# C-E-R-T-I-F-I-C-A-T-E

*This is to certify that Mr. / Ms.* Rayhan.K

*with USN* 1SV15EC022 *has satisfactorly completed the*

*course assignment in the subject of* Signal and System

Signature of the Student

Staff in Charge

Head of the Department

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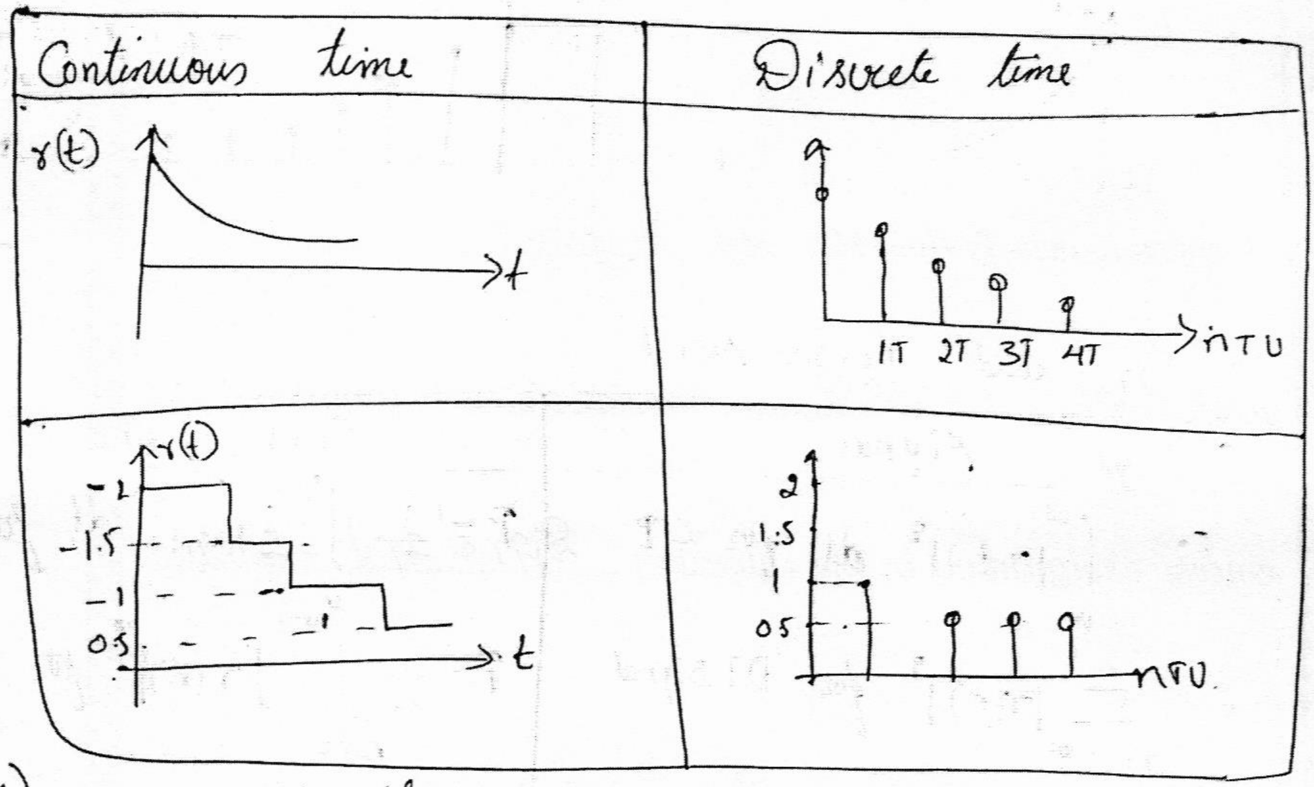
# Assignment

## Define Signal

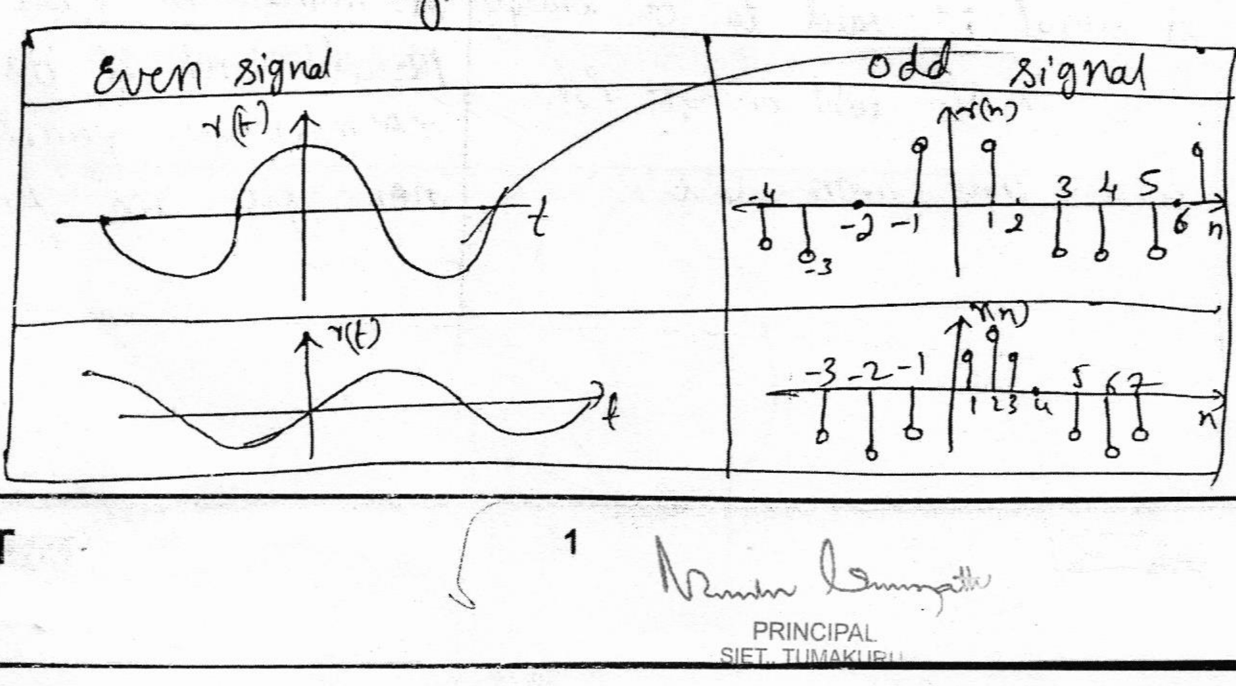
Signal is formally defined as a function of one or more variable that convey information of physical phenomena.

With an example distinguish b/w the following

i) Continuous time & discrete time signal




(ii) Even and odd signal



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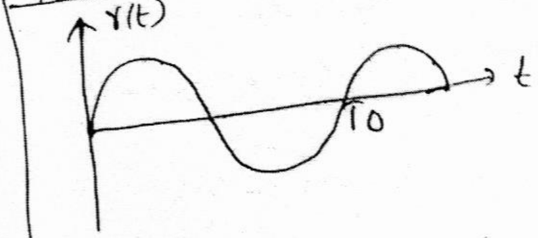
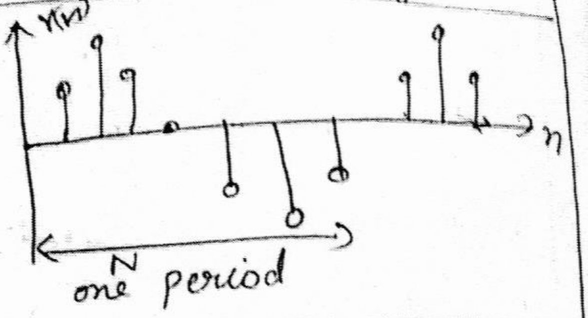
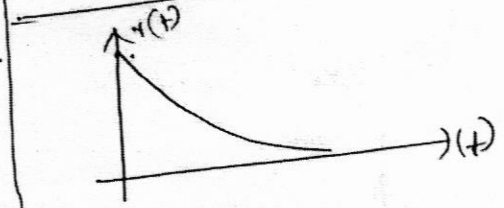
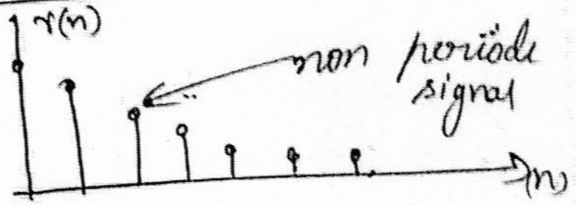
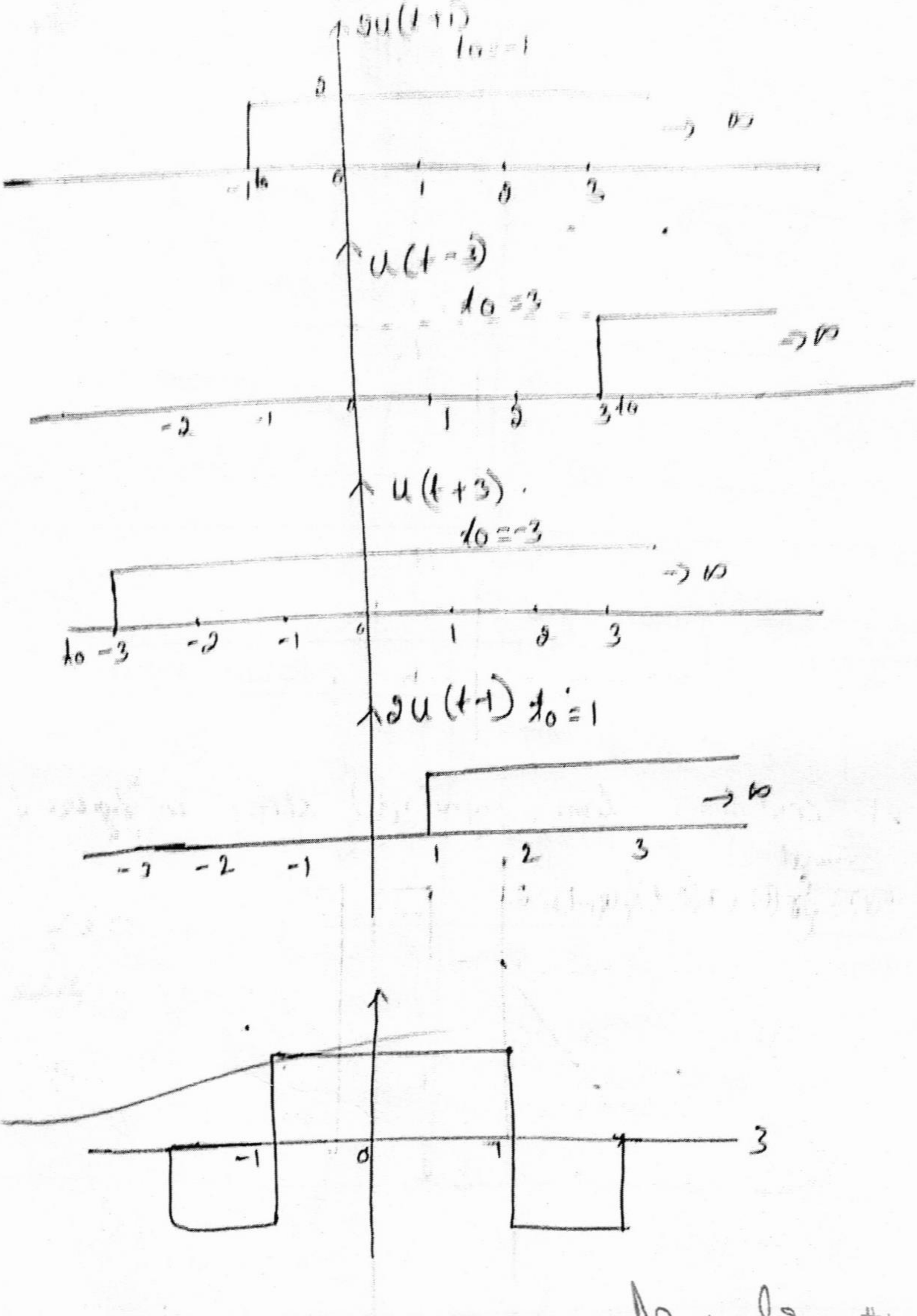
Periodic and non periodic sig...

| Periodic signal | non periodic signal |
|---|---|
|  |  |
|  |  |

Energy and power signal

| Energy signal | Power signal |
|---|---|
| $E = \int_{-\infty}^{\infty} \lvert x(t)\rvert^2 \, dt$ for CP<br>$E = \sum_{n=-\infty}^{\infty} \lvert x(n)\rvert^2$ for DT signal | $P = \frac{1}{T}\int_0^T \lvert x(t)\rvert^2 \, dt$ for CT<br>$P = \frac{1}{N}\int_{n=0}^{N-1} \lvert x(n)\rvert^2$ for DT |
| A signal is said to be energy signal if its total energy is finite and non zero | A signal is said to be power signal if its is normalized Power is non zero and finite |

Sketch the signal $x(t) = -u(t+3) + 2u(t+1) - 2u(t-1) + u(t-3)$

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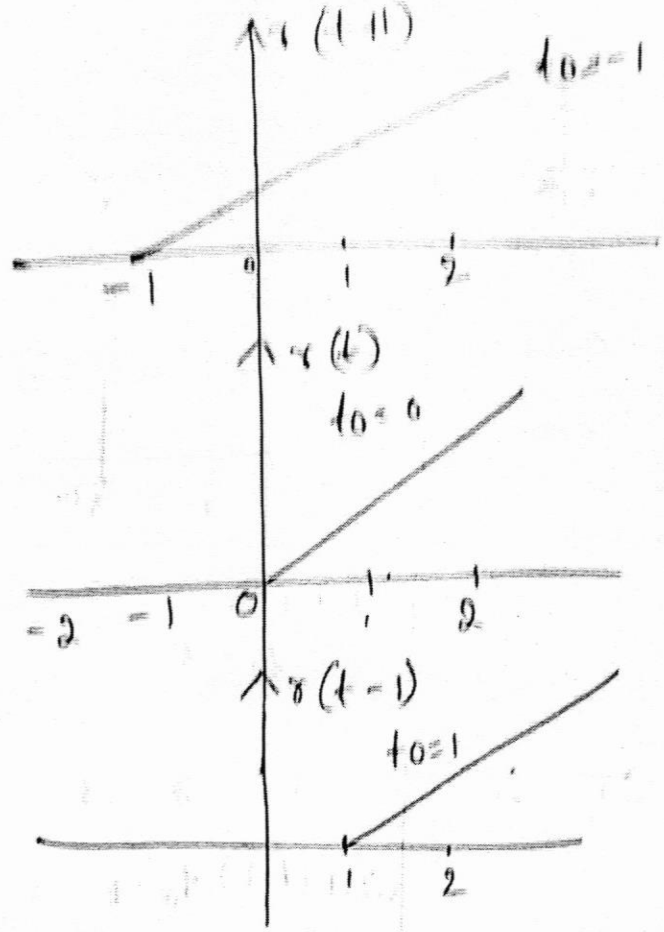
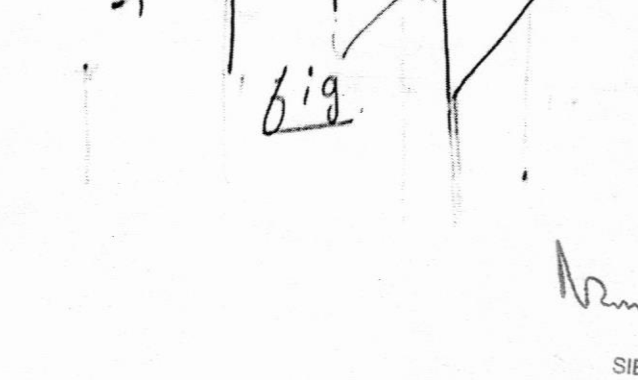
b) Sketch the signal $r(t) - r(t+1) = r(t) + r(t-1)$




4] a) A continuous time signal $x(t)$ shown in figure 1 Draw the signal
$y(t) = \{x(t) + x(2-t)\} u(1-t)$




fig

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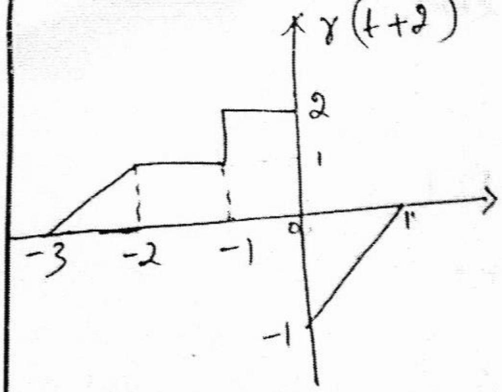
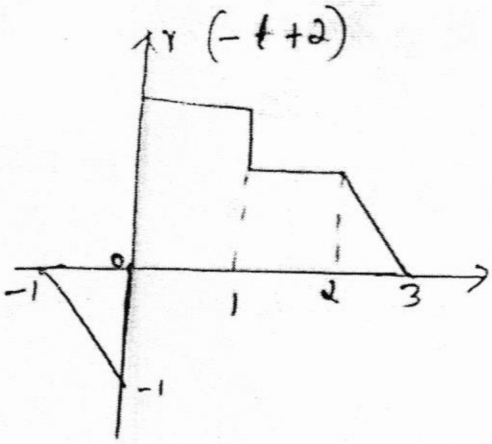
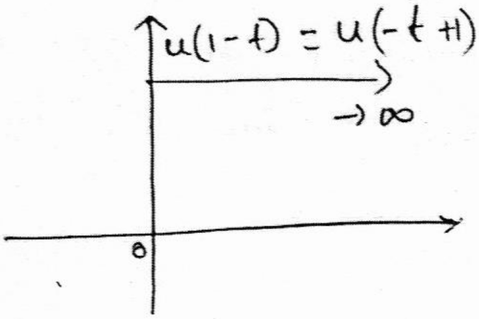
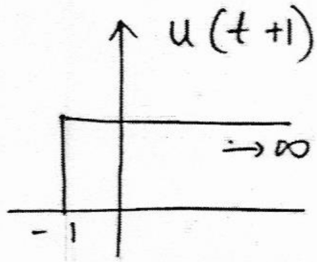
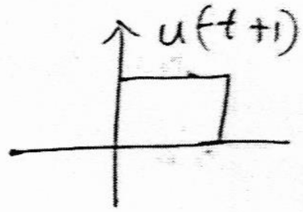



| | $r(t)$ | | $r(-t+2)$ | result |
|---|---|---|---|---|
| -1 to 0 | $r(t)$ | + | $-r(t)$ | 0 |
| 0 to 1 | 1 | + | 2 | 3 |
| 1 to 2 | 2 | + | 1 | 3 |
| 2 to 3 | $r(t)$ | + | $-r(t)$ | 0 |

Explain linearity, causal, Memory, time invarciability, invertibility, stability, property of system

Linearity :- A system is said to be linear if it satisfies the superposition principles

Causal :- A system is said to be causal which represent value of o/p depends on present or past value of the i/p or it depends on both present & past value of the input signal. If the o/p depends on when even more future value then the system is said to be causal

eg:- $y(n) = \frac{1}{3}\left[x(n) + x(n-1) + x(n-2)\right] \rightarrow$ Causal

$y(n) = \frac{1}{3}\left[x(n) + x(n-1) + x(n+2)\right] \rightarrow$ non Causal

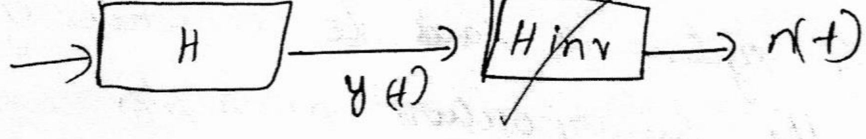
**Time invariance** :→ A system is said to be time invariance if a time delay or time advances of the i/p signal leads to the identical time shift in the o/p signal

i.e

$$\left.\begin{aligned} x(t) &\longrightarrow y(t) \\ x(t-m) &\rightarrow y(t-m) \\ x(t+m) &\rightarrow y(t+m) \end{aligned}\right\} \text{Time invariant system}$$

$$\left.\begin{aligned} x(t-m) &\rightarrow y(t) \\ x(t+m) &\rightarrow y(t) \end{aligned}\right\} \text{Time Variant}$$

**Invertibility** :→ The system is said to be invertible if the i/p of the system can be recover from the o/p




$H \cdot H_{inv} = I$

$x(t) = H_{inv}\{y(t)\}$

$x(t) = (H_{inv} H)\{x(t)\}$

**Stability Property:** An arbitrary relax system is said to be BIBO if and only if the bounded i/p produces a bounded o/p. Let y(t) be the response. The term bounded input refers to the finite value of i/p refers to the finite value of the i/p signal x(t) for any value of (t) which is given by

$$|x(t)| \le M_x < \infty$$

where $M_n$ is a constant after computing magnitude of $r(t)$.

**Memory:** The continuous time system is said to be static or memory less instantaneous, if the o/p depends upon the present i/p only.

Determine whether the system is linear, time invariant, memoryless, causal, stable $T\{r(n)\} = g(n)\,r(n)$

$$T\{r(n)\} = g(n)\,r(n)$$

$$y(n) = g(n)\,r(n)$$

$y(n-n_0) = g(n)\,r(n-n_0) \Rightarrow$ o/p is time invariance

$T\{r(n)\} = g(n)\,r(n) \Rightarrow$ depends only present. The system is said to be memory less

$T\{r(n)\} = g(n)\,r(n) \Rightarrow$ depends only on present hence it is causal.

$T\{r(n)\} = g(n)\,r(n) \Rightarrow |g(n)\,r(n)| \le M_n < \infty = |g(n)|\,|r(n)| \le M_n < \infty$

The system is stable if g(n) & r(n) is stable

Determine & sketch the even com... signal shown in fig

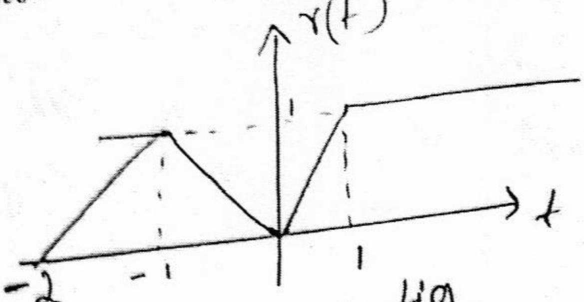
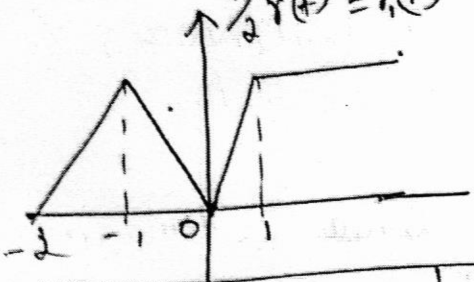


fig.

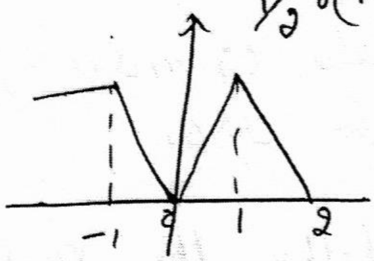
$$r_e(t) = \frac{1}{2}\left[r(t) + r(-t)\right] = \frac{1}{2} r(t) + \frac{1}{2} r(-t)$$

$$r_o(t) = \frac{1}{2}\left[r(t) - r(-t)\right] = \frac{1}{2} r(t) - \frac{1}{2} r(-t)$$

$\frac{1}{2} r(t) = r_1(t)$

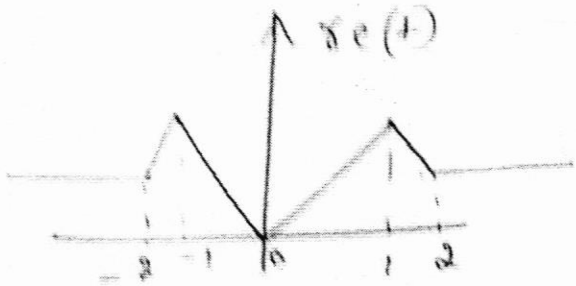


$\frac{1}{2} r(-t) = r_2(t)$

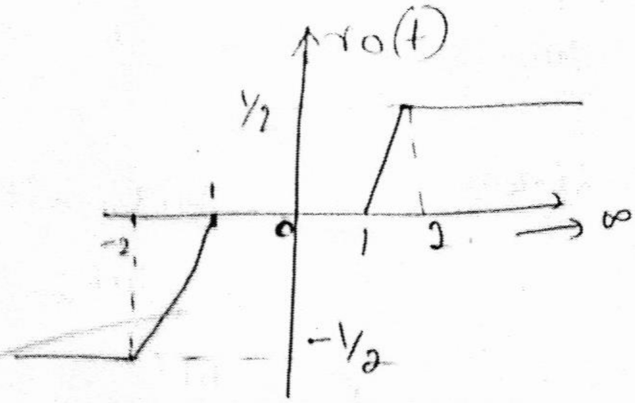


| Time duration | $r_1(t)$ | $r_2(t)$ | $r_e(t)$ | $r_o(t)$ |
|---|---|---|---|---|
| $-\infty$ to $-2$ | 0 | $1/2$ | $1/2$ | $-1/2$ |
| $-2$ to $-1$ | $r(t)$ $(0 \to 1/2)$ | $1/2$ | $r(t)$ $(1/2$ to $1)$ | $r(t)$ $(-1/2$ to $0)$ |
| $-1$ to $0$ | $-r(t)$ $(1/2 \to 0)$ | $-r(t)$ $(-1/2 \to 0)$ | $-r(t)$ $(1$ to $0)$ | 0 |
| $0$ to $1$ | $r(t)$ $(0 \to 1/2)$ | $r(t)$ $0 \to 1/2$ | $r(t)$ $(0$ to $1)$ | 0 |
| $1$ to $2$ | $1/2$ | $-r(t)$ | $-r(t)$ $(1$ to $1/2)$ | $r(t)$ $(0$ to $1/2)$ |
| $2$ to $\infty$ | $1/2$ | 0 | $1/2$ | $1/2$ |

Even function




odd function

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# Assignment

## Properties of discrete time fourier series

**Linearity**

$$x(n) \xleftrightarrow{DTFS} X(k)$$

$$y(n) \xleftrightarrow{DTFS} Y(k)$$

$$\underbrace{a x(n) + b y(n)}_{z(n)} \xleftrightarrow{DTFS} \underbrace{a X(k) + b Y(k)}_{Z(k)}$$

$$X(k) = \frac{1}{N} \sum_{n=\langle N \rangle} x(n) \, e^{-jk\Omega_0 n}$$

$$Z(k) = \frac{1}{N} \sum_{n=\langle N \rangle} z(n) \, e^{-jk\Omega_0 n}$$

$$Z(k) = \frac{1}{N} \left[ a \sum_{n=\langle N \rangle} x(n) \, e^{-jk\Omega_0 n} + b \sum_{n=\langle N \rangle} y(n) \, e^{-jk\Omega_0 n} \right]$$

$$\boxed{Z(k) = a X(k) + b Y(k)}$$

LHS = R.HS. Hence proved

**Periodicity**

If $x(n) \xleftrightarrow{DTFS} X(k)$ with period 'N' then

$$\boxed{X(k+N) = X(k)}$$



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Proof $X(k) = \frac{1}{N} \sum_{n=\langle N \rangle} x(n) e^{-jk\Omega_0 n}$

let k be replaced with k+N Then we have

$$x(k+N) = \frac{1}{N} \sum_{n=\langle N \rangle} x(n) e^{-j(k+N)\Omega_0 n}$$

$$= \frac{1}{N} \sum_{n=\langle N \rangle} x(n) e^{-jk\Omega_0 n} \cdot e^{-jN\Omega_0 n}$$

Here $\Omega_0 = \frac{2\pi}{N}$ Hence, $e^{-jN\Omega_0 n} = e^{-jN\frac{2\pi}{N}n} = e^{-j2\pi n} = 1$ (always)

$$\therefore X(k+N) = \frac{1}{N} \sum_{n=\langle N \rangle} x(n) e^{-jk\Omega_0 n} \times 1 = X(k)$$

$$\therefore \boxed{X(k+N) = X(k)}$$

③ Time Shift

If $x(n) \xleftrightarrow{DTFS} X(k)$ then

$$\boxed{y(n) = x(n-n_0) \xleftrightarrow{DTFS} Y(k) = e^{-jk\Omega_0 n_0} X(k)}$$

Proof:- $\underline{Y(k) = \frac{1}{N} \sum_{n=\langle N \rangle} y(n) e^{-jk\Omega_0 n}} = \frac{1}{N} \sum_{n=\langle N \rangle} x(n-n_0) e^{-jk\Omega_0 n}$

let $n - n_0 = m$, then

$$Y(k) = \frac{1}{N} \sum_{m=\langle N \rangle} x(m) e^{-jk\Omega_0 (m+n_0)}$$

$$= e^{-jk\Omega_0 n_0} \frac{1}{N} \sum_{m=\langle N \rangle} x(m) e^{-jk\Omega_0 m} \Rightarrow \boxed{Y(k) = e^{-jk\Omega_0 n_0} X(k)}$$

Proved

## Frequency shift

If $x(n) \xleftrightarrow{DTFS} X(k)$ then,

$y(n) = e^{jK_0 \Omega_0 n} x(n) \xleftrightarrow{DTFS} Y(k) = X(K - K_0)$

Proof: $Y(k) = \frac{1}{N} \sum_{n=\langle N\rangle} y(n) e^{-jK\Omega_0 n} = \frac{1}{N} \sum_{n=\langle N\rangle} e^{jK\Omega_0 n} x(n) e^{-jK\Omega_0 n}$

$$= \frac{1}{N} \sum_{n=\langle N\rangle} x(n) e^{-j(k-K_0)\Omega_0 n}$$

$$\boxed{Y(k) = X(k - K_0)}$$

## Convolution:-

If $x(n) \xleftrightarrow{DTFS} X(k)$ and $y(n) \xleftrightarrow{DTFS} Y(k)$

then, $\boxed{z(n) = x(n) * y(n) \xleftrightarrow{DTFS} N\, X(k) Y(k)}$

Proof: $Z(k) = \frac{1}{N} \sum_{n=\langle N\rangle} z(n) e^{-jK\Omega_0 n} = \frac{1}{N} \sum_{n=\langle N\rangle} [x(n) * y(n)] e^{-jK\Omega_0 n}$

$$= \frac{1}{N} \sum_{n=\langle N\rangle} \left\{ \sum_{l=\langle N\rangle} x(l)\, y(n-l) \right\} e^{-jk\Omega_0 n}$$

$$= \frac{1}{N} \sum_{l=\langle N\rangle} x(l) \sum_{n=\langle N\rangle} y(n-l) e^{-jK\Omega_0 n}$$

Let $n - l = m$, we get

$$Z(k) = \frac{1}{N} \sum_{l=\langle N\rangle} x(l) \sum_{m=\langle N\rangle} y(m) e^{-jK\Omega_0 (m+l)}$$

$$= \frac{1}{N} \sum_{l=\langle N\rangle} r(l) \sum_{m=\langle N\rangle} y(m) e^{-jk\Omega_o m} e^{-jk\Omega_o l}$$

$$z(k) = \frac{1}{N} \sum_{l=\langle N\rangle} r(l) e^{-jk\Omega_o l} \sum_{m=\langle N\rangle} y(m) e^{-jk\Omega_o m}.$$

$$Z(k) = X(k) \cdot N\, Y(k)$$

$$\boxed{Z(k) = N\, X(k) \cdot Y(k)}$$

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Assignment

State and prove linearity and Time shifting property of DTFS

Linearity: $x(n) \xleftrightarrow{DTFS} X(k)$ and $y(n) \xleftrightarrow{DTFS} Y(k)$

then, $z(n) = ax(n) + by(n) \xleftrightarrow{DTFS} Z(k) = aX(k) + bY(k)$

Proof
$$Z(k) = \frac{1}{N} \sum_{n=\langle N\rangle} z(n) e^{-jk\Omega_0 n}$$
$$= \frac{1}{N} \sum_{n=\langle N\rangle} [ax(n) + by(n)] e^{-jk\Omega_0 n}$$
$$= a \frac{1}{N} \sum_{n=\langle\rangle} x(n) e^{-jk\Omega_0 n} + b \frac{1}{N} \sum_{n=\langle N\rangle} y(n) e^{-jk\Omega_0 n}$$

$$Z(k) = aX(k) + bY(k)$$

Time Shift:
$x(n) \xleftrightarrow{DTFS} X(k)$ then
$$y(n) = x(n-n_0) \xleftrightarrow{DTFS} Y(k) = e^{-jk\Omega_0 n_0} X(k)$$

Proof:
$$Y(k) = \frac{1}{N} \sum_{n=\langle N\rangle} y(n) e^{-jk\Omega_0 n}$$
$$= \frac{1}{N} \sum_{n=\langle N\rangle} x(n-n_0) e^{-jk\Omega_0 n}$$

Let $n - n_0 = m$, then
$$Y(k) = \frac{1}{N} \sum_{m=\langle N\rangle} x(m) e^{-jk\Omega_0 (m+n_0)}$$



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$$= e^{-jk\Omega_0 n_0} \frac{1}{N} \sum_{m=\langle N\rangle} x(m)\, e^{-jk\Omega_0 m}$$

$$Y(k) = e^{-jk\Omega_0 n_0} X(k)$$

Consider the signal $x(n) = 2 + 2\cos\frac{\pi}{4}n + \cos\frac{\pi}{2}n + \frac{1}{2}\cos\frac{3\pi}{4}n$

Using DTFS i) Determine & sketch its power density spectrum

ii) Evaluate the power of the signal

$$\left.\begin{aligned} \Omega_{01} &= \frac{\pi}{4} \\ \Omega_{02} &= \pi/2 \\ \Omega_{03} &= 3\pi/4 \end{aligned}\right\} \quad \begin{aligned} \Omega_0 &= \gcd\{\Omega_{01}, \Omega_{02}, \Omega_{03}\} \\ &= \gcd\{\pi/4, \pi/2, 3\pi/4\} \\ &= \gcd\{\pi/4, \pi/2, 3\pi/4\} \end{aligned}$$

$\Rightarrow \pi/4 = \Omega_0$

where $\Omega_0 = 2\pi/N$

$\therefore \pi/4 = 2\pi/N$

$$\boxed{N=8}$$

$$x(n) = 2e^{j0} + 2\left[\frac{e^{j(1)\pi/4 n} + e^{j(-1)\pi/4 n}}{2}\right] + \frac{e^{j(2)\pi/4 n} + e^{j(-2)\pi/4}}{2}$$

$$+ \frac{1}{2}\left(2\frac{e^{j(3)\pi/4 n} + e^{j(-3)\pi/4}}{2}\right)$$

$$x(n) = 2e^{j0} + e^{j\pi/4 n} + e^{-j\pi/4 n} + \frac{1}{2}e^{j2\pi/4 n} + \frac{1}{2}e^{j2\pi/4 n} + \frac{1}{4}e^{j3\pi/4 n} + \frac{1}{4}e^{-j\frac{3\pi}{4}n}$$

$$x(n) = \sum_{k=\langle N\rangle} X(k)\, e^{jk\Omega_0 n}$$

Comparing with the standard equation

$X(0) = 2$ $\quad X(1) = 1$

$X(-1) = 1$ $\quad X(2) = 1/2$

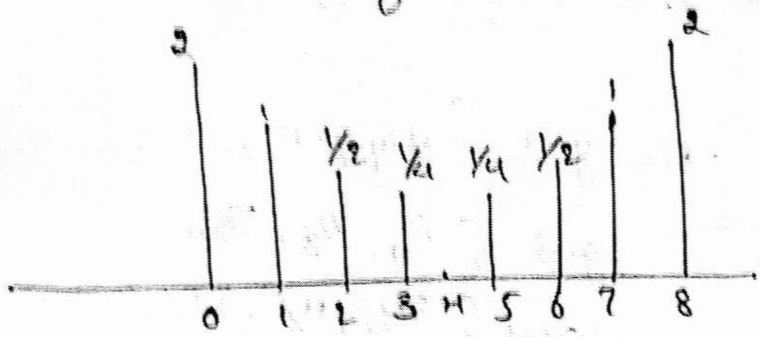
$X(-2) = 1/2$ $\quad X(3) = 1/4$

$X(-3) = 1/4$ $\quad X(7) = 1, X(6) = 1/2, \quad X(5) = 1/4$

$X(8) = 2$ $\quad X(10) = 1/2, \quad X(11) = 1/4$

$X(9) = 1$

Take an interval from 0 to 7 & draw the magnitude




Total power can be obtained by parallel theorem

$$P = \frac{1}{N} \sum_{n=\langle N\rangle} |x(n)|^2$$

$$= \sum_{k=\langle N\rangle} |X(k)|^2$$

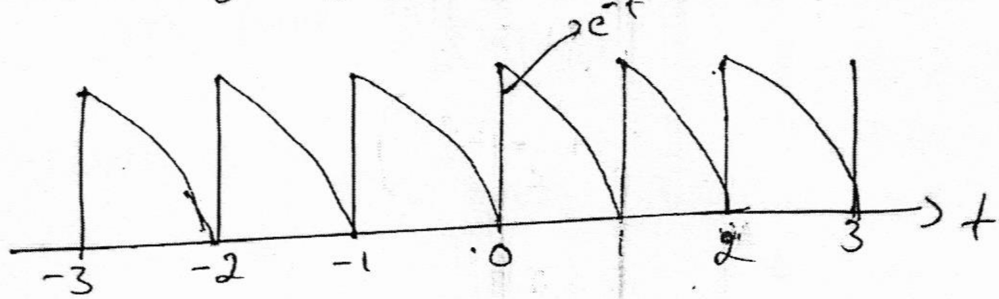
$$P = \sum_{k=0}^{7} |X(k)|^2$$

$$P = (X(0))^2 + (X(1))^2 + (X(2))^2 + (X(3))^2 + (X(3))^2 + (X(4))^2 + (X(5))^2$$
$$+ (X(6))^2 + (X(7))^2$$

$$P = 4+1+\frac{1}{4}+\frac{1}{16}+0+\frac{1}{16}+\frac{1}{4}+1$$

$$P = 6.625 \text{ watts}$$

For the signal x(t) shown in fig find the fourier series representation & draw its magnitude and phase spectra




$$T=1, \quad \omega_0 = \frac{2\pi}{T} = \frac{2\pi}{1} = 2\pi$$

$$x(k) = \frac{1}{T}\int_{\langle T\rangle} x(t)\, e^{-jk\omega_0 t}\, dt$$

$$x(k) = \frac{1}{1}\int_0^1 e^{-t}\, e^{-jk\omega_0 t}\, dt$$

$$x(k) = \int_0^1 e^{-(1+jk\omega_0)t}\, dt$$

$$= \left.\frac{e^{-(1+jk\omega_0)t}}{-(1+jk\omega_0)}\right|_0^1$$

$$= \frac{e^{-(1+jk\omega_0)}}{-(1+jk\omega_0)} + \frac{1}{(1+jk\omega_0)}$$

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$$x(k) = \frac{e^{-(1+jk\omega_0)}}{(1+jk\omega_0)}$$

$$x(k) = \frac{1 - e^{-(1+jk1\omega_0)}}{1+jk1\omega_0}$$

$$= \frac{1 - e^{-(1+jk2\pi)}}{1+jk2\pi}$$

$$= \frac{1 - [e^{-1} e^{-jk2\pi}]}{1+jk2\pi}$$

$$x(k) = \frac{1 - e^{-1}}{1+jk2\pi}$$

$k = 0, 1, 2, 3, 4, -1, -2, -3, -4$

$$X(0) = \frac{1-e^{-1}}{1+0} = \frac{1-e^{-1}}{1} = 0.034$$

$$x(k) = a + jb$$

$$= \frac{(1-e^{-1})(1-jk2\pi)}{1-j2k^2 2j\pi^2}$$

$$= \frac{(1-e^{-1})(1-jk2\pi)}{1+k^2 4\pi^2}$$

$$= \frac{0.6321\,(1-jk2\pi)}{1+k^2 4\pi^2}$$

$$x(k) = \frac{0.6321 - j3.9018}{1+4\pi^2}$$

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$$x(1) = \frac{0.6321 - j\,3.4018}{1 + 4\pi^2}$$

$$x(1) = 0.0156 - j\,0.0963$$

$$|x(1)| = 0.0156 - j\,0.0963$$

$$= \sqrt{0.0156^2 + 0.0963^2}$$

$$|x(1)| = 0.1$$

$$x(1) = \tan^{-1} b/a$$

$$= \tan^{-1}\left(\frac{-0.0963}{0.0156}\right)$$

$$= -1.41$$

$$x(2) = \frac{0.6321 - j\,7.8036}{1 + 16\pi^2} = 3.97 \times 10^{-3} - j\,0.0491$$

$$x(2) = \sqrt{0.0039^2 + 0.0491^2}$$

$$|x(2)| = 0.05$$

$$x(2) = \tan^{-1}(b/a)$$

$$= \tan^{-1}\left(\frac{-0.0491}{0.0039}\right)$$

$$\alpha(2) = \underline{-1.49}$$

Find the fourier series coeffient of the signal $x(t)$ in the figure and draw the spectra

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When k is even

$k = \pm 2, \pm 4, \pm 6$

$$X(k) = \frac{1 - \cos k\pi}{k^2 \pi^2}$$

$$X(k) = 0$$

When (k) is odd

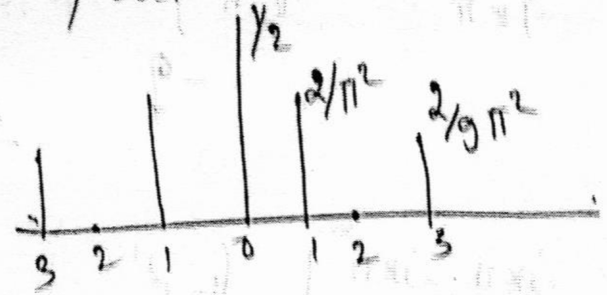
$k = \pm 1, \pm 3, \pm 5 \ldots$

$$X(k) = \left(\frac{1 - \cos k\pi}{k^2 \pi^2}\right) \Rightarrow X(1) = \left(\frac{1 - \cos \pi}{\pi^2}\right)$$

$$X(1) = 2/\pi^2$$

$$X(3) = \frac{2}{9\pi^2}$$

The spectra is as shown in the below figure




) Find the DTFT of the signal, $x(n) = \alpha^n u(n)$; $|\alpha| < 1$ Draw its magnitude spectrum

) Proof: we have the DTFT

$$X(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\Omega n}$$

$$= \sum_{n=0}^{\infty} \alpha^n e^{-j\Omega n}$$

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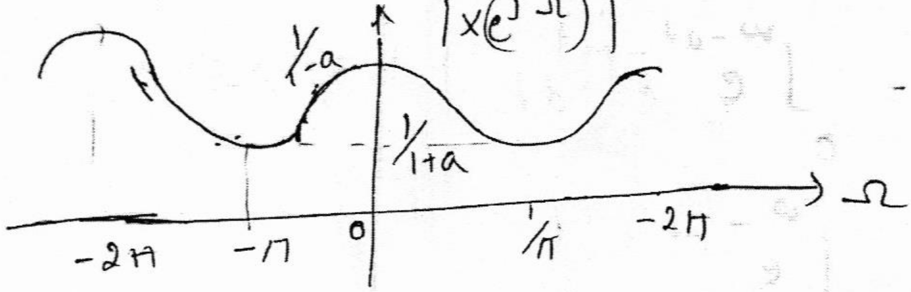
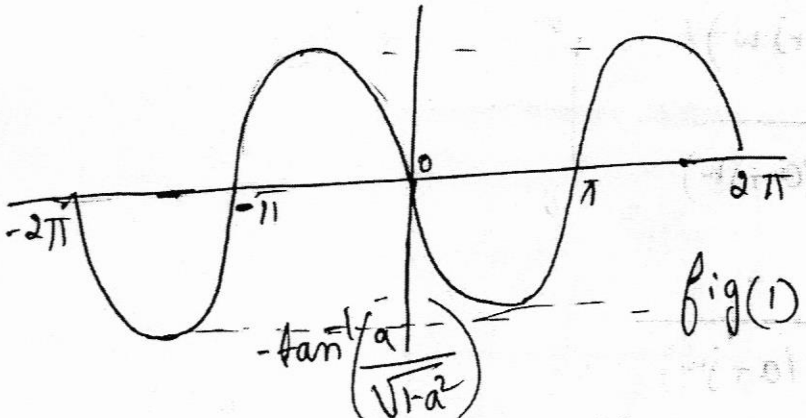
$$= \sum_{n=0}^{\infty} \left(\alpha e^{-j\Omega}\right)^n$$

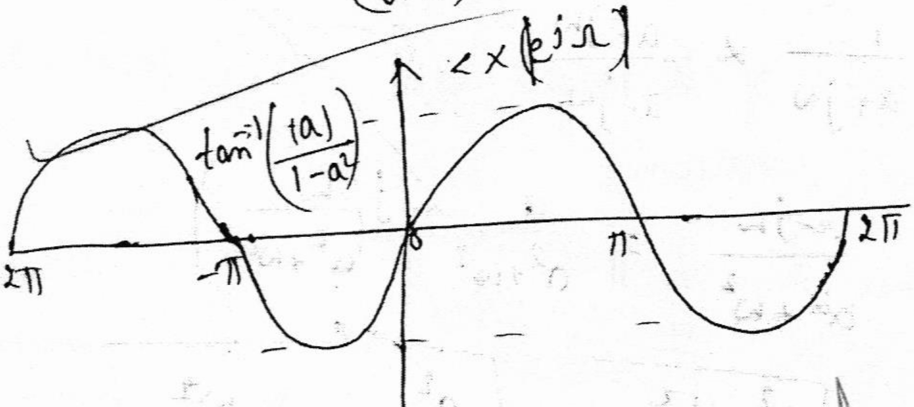
$$x(e^{j\Omega}) = \frac{1}{1-\alpha e^{-j\Omega}}$$

The magnitude and phase of $x(e^{j\Omega})$ is shown in the figure 1 and fig 2 $a>0$ and $a<0$ respectively.
These spectra are periodic with period $2\pi$




fig(1)

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6) Obtain the Fourier transform of $x(t) = e^{-at}u(t)$; also Draw its magnitude and Phase spectrum

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{\infty} e^{-at} u(t)\, e^{-j\omega t}\, dt$$

$$= \int_{0}^{\infty} e^{-at}\, e^{-j\omega t}\, dt$$

$$= \int_{0}^{\infty} e^{-(a+j\omega)t}\, dt$$

$$= \left. \frac{e^{-(a+j\omega)t}}{-(a+j\omega)} \right|_{0}^{\infty}$$

$$= 0 + \frac{1}{(a+j\omega)}$$

$$= \frac{1}{a+j\omega} \times \frac{a-j\omega}{a-j\omega}$$

$$= \frac{a-j\omega}{a^2+\omega^2} = \frac{a}{a^2+\omega^2} - j\left(\frac{\omega}{a^2+\omega^2}\right)$$

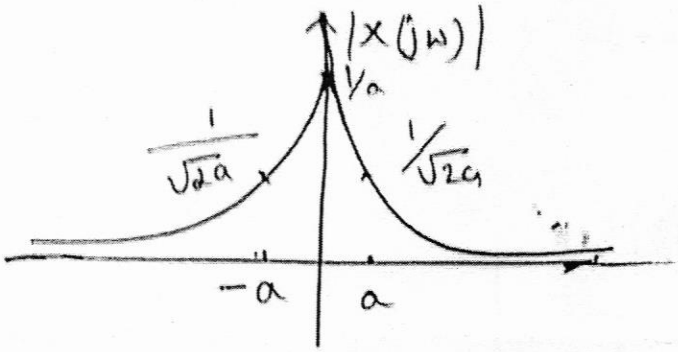
$$|X(j\omega)| = \sqrt{a^2+b^2} = \sqrt{\frac{a^2}{(a^2+\omega^2)^2} + \frac{\omega^2}{(a^2+\omega^2)^2}}$$

$$= \sqrt{\frac{a^2+\omega^2}{(a^2+\omega^2)^2}}$$

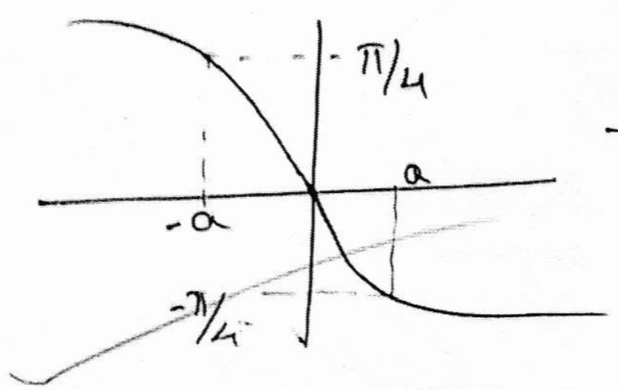
$$|X(j\omega)| = \frac{1}{\sqrt{a^2+\omega^2}}$$

$$\angle |X(j\omega)| = \tan^{-1}(b/a) = \tan^{-1}\left(\frac{\frac{-\omega}{a^2+b^2}}{\frac{a}{a^2+b^2}}\right)$$

$$\boxed{\angle X(j\omega) = \tan^{-1}\frac{-\omega}{a}}$$




→ magnitude




→ phase

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