 & 0 & 2 & 3 & 1 & 0 \\ 0 & -1 & -1 & 0 & 0 & 2 & 3 & 1 \\ 1 & 0 & 3 & 0 & 0 & 2 & 3 & 1 \\ 3 & 1 & 0 & -1 & 3 & 0 & 0 & 2 \\ 2 & 3 & 1 & 0 & -1 & 3 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \\ 3 \\ 2 \\ 2 \\ 0 \\ 4 \\ 6 \end{bmatrix}$$

$$y_1(n) = \{5, 2, 3, 2, 2, 0, 4, 6\}$$

$$\begin{bmatrix} y_2(0) \\ y_2(1) \\ y_2(2) \\ y_2(3) \\ y_2(4) \\ y_2(5) \\ y_2(6) \\ y_2(7) \end{bmatrix} = \begin{bmatrix} 3 & 0 & 0 & 1 & 2 & 1 & 0 & 2 \\ 2 & 3 & 0 & 0 & 1 & 2 & 1 & 0 \\ 0 & 2 & 3 & 0 & 0 & 1 & 2 & 1 \\ 1 & 0 & 2 & 3 & 0 & 0 & 1 & 2 \\ 2 & 1 & 0 & 2 & 3 & 0 & 0 & 1 \\ 1 & 2 & 1 & 0 & 2 & 3 & 0 & 0 \\ 0 & 1 & 2 & 1 & 0 & 2 & 3 & 0 \\ 0 & 0 & 1 & 2 & 1 & 0 & 2 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \\ 5 \\ 3 \\ 4 \\ 3 \\ 3 \\ 1 \end{bmatrix}$$

$$y_2(n) = \{3, 5, 5, 3, 3, 4, 3, 1\}$$

$$y_1(n) = \{\cancel{5}, \cancel{2}, 3, 2, 2, 0, 4, 6\}$$

$$y_2(n) = \{\cancel{3}, \cancel{5}, 5, 3, 3, 4, 3, 1\}$$

$$y(n) = \{3, 2, 2, 0, 4, 6, 5, 3, 3, 4, 3, 1\}$$

Add Method :-  $x_1(n) = \{3, -1, 0, 1, 3, 2, 0, 0\}$

$$x_2(n) = \{0, 1, 2, 1, 0, 0, 0, 0\}$$

$$w(n) = \{1, 1, 1, 0, 0, 0, 0, 0\}$$

$$\begin{bmatrix} y_1(0) \\ y_1(1) \\ y_1(2) \\ y_1(3) \\ y_1(4) \\ y_1(5) \\ y_1(6) \\ y_1(7) \end{bmatrix} = \begin{bmatrix} 3 & 0 & 0 & 2 & 3 & 1 & 0 & 0 \\ -1 & 3 & 0 & 0 & 2 & 3 & 1 & 0 \\ 0 & -1 & 3 & 0 & 0 & 2 & 3 & 1 \\ 1 & 0 & -1 & 3 & 0 & 0 & 2 & 3 \\ 3 & -1 & 0 & -1 & 3 & 0 & 0 & 2 \\ 2 & 3 & 1 & 0 & -1 & 3 & 0 & 0 \\ 0 & 2 & 3 & 1 & 0 & -1 & 3 & 0 \\ 0 & 0 & 2 & 3 & 1 & 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 2 \\ 0 \\ 4 \\ 6 \\ 5 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} y_2(0) \\ y_2(1) \\ y_2(2) \\ y_2(3) \\ y_2(4) \\ y_2(5) \\ y_2(6) \\ y_2(7) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 2 & 1 \\ 1 & 0 & 0 & 0 & 6 & 0 & -1 & 2 \\ 2 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 2 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 2 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 6 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 3 \\ 4 \\ 3 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$$y_2(n) = \{0, 1, 3, 4, 3, 1, 0, 0\}$$

$$y_1(n) = \begin{bmatrix} 3 & 2 & 2 & 0 & 4 & 6 & 5 & 2 \end{bmatrix}$$

$$y_2(n) = \begin{bmatrix} 0 & 1 & 3 & 4 & 3 & 1 & 0 & 0 \end{bmatrix}$$

$$y(n) = \{3, 2, 2, 0, 4, 6, 5, 3, 3, 4, 3, 1, \dots\}$$

2) Derive an Expression for decimation in time (DIT) FFT algorithm.

$$x(n) = \{x(0), x(1), x(2), \dots, x(N-1)\}$$

Even indexed  $\{x(0), x(2), x(4), \dots, x(N-2)\}$

odd indexed  $\{x(1), x(3), x(5), \dots, x(N-1)\}$

WRT. DFT eqn.

$$X(K) = \sum_{n=0}^{N-1} x(n) W_N^{nK} \quad \text{--- (1)}$$

Separate  $x(n)$  into even & odd values of  $x(n)$

$$X(K) = \sum_{n=\text{even}} x(n) W_N^{nK} + \sum_{n=\text{odd}} x(n) W_N^{nK}$$

$$X(K) = \sum_{n=0}^{\frac{N}{2}-1} x_e(2n) W_N^{2nK} + \sum_{n=0}^{\frac{N}{2}-1} x_o(2n+1) W_N^{(2n+1)K}$$

$$W_N^{2nK} = \frac{W_N^{nK}}{2} \Rightarrow W_N^{nK} = e^{-j \frac{2\pi}{N} nK}$$

$$W_N^{2nK} = e^{-j \frac{2\pi}{N} 2nK} = e^{-j \frac{2\pi}{N/2} nK} = W_{N/2}^{nK}$$

$$X(K) = \underbrace{\sum_{n=0}^{\frac{N}{2}-1} x_e(2n) W_{N/2}^{nK}}_{\frac{N}{2} \text{ point of DFT of even Seq.}} + W_N^K \underbrace{\sum_{n=0}^{\frac{N}{2}-1} x_o(2n+1) W_{N/2}^{nK}}_{\frac{N}{2} \text{ DFT of odd Seq.}}$$

$\frac{N}{2}$  point of DFT of even Seq.       $\frac{N}{2}$  DFT of odd Seq.

$$X(K) = G(K) + W_N^K H(K) \rightarrow \text{(A)}$$

where  $G(K) = \sum_{n=0}^{\frac{N}{2}-1} x_e(2n) W_{N/2}^{nK}$

$$H(K) = \sum_{n=0}^{\frac{N}{2}-1} x_o(2n+1) W_{N/2}^{2nK}$$

Since  $G(K)$  &  $H(K)$  are periodic with period  $\frac{N}{2}$

$$\therefore G\left(k + \frac{N}{2}\right) = G(k) \quad \& \quad H\left(k + \frac{N}{2}\right) = H(k)$$

So, eqn (A) becomes

$$x\left(k + \frac{N}{2}\right) = G\left(k + \frac{N}{2}\right) \omega_N^{k + N/2} H\left(k + \frac{N}{2}\right)$$

$$x\left(k + \frac{N}{2}\right) = G(k) + \omega_N^k \omega_N^{N/2} H(k)$$

$$\omega_N^{N/2} = e^{-j \cdot 2\pi/N \cdot (N/2)} = e^{-j\pi} = (-1)$$

$$\therefore x\left(k + \frac{N}{2}\right) = G(k) - \omega_N^k H(k) \rightarrow \textcircled{B} \quad 0 \leq k \leq \frac{N}{2} - 1$$

Consider  $N=8$ . then eqn (A)

$$\text{when } k=0, \quad x(0) = G(0) + \omega_8^0 H(0)$$

$$k=1, \quad x(1) = G(1) + \omega_8^1 H(1)$$

$$k=2, \quad x(2) = G(2) + \omega_8^2 H(2)$$

$$k=3, \quad x(3) = G(3) + \omega_8^3 H(3)$$

from eqn (B) i.e.,

$$x\left(k + \frac{N}{2}\right) = G(k) - \omega_N^k H(k)$$

$$\text{when } k=0, \quad x\left(\frac{N}{2}\right) = G(0) - \omega_8^0 H(0)$$

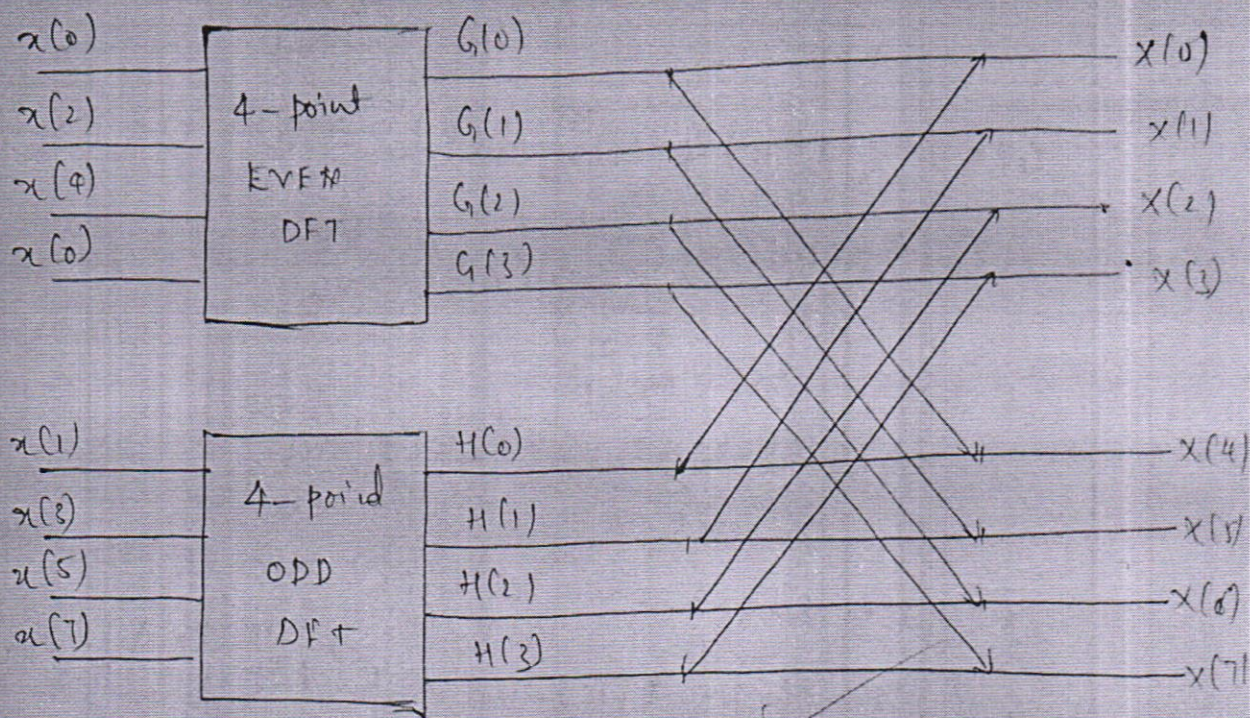
$$x(4) = G(0) - \omega_8^0 H(0)$$

$$k=1, \quad x(5) = G(1) - \omega_8^1 H(1)$$

$$k=2, \quad x(6) = G(2) - \omega_8^2 H(2)$$

$$k=3, \quad x(7) = G(3) - \omega_8^3 H(3)$$





I<sup>st</sup> Stage decomposition

II<sup>nd</sup> Stage decomposition:-

$$\left\{ G(k) = \sum_{n=0}^{N/2-1} x_e(2n) \omega_{N/2}^{nk} \right\}$$

$$G(k) = \sum_{n=0}^{N/4-1} x_{ee}(2n) \omega_{N/2}^{nk} + \sum_{n=0}^{N/4-1} x_{eo}(2n) \omega_{N/2}^{nk}$$

Substituting  $n=2u$  for 1<sup>st</sup> term &  $n=2u+1$  for

II<sup>nd</sup> term.

$$G(k) = \sum_{u=0}^{N/4-1} x_{ee}(4u) \omega_{N/2}^{2uk} + \sum_{u=0}^{N/4-1} x_{eo}[2(2u+1)] \omega_{N/2}^{(2u+1)k}$$

$$G(k) = \sum_{u=0}^{N/4-1} x_{ee}(4u) \omega_{N/4}^{uk} + \sum_{u=0}^{N/4-1} x_{eo}[4u+2] \omega_{N/2}^{2uk} \omega_{N/2}^k$$

$$G(k) = \sum_{n=0}^{N/4-1} x(4n) W_{N/4}^{nk} + W_{N/2}^k \sum_{n=0}^{N/4-1} x(4n+2) W_{N/4}^{nk}$$

$$G(k) = A(k) + W_{N/2}^{nk} B(k) \rightarrow \textcircled{C} \quad 0 \leq k \leq \frac{N}{4} - 1$$

$$\text{where } A(k) = \sum_{n=0}^{N/4-1} x(4n) W_{N/4}^{nk}$$

$$B(k) = \sum_{n=0}^{N/4-1} x(4n+2) W_{N/4}^{nk}$$

$A(k)$  &  $B(k)$  are periodic with the period  $N/4$

$$A\left(k + \frac{N}{4}\right) = A(k) \quad B\left(k + \frac{N}{4}\right) = B(k)$$

Eq<sup>n</sup>  $\textcircled{C}$ .

$$G\left(k + \frac{N}{4}\right) = A\left(k + \frac{N}{4}\right) + W_{N/2}^{k + N/4} B\left(k + \frac{N}{4}\right) \quad 0 \leq k \leq \frac{N}{4} - 1$$

$$\therefore G\left(k + \frac{N}{4}\right) = A(k) + W_{N/2}^k W_{N/2}^{N/4} B(k)$$

$$W_{N/2}^{N/4} = e^{-j \frac{2\pi}{N/2} \left[ \frac{N}{4} \right]} = e^{-j \frac{4\pi}{N} \left[ \frac{N}{4} \right]} = e^{-j\pi} = -1$$

$$G\left(k + \frac{N}{4}\right) = A(k) - W_{N/2}^k B(k) \rightarrow \textcircled{D} \quad 0 \leq k \leq \frac{N}{4} - 1$$

from eq<sup>n</sup>  $\textcircled{C}$ .  
where

$$k=0, \quad G(2) = A(0) + W_4^0 B(0)$$

$$k=1, \quad G(3) = A(1) + W_4^1 B(1)$$

$$k=2, \quad G(4) = A(2) + W_4^2 B(2)$$

$$k=3, \quad G(5) = A(3) + W_4^3 B(3)$$

$$H(k) = \sum_{n=0}^{N/2-1} x_0(2n+1) \omega_{N/2}^{nk} \quad 0 \leq k \leq N/2 - 1$$

Decimation  $H(k)$  into even & odd sequence  $(2n+1)k$

$$H(k) = \sum_{n=0}^{N/4-1} x[2(2n+1)] \omega_{N/2}^{2nk} + \sum_{n=0}^{N/4-1} [2(2n+1)+1] \omega_{N/2}^{(2n+1)k}$$

$$= \sum_{n=0}^{N/4-1} x(4n+1) \omega_{N/4}^{nk} + \sum_{n=0}^{N/4-1} x(4n+2+1) \omega_{N/2}^{2nk} \omega_{N/2}^k$$

$$H(k) = \sum_{n=0}^{N/4-1} x(4n+1) \omega_{N/4}^{nk} + \omega_{N/8}^k \sum_{n=0}^{N/4-1} x(4n+3) \omega_{N/4}^{nk}$$

$$H(k) = C(k) + \omega_{N/2}^k D(k)$$

where,  $k=0$ ,  $H(0) = C(0) + \omega_4^0 D(0)$

$k=1$ ,  $H(1) = C(1) + \omega_4^1 D(1)$

$k=2$ ,  $H(2) = C(2) + \omega_4^2 D(2)$

$k=3$ ,  $H(3) = C(3) + \omega_4^3 D(3)$

$C(k)$  &  $D(k)$  are periodic with  $N/4$ .

$$C(k + N/4) = C(k) \quad D(k + N/4) = D(k)$$

$$\therefore H(k + \frac{N}{4}) = C(k + \frac{N}{4}) + \omega_{N/2}^{k + N/4} D(k + \frac{N}{4})$$

$$H(k + \frac{N}{4}) = C(k) + \omega_{N/2}^{N/4} \omega_{N/2}^k D(k)$$

$$\omega_{N/2}^{N/4} = e^{-j \frac{2\pi}{N/2} \cdot \frac{N}{4}} = e^{-j \frac{2\pi}{N} \cdot \frac{N}{4}} = e^{-j\pi} = -1$$

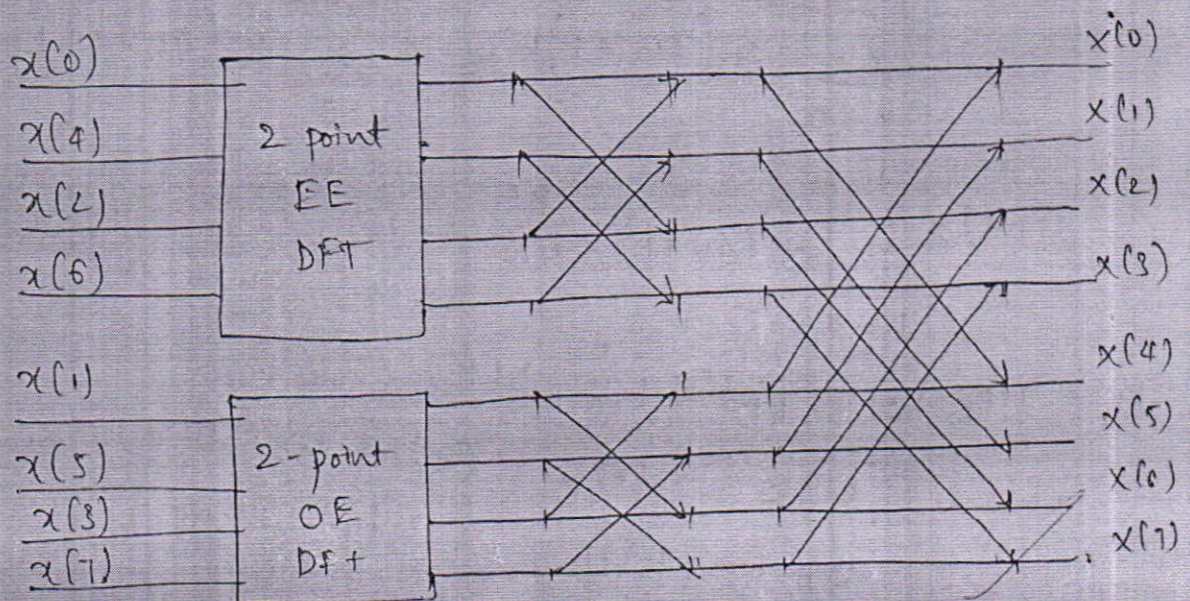
$$H\left(k + \frac{N}{4}\right) = C(k) - W_4^k D(k)$$

$$k=0, H(2) = C(0) - W_4^0 D(0)$$

$$k=1, H(3) = C(1) - W_4^1 D(1)$$

$$k=2, H(4) = C(2) - W_4^2 D(2)$$

$$k=3, H(5) = C(3) - W_4^3 D(3)$$



II-Stage deComposition

Stage 03:  $W_N^k, A(k) = \sum_{n=0}^{N/4-1} x(4n) W_{N/4}^{nk} \left\{ \frac{N}{4} = \frac{8}{4} = 2 \right\}$

$$A(k) = \sum_{n=0}^1 x(4n) W_2^{nk}$$

$$A(k) = x(0) \cdot W_2^0 + x(4) W_2^k \quad \left\{ W_2^0 = e^{-j \frac{2\pi}{2} (0)} = e^0 = 1 \right.$$

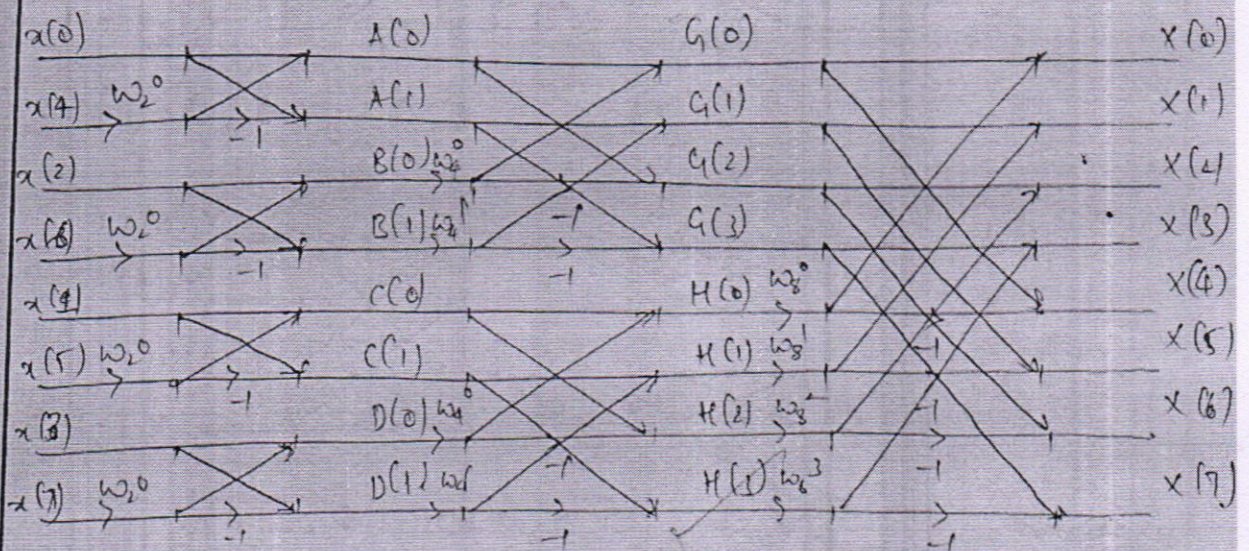
when  $k=0, A(0) = x(0) + W_2^0 x(4)$

$k=1, A(1) = x(0) + x(4)$

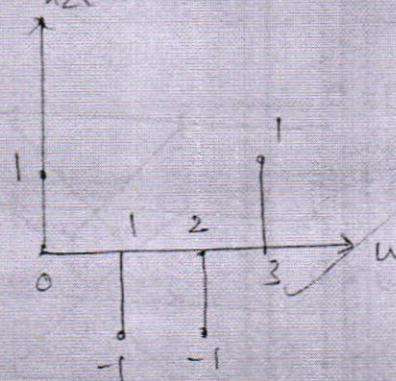
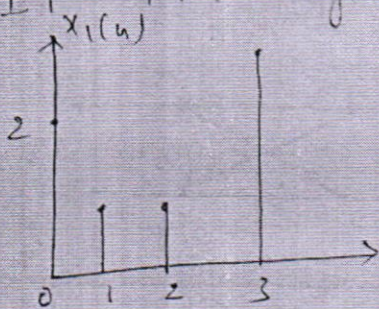
$$K=1, \quad A(1) = x(0) + \omega_2^1 x(4)$$

$$A(1) = x(0) - \omega_2^0 x(4)$$

$$K=2, \quad A(2) = x(0) + \omega_2^2 x(4)$$



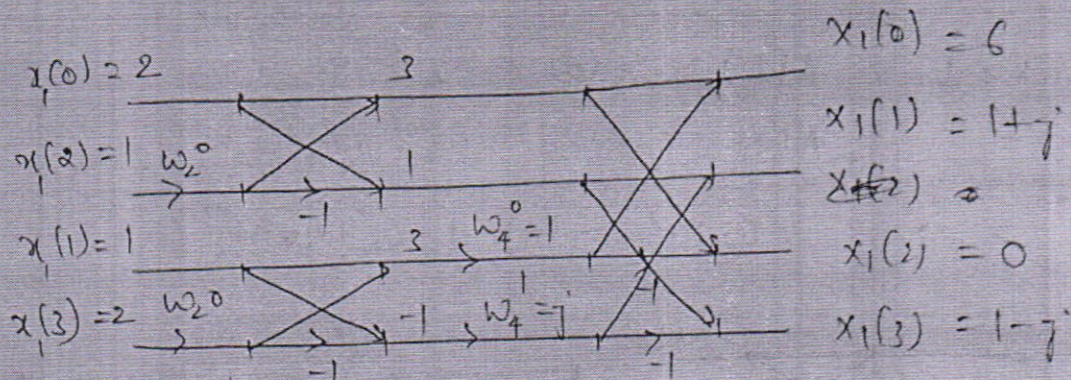
3. Given the sequences  $x_1(n)$  &  $x_2(n)$  below. Compute the Circular Convolution  $x_1(n) \otimes x_2(n)$  for  $N=4$  use DIT-FFT algorithm.



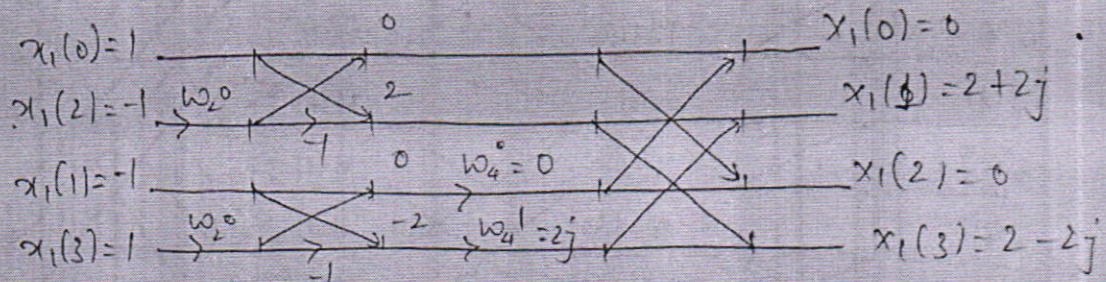
Sol<sup>y</sup>:-

$$x_1(n) = \{2, 1, 1, 2\}$$

$$x_2(n) = \{1, -1, -1, 1\}$$



$$x_1(k) = \{ 6, 1+j, 0, 1-j \}$$

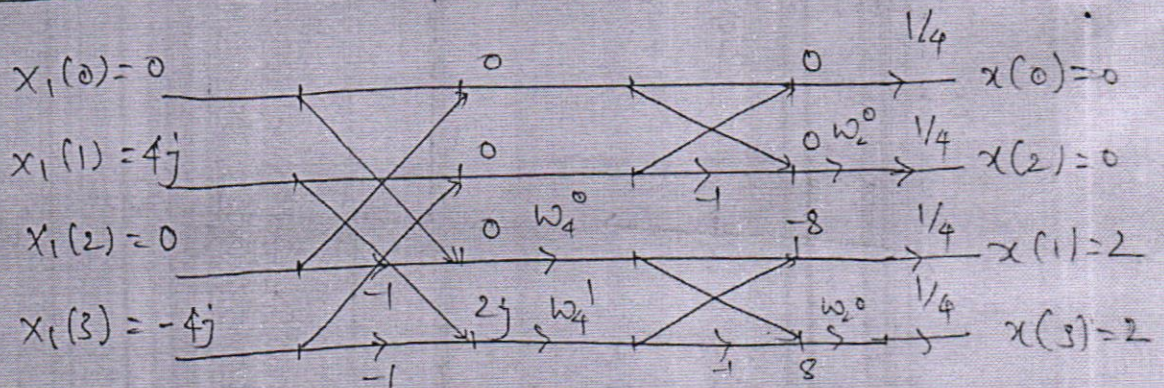


$$x_2(k) = \{ 0, 2+2j, 0, 2-2j \}$$

$$x_3(k) = x_1(k) x_2(k)$$

$$= \{ 6, 1+j, 0, 1-j \} \{ 0, 2+2j, 0, 2-2j \}$$

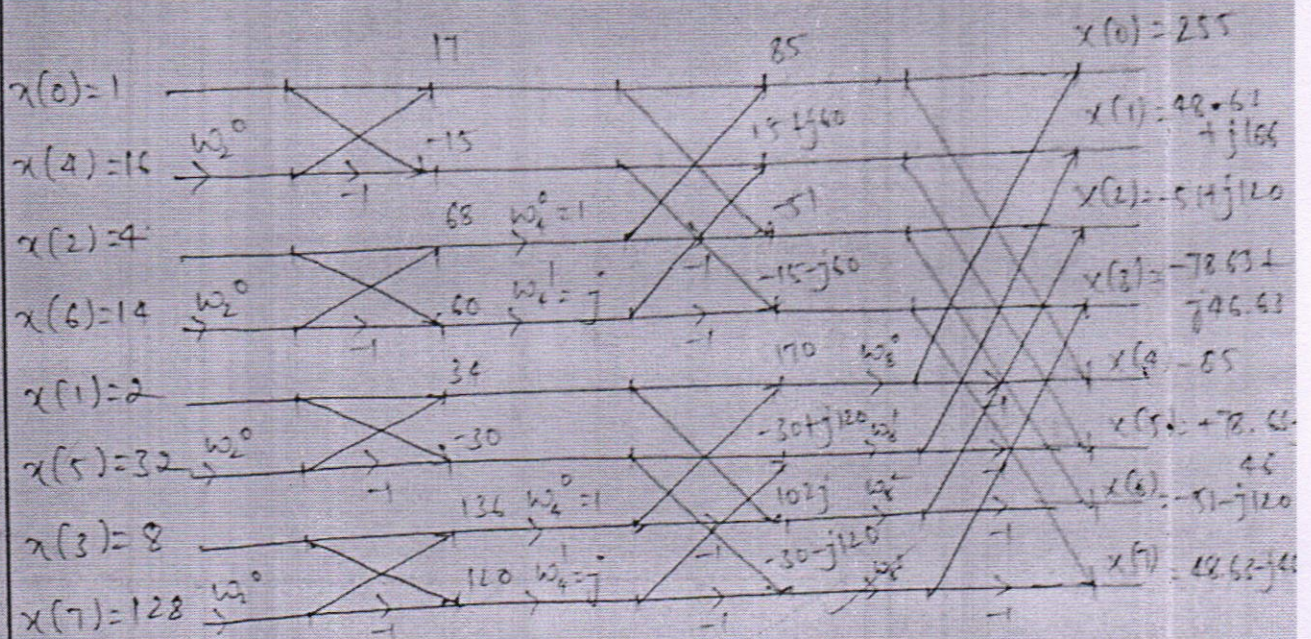
### IDFT



$$x_1(u) \otimes x_2(u) = \{ 0, -2, 0, 2 \}$$

4) Given  $x(n) = a^n$  &  $N=8$ . Find  $X(k)$  using DIF-FFT algorithm.

$\Rightarrow x(n) = \{1, 2, 4, 8, 16, 32, 64, 128\}$



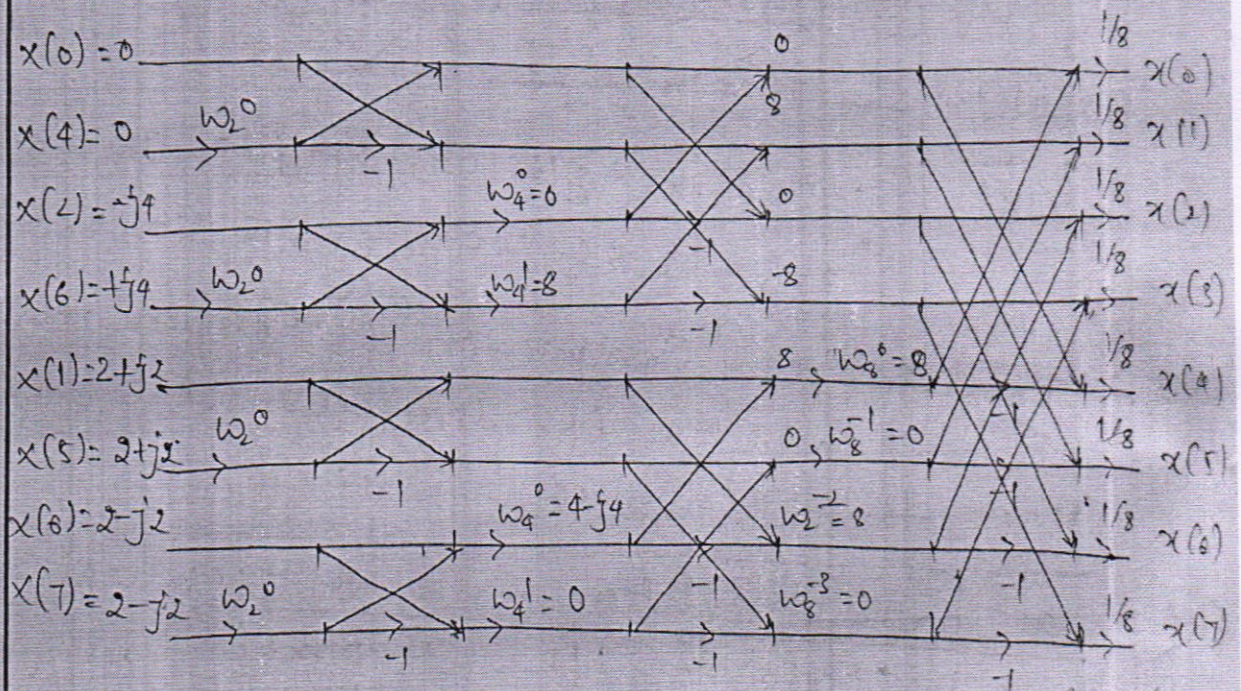
$X(k) = \{255, 48.63 + j166.05, -51 + j120, -78.63 + j46.63, -85, -78.63 - j46.63, -51 - j120, 48.63 - j166.05\}$

5.) Tabulate the Comparison of Complex multiplication and addition for direct computation of DFT. Also the FFT algorithm for  $N=16, 32, 128$ .

No.	No. of addition		No. of Multiplication	
	Direct DFT $(N(N-1))$	FFT $N \log_2 N$	Direct DFT $N^2$	$\frac{N}{2} \log_2 N$
16	240	64	256	32
32	992	166	1024	80
128	16256	896	16384	448

6) first five. Samples of the 8-point DFT of a real valued Sequence is given by  $x(0)=0$ ,  $x(1)=2+j2$ ,  $x(2)=-j4$ ,  $x(3)=2-j2$ ,  $x(4)=0$ . Determine the Remaining point hence find the original Sequence  $x(n)$ . using DIF-FFT.

$$\Rightarrow \begin{aligned} x(5) &= x^*(3) = 2+j2 \\ x(6) &= x^*(2) = -j4 \\ x(7) &= x^*(1) = 2-j2 \end{aligned}$$



$$x(n) = \{1, 1, -1, -1, -1, 1, +1, -1\}$$



## Assignment - 03

1) Direct form I, II, Cascade & Parallel form for the given T.F.  $H(z) = \frac{8z^3 - 4z^2 + 4z - 2}{(z - 1/2)(z^2 - z + 1/4)}$

Sol<sup>n</sup>: Numerator.  $8z^3 - 4z^2 + 4z - 2 = 0$  Can be expressed as

when  $z=0, \Rightarrow 2$

when  $z = 1/2 \Rightarrow 8(1/2)^3 - 4(1/2)^2 + 4(1/2) - 2 = 0$

$8(1/8) - 4(1/4) + 2 - 2 = 0$

$1 - 1 + 2 - 2 = 0$

$\frac{1}{2} = 0.5$

8	-4	4	-2
0	4	0	2

$8 \quad 0 \quad 4 \quad 0 \Rightarrow 8z^2$

$z = \frac{1}{2} \quad 8z^2 + 4$

$(z - 1/2)(8z^2 + 4) = 0$

$(z - \frac{1}{2})$

$8z^3 + 4z - 4z^2 - 2 = 0$

$8z^3 - 4z^2 + 4z - 2 = 0$

$(z - 1/2)(8z^2 + 4)$

$(z - 1/2)4(2z^2 + 1)$

$4(z - 1/2)(2z^2 + 1)$

$2(2z - 1)(2z^2 + 1)$

$$H(z) = \frac{z(2z-1)}{(z-1/4)} \cdot \frac{2z^2+1}{(z^2-z+1/2)} = \frac{z}{z} \cdot \frac{z^2}{z^2}$$

$$H(z) = \frac{z(2-z^{-1})}{(1-1/4z^{-1})} \cdot \frac{2+z^{-2}}{(1-z^{-1}+1/2z^{-2})}$$

$$H_1(z) \quad H_2(z)$$

$$\frac{w(z)}{x(z)} \cdot \frac{y(z)}{w(z)} = \frac{2}{(1-1/4z^{-1})} \cdot \frac{(2-z^{-1})}{1}$$

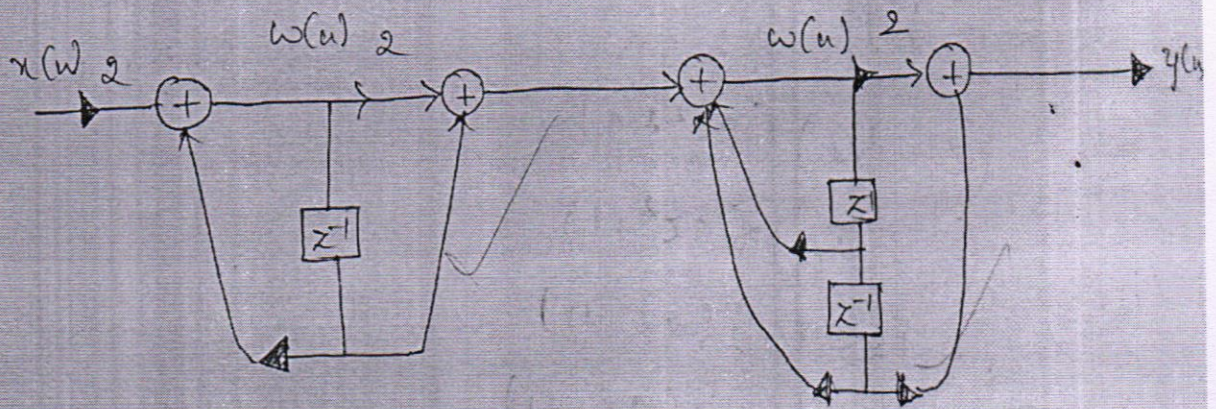
$\frac{w(z)}{x(z)} = \frac{2}{1-1/4z^{-1}}$	$\frac{y(z)}{w(z)} = \frac{2-z^{-1}}{1}$	$\frac{w(z)}{y(z)} \cdot \frac{y(z)}{w(z)} = \frac{1}{(1-z^{-1}+1/2z^{-2})}$
$w(z) - \frac{1}{4}z^{-1}w(z) = 2x(z)$	$y(z) = 2w(z) - z^{-1}w(z)$	$\frac{w(z)}{x(z)} = \frac{1}{1-z^{-1}+1/2z^{-2}}$
$w(n) = 2x(n) + \frac{1}{4}w(n-1)$	$y(n) = 2w(n) - w(n-1)$	

$$w(z) - z^{-1}w(z) + \frac{1}{2}z^{-2}w(z) = x(z)$$

$$w(n) = x(n) + w(n-1) - \frac{1}{2}w(n-2)$$

$$Y(z) = 2W(z) + z^{-2}W(z)$$

$$y(n) = 2w(n) + w(n-2)$$



parallel form:

$$H(z) = \frac{8z^3 - 4z^2 + 4z - 2}{(z - 1/4)(z^2 - z + 1/2)}$$

$$\begin{aligned} (z - 1/4)(z^2 - z + 1/2) &= z^3 - z^2 + \frac{1}{2}z - \frac{1}{4}z^2 + \frac{1}{4}z - \frac{1}{8} \\ &= z^3 - (1 + \frac{1}{4})z^2 + (\frac{1}{2} + \frac{1}{4})z - \frac{1}{8} \\ &= z^3 - \frac{5}{4}z^2 + \frac{3}{4}z - \frac{1}{8} \end{aligned}$$

$$H(z) = \frac{8z^3 - 4z^2 + 4z - 2}{z^3 - \frac{5}{4}z^2 + \frac{3}{4}z - \frac{1}{8}}$$

Direct form I:

$$H(z) = \frac{8z^3 - 4z^2 + 4z - 2}{(z - 1/4)(z^2 - z + 1/2)} \begin{array}{l} \div z^3 \\ \div z^2 \end{array}$$

$$H(z) = \frac{8 - 4z^{-1} + 4z^{-2} - 2z^{-3}}{\left(1 - \frac{1}{4}z^{-1}\right)\left(1 - z^{-1} + \frac{1}{2}z^{-2}\right)}$$

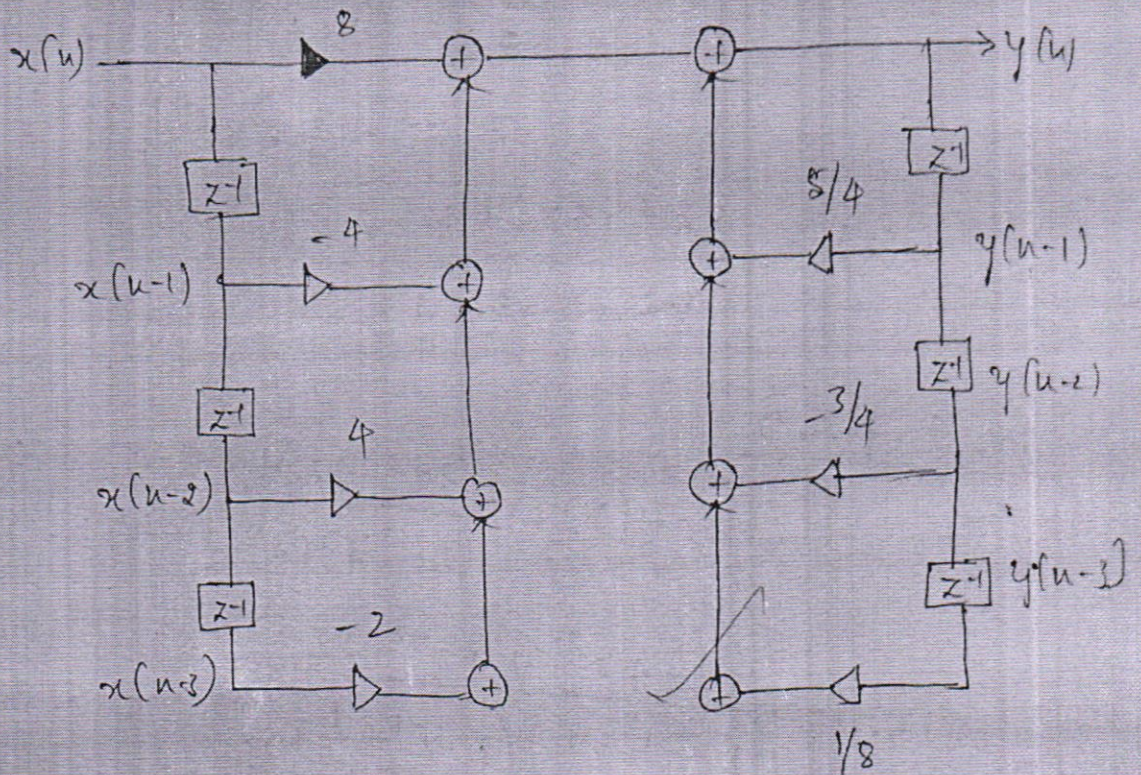
$$H(z) = \frac{8 - 4z^{-1} + 4z^{-2} - 2z^{-3}}{1 - z^{-1} + \frac{1}{2}z^{-2} - \frac{1}{4}z^{-1} + \frac{1}{4}z^{-2} - \frac{1}{8}z^{-3}}$$

$$\frac{Y(z)}{X(z)} = \frac{8 - 4z^{-1} + 4z^{-2} - 2z^{-3}}{1 - \frac{5}{4}z^{-1} + \frac{3}{4}z^{-2} - \frac{1}{8}z^{-3}}$$

$$Y(z) \left[ 1 - \frac{5}{4}z^{-1} + \frac{3}{4}z^{-2} - \frac{1}{8}z^{-3} \right] = 8X(z) - 4z^{-1}X(z) + 4z^{-2}X(z) - 2z^{-3}X(z)$$

Take Inverse Z-transform.

$$y(n) = 8x(n) - 4x(n-1) + 4x(n-2) - 2x(n-3) + \frac{5}{4}y(n-1) - \frac{3}{4}y(n-2) + \frac{1}{8}y(n-3)$$



Direct form II :-

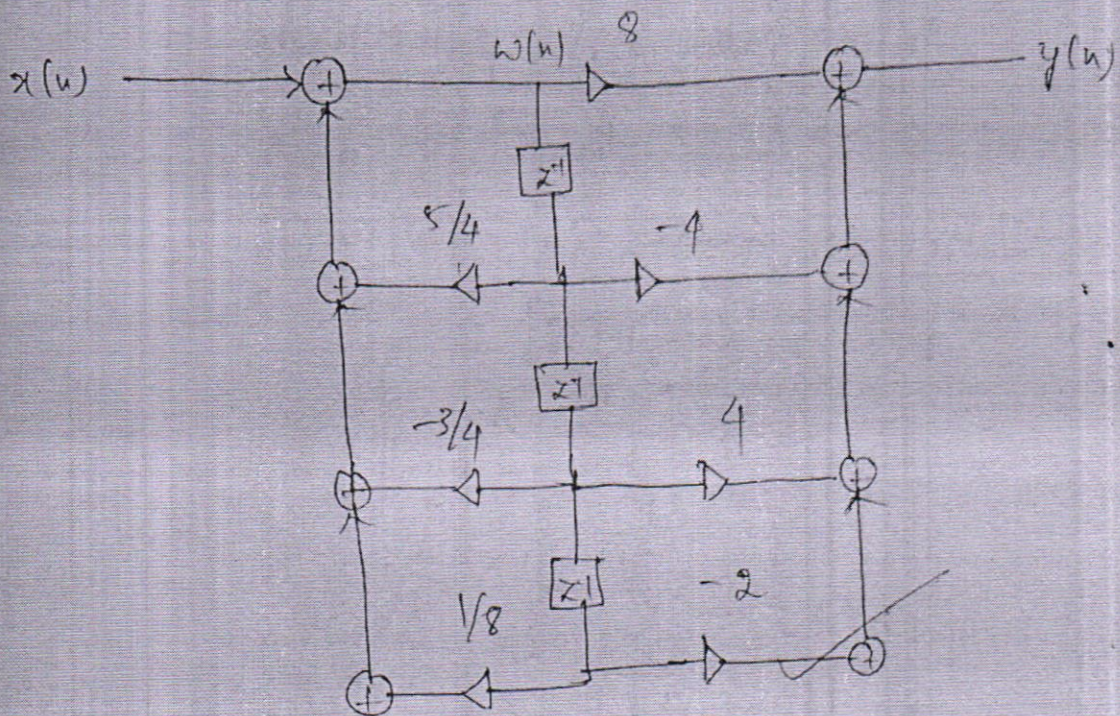
$$\frac{W(z)}{X(z)} \cdot \frac{Y(z)}{W(z)} = \frac{1}{1 - \frac{5}{4}z^{-1} + \frac{3}{4}z^{-2} - \frac{1}{8}z^{-3}} \cdot \frac{8 + 4z^{-1} + 4z^{-2} - 2z^{-3}}{1}$$

$$W(z) - \frac{5}{4}z^{-1}W(z) + \frac{3}{4}z^{-2}W(z) - \frac{1}{8}z^{-3}W(z) = X(z)$$

$$w(n) = x(n) + \frac{5}{4}w(n-1) - \frac{3}{4}w(n-2) + \frac{1}{8}w(n-3)$$

$$Y(z) = 8W(z) - 4z^{-1}W(z) + 4z^{-2}W(z) - 2z^{-3}W(z)$$

$$y(n) = 8w(n) - 4w(n-1) + 4w(n-2) - 2w(n-3)$$



2) Impulse Invariant method.

Sol<sup>n</sup> In this technique, the derived impulse response of the digital filter is obtained by uniformly sampling the impulse response of the equivalent analog filter, putting the  $t = nT$ .

$$h(n) = h_a(nT) \quad n = 0, 1, 2, 3, \dots$$

take Laplace transform for  $h(t)$

$$H(s) = \mathcal{L}[h(t)]$$

$$H(s) = \sum_{k=1}^N \frac{A_k}{s - p_k} \rightarrow \textcircled{1}$$

$$H(s) = \frac{A_1}{s - p_1} + \frac{A_2}{s - p_2} + \dots + \frac{A_N}{s - p_N}$$

$A_k \rightarrow$  Gain factor for  $k^{\text{th}}$  pole.

$s \rightarrow p_k$  is the  $k^{\text{th}}$  pole

Take the inverse L.T. for eqn (1)

$$h(t) = \mathcal{L}^{-1} [H(s)]$$
$$= \mathcal{L}^{-1} \left[ \sum_{k=1}^N \frac{A_k}{s - p_k} \right]$$

$$h(t) = \sum_{k=1}^N \mathcal{L}^{-1} \frac{A_k}{s - p_k}$$

$$h(t) = \sum_{k=1}^N A_k e^{p_k t} \rightarrow (2)$$

The impulse response of the digital filter is obtained by uniformly sampling the impulse response of the analog filter.

i.e.,  $h(n) = h(t) \Big|_{t=nT}$

$$= \sum_{k=1}^N A_k e^{p_k nT}$$

$$h(n) = \sum_{k=1}^N A_k p_k nT \quad - (3)$$

eqn ③ Can. be obtained by taking the z-transform

$$\text{i.e., } H(z) = \mathcal{Z}[h(n)]$$

$$= \sum_{n=-\infty}^{\infty} h(n) z^{-n}$$

$$= \sum_{n=0}^{\infty} \sum_{k=1}^N A_k e^{p_k n T} z^{-n}$$

$$= \sum_{k=1}^N \sum_{n=0}^{\infty} [A_k [e^{p_k T} z^{-1}]^n]$$

$$= \sum_{k=1}^N A_k \sum_{n=0}^{\infty} [e^{p_k T} z^{-1}]^n \checkmark$$

A/c. to STD. Summation i.e.,

$$\sum_{n=0}^{\infty} a^n = \frac{1}{1-a}$$

$$H(z) = \sum_{k=1}^N A_k \frac{1}{1 - e^{p_k T} z^{-1}} \checkmark \rightarrow \textcircled{4}$$

Comparing eqn ① & ③. i.e.  $H(s)$  &  $H(z)$

$$H(s) = \sum_{k=1}^N \frac{A_k}{s - p_k} \quad \text{and} \quad H(z) = \sum_{k=1}^N \frac{A_k}{1 - e^{p_k T} z^{-1}}$$

we say that  $\frac{1}{s - p_k} \rightarrow \frac{1}{1 - e^{p_k T} z^{-1}} \rightarrow (5)$

where  $T \rightarrow$  Sampling time period

3) problem. by using. impulse invariant.

T.F  $H(s) = \frac{1}{s^2 + \sqrt{2}s + 1}$ . To find assume that

3dB. Cut off frequency. of 150 KHz and a Sampling frequency of 1.28 KHz.

$\Rightarrow$  Given.  $f_c = 150 \text{ KHz}$   $\rightarrow \omega_c = 2\pi f_c = 300\pi \text{ rad/Sec}$

Sampling period  $T = \frac{1}{f_s} = 7.8125 \times 10^{-4} \text{ Sec.}$

$$H(s) = H_2(s) \Bigg|_{s \rightarrow \frac{s}{\omega_c}} = H_2(s) \Bigg|_{s \rightarrow \frac{s}{300\pi}}$$

$$H_a(s) = \frac{1}{\left(\frac{s}{300\pi}\right)^2 + \sqrt{2} \left(\frac{s}{300\pi}\right) + 1}$$

$$= \frac{888.26 \times 10^3}{s^2 + 1332.86 s + 888.26 \times 10^3}$$

$$= \frac{888.26 \times 10^3}{(s + 666.43)^2 + 666.43^2}$$

$$H_a(s) = \frac{888.26 \times 10^3}{666.43} \cdot \frac{666.43}{(s + 666.43)^2 + (666.43)^2}$$



$$= 1332.86 \frac{666.43}{(s+666.43)^2 + (666.42)^2}$$

$$H(z) = 1332.86 \frac{e^{-666.93T} \sin(666.43T) z^{-1}}{1 - 2e^{-666.43T} \cos(666.43T) z^{-1} + e^{-2(666.43)T} z^{-2}}$$

put  $T = 7.8125 \times 10^{-4}$

$$H(z) = \frac{393.92 z^{-1}}{1.10308 z^{-1} + 0.3532 z^{-2}}$$

4. Explain. Bilinear transformation derivation.

The Bilinear transformation is used for transforming an analog filter to a digital filter. Bilinear transformation can be linked to the trapezoidal formula for numerical integration.

Consider the  $n^{\text{th}}$  order differential Eqn of an analog system.

$$\frac{dy(t)}{dt} = x(t) \rightarrow \textcircled{1}$$

Integration on B.S.

$$\int_{(n-1)T}^{nT} \frac{dy(t)}{dt} dt = \int_{(n-1)T}^{nT} x(t) dt$$

$$\left[ y(t) \right]_{(n-1)T}^{nT} = \int_{(n-1)T}^{nT} x(t) dt$$

$$\left[ y(nT) - y((n-1)T) \right] = \int_{(n-1)T}^{nT} x(t) dt$$

A/c. to trapezoidal rule

$$\int_a^b f(x) dx = \frac{b-a}{2} [f(a) + f(b)]$$

$$y(nT) - y((n-1)T) = \frac{nT - (n-1)T}{2} [x((n-1)T) + x(nT)]$$

$$= \frac{nT - nT + T}{2} [x(nT) + x((n-1)T)]$$

$$y(nT) - y((n-1)T) = \frac{T}{2} [x(nT) + x((n-1)T)]$$

But we know.  $y(n) = y(t) \Big|_{t=nT}$

$$\text{i.e. } y(n) = y(nT)$$

$$y(n) - y(n-1) = \frac{T}{2} [x(n) + x(n-1)]$$

Z-Transform on both sides

$$Y(z) - z^{-1} Y(z) = \frac{T}{2} [X(z) + z^{-1} X(z)]$$

$$Y(z) [1 - z^{-1}] = \frac{T}{2} [1 + z^{-1}] X(z)$$

$$Y(z) [1 - z^{-1}] \frac{2}{T} \frac{1}{2} [1 + z^{-1}] = X(z)$$

$$\frac{2}{T} \frac{1 - z^{-1}}{1 + z^{-1}} Y(z) = X(z) \rightarrow (2)$$

$$\frac{dy(t)}{dt} = x(t) \checkmark$$

Take LT

$$s Y(s) = X(s) \rightarrow (3)$$

Comparing (2) & (3)

$$s Y(s) = \frac{2}{T} \frac{1 - z^{-1}}{1 + z^{-1}} \rightarrow (4)$$

T - Sampling time period

The Relation b/w S & z is known as

Bilinear Transformation.

$$s = \frac{2}{T} \frac{(1 - z^{-1})}{(1 + z^{-1})}$$

$$\frac{sT}{2} = \frac{1 - z^{-1}}{1 + z^{-1}} \checkmark$$

$$\frac{ST}{2}(1+z^{-1}) = 1-z^{-1}$$

$$\frac{ST}{2}\left(1 + \frac{1}{z}\right) = \frac{z-1}{z}$$

$$\frac{ST}{2}z + \frac{ST}{2} = z-1$$

$$1 + \frac{ST}{2} = z - \frac{ST}{2}z$$

$$= z\left(1 - \frac{ST}{2}\right)$$

$$z = \frac{1 + \frac{ST}{2}}{1 - \frac{ST}{2}}$$

Relation b/w. analog. frequency and digital frequency.

$$S = \frac{z}{T} \frac{1-z^{-1}}{1+z^{-1}}$$

but  $S = j\omega$  and  $z = e^{j\omega T}$

$$j\omega = \frac{z}{T} \frac{1 - e^{-j\omega T}}{1 + e^{-j\omega T}}$$

$$j\Omega = \frac{2}{T} \frac{e^{-j\omega/2} [e^{j\omega/2} - e^{-j\omega/2}]}{e^{j\omega/2} [e^{j\omega/2} + e^{-j\omega/2}]}$$

$$j\Omega = \frac{2}{T} \frac{2j \left[ \frac{e^{j\omega/2} - e^{-j\omega/2}}{2j} \right]}{2 \left[ \frac{e^{j\omega/2} + e^{-j\omega/2}}{2} \right]}$$

$$\Omega = \frac{2}{T} \frac{\sin \frac{\omega}{2}}{\cos \frac{\omega}{2}}$$

$$\Omega = \frac{2}{T} \tan \frac{\omega}{2}$$

Analog frequency

warping digital frequency.

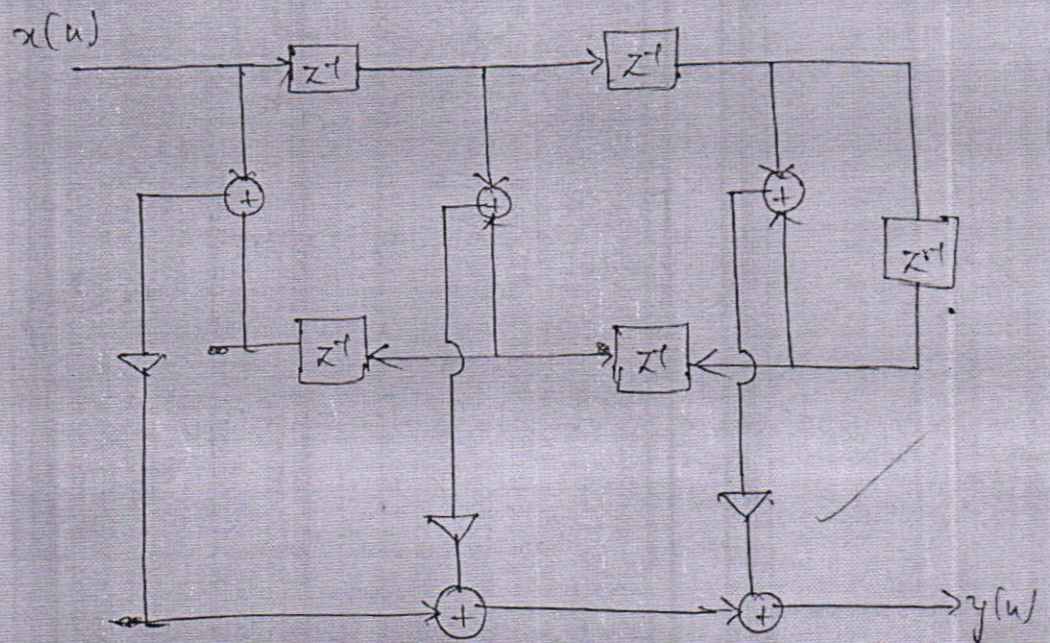
$$\omega = 2 \tan^{-1} \left( \frac{\Omega T}{2} \right)$$

for. Small value. of  $\omega = \Omega T$ .

2) Realize the linear FIR filter having the following impulse response  $h(n) = \delta(n) - \frac{1}{4} \delta(n-1) + \frac{1}{2} \delta(n-2) + \frac{1}{2} \delta(n-3) - \frac{1}{4} \delta(n-4) + \delta(n-5)$ .

$$\Rightarrow H(z) = 1 + \frac{1}{4}z^{-1} + \frac{1}{2}z^{-2} + \frac{1}{2}z^{-3} - \frac{1}{4}z^{-4} + 1 \cdot z^{-5}$$

$$\frac{Y(z)}{X(z)} = 1[1 + z^{-5}] - \frac{1}{4}[z^{-1} + z^{-4}] + \frac{1}{2}[z^{-2} + z^{-3}]$$



FIR filter

$$H(e^{j\omega}) = e^{j\omega \left(\frac{N-1}{2}\right)} \sum_{n=0}^{\frac{N-1}{2}-1} 2h(n) \cos \omega \left[n - \left(\frac{N-1}{2}\right)\right]$$

for even.

$$H(e^{j\omega}) = e^{j\omega \left(\frac{N-1}{2}\right)} \left\{ h\left(\frac{N-1}{2}\right) + \sum_{n=0}^{\left(\frac{N-3}{2}\right)} h(n) \cos \left[\omega \left(n - \left(\frac{N-1}{2}\right)\right)\right] \right\}$$

for odd.