CRITERION 1- CURRICULAR ASPECTS

Criteria 1.1

Curriculum Planning and Implementation

COURSE FILE

CIVIL

Department Of Civil Engineering

Date: 25/07/2019

OFFICE ORDER

Asst. Professor **Mr. C Nagaraja** is appointed as course instructor for following courses.

Sl No	Course Code	Course	Remarks
1	18CIV14	Elements of Civil Engineering & Mechanics	
2	18CV32	Strength of Materials	
3	17CV54	Computer Aided Building Planning & Drawing	Shared with Mr. Manogna H N
4	18CCS31	Analysis and Design of Shell Roof Structures – Classical and FE Approach	

(Dr. G. Mahesh Kumar)
HOD

SHRIDEVI

SHRIDEVI INSTITUTE OF ENGINEERING & TECHNOLOGY, TUMAKURU-572 106

DEPARTMENT OF CIVIL ENGINEERING

CALENDAR OF EVENTS

FS 543667

An ISO 9001:2015 Certified Institution

III & V Semester B E July 2019 to November 2019 Session

JULY

Date	Day	Activities
25/7/19	Thu	Commencement
26/7/19	Fri	
27/7/19	Sat	
28/7/19	Sun	
*29/7/19	Mon	# HODs Meeting
30/7/19	Tue	$ Deptl Staff Meeting
31/7/19	Wed	
No of Working Days: 06		

Total No of Working Days: 100

Practical Examinations
3/12/19
To
13/12/19

Theory Examinations
16/12/19
To
7/2/20

A positive attitude causes a chain reaction of positive thoughts, events and out comes. It is a catalyst and it sparks extraordinary results. Everyone should develop a positive attitude to learn and deliver.

AUGUST

Date	Day	Activities
1/8/19	Thu	
2/8/19	Fri	
3/8/19	Sat	
4/8/19	Sun	
*5/8/19	Mon	HODs Meeting
6/8/19	Tue	Deptl Staff Meeting
7/8/19	Wed	
8/8/19	Thu	
9/8/19	Fri	
10/8/19	Sat	
11/8/19	Sun	
*12/8/19	Mon	Bakrid
13/8/19	Tue	HODs Meeting
14/8/19	Wed	Deptl Staff Meeting
15/8/19	Thu	Independence Day
16/8/19	Fri	
17/8/19	Sat	
18/8/19	Sun	
*19/8/19	Mon	HODs Meeting
20/8/19	Tue	Deptl Staff Meeting
21/8/19	Wed	
22/8/19	Thu	
23/8/19	Fri	
24/8/19	Sat	
25/8/19	Sun	
*26/8/19	Mon	HODs Meeting
27/8/19	Tue	Deptl Staff Meeting
28/8/19	Wed	
29/8/19	Thu	
30/8/19	Fri	
31/8/19	Sat	
No of Working Days: 25		

SEPTEMBER

Date	Day	Activities
1/9/19	Sun	
* 2/9/19	Mon	Vinayaka Chathurthi
3/9/19	Tue	HODs Meeting
4/9/19	Wed	Deptl Staff Meeting
5/9/19	Thu	IA Test I
6/9/19	Fri	IA Test I
7/9/19	Sat	IA Test I
8/9/19	Sun	
* 9/9/19	Mon	HODs Meeting
10/9/19	Tue	Muharram
11/9/19	Wed	Deptl Staff Meeting
12/9/19	Thu	
13/9/19	Fri	
14/9/19	Sat	
15/9/19	Sun	
* 16/9/19	Mon	HODs Meeting
17/9/19	Tue	Deptl Staff Meeting
18/9/19	Wed	
19/9/19	Thu	
20/9/19	Fri	
21/9/19	Sat	
22/9/19	Sun	
* 23/9/19	Mon	HODs Meeting
24/9/19	Tue	Deptl Staff Meeting
25/9/19	Wed	
26/9/19	Thu	
27/9/19	Fri	
28/9/19	Sat	Mahalaya Amavasye
29/9/19	Sun	
* 30/9/19	Mon	
No of Working Days: 22		

OCTOBER

Date	Day	Activities
1/10/19	Tue	
2/10/19	Wed	Gandhi Jayanthi
3/10/19	Thu	
4/10/19	Fri	
5/10/19	Sat	
6/10/19	Sun	
* 7/10/19	Mon	Ayudha Pooja
8/10/19	Tue	Vijayadashami
9/10/19	Wed	HODs Meeting
10/10/19	Thu	Deptl Staff Meeting
11/10/19	Fri	
12/10/19	Sat	
13/10/19	Sun	Valmiki Jayanthi
* 14/10/19	Mon	# IA Test II
15/10/19	Tue	$ IA Test II
16/10/19	Wed	IA Test II
17/10/19	Thu	
18/10/19	Fri	
19/10/19	Sat	
20/10/19	Sun	
* 21/10/19	Mon	HODs Meeting
22/10/19	Tue	Deptl Staff Meeting
23/10/19	Wed	
24/10/19	Thu	
25/10/19	Fri	
26/10/19	Sat	
27/10/19	Sun	Naraka Chathurdashi
* 28/10/19	Mon	HODs Meeting
29/10/19	Tue	Balipadyami
30/10/19	Wed	Deptl Staff Meeting
31/10/19	Thu	
No of Working Days: 23		

NOVEMBER

Date	Day	Activities
1/11/19	Fri	Kannada Rajyotsava
2/11/19	Sat	
3/11/19	Sun	
* 4/11/19	Mon	
5/11/19	Tue	
6/11/19	Wed	
7/11/19	Thu	
8/11/19	Fri	
9/11/19	Sat	
10/11/19	Sun	
* 11/11/19	Mon	HODs Meeting
12/11/19	Tue	Deptl Staff Meeting
13/11/19	Wed	
14/11/19	Thu	
15/11/19	Fri	Kanaka Jayanthi
16/11/19	Sat	
17/11/19	Sun	
* 18/11/19	Mon	HODs Meeting
19/11/19	Tue	Deptl Staff Meeting
20/11/19	Wed	
21/11/19	Thu	IA Test III
22/11/19	Fri	IA Test III
23/11/19	Sat	IA Test III
24/11/19	Sun	
* 25/11/19	Mon	HODs Meeting
26/11/19	Tue	Deptl Staff Meeting
27/11/19	Wed	
28/11/19	Thu	
29/11/19	Fri	
30/11/19	Sat	Closure
No of Working Days: 24		

Maheshkumar 25/07/19

Dr. G Mahesh Kumar
Head of the Department

(Dr T Hemadri Naidu)
Principal

SHRIDEVI
EDUCATION

SHRIDEVI INSTITUTE OF ENGINEERING AND TECHONOLOGY, TUMKUR-572106
(Recognised by Govt. of Karnataka, Affiliated to VTU, Belagavi and approved by AICTE, New Delhi)
Sira Road, Tumkuuru – 572 106. Karnataka.

FS 543667
An ISO 9001:2015 Certified Institution

Department of Civil Engineering

Academic Year 2019– 2020 (Odd Semester)

TIME TABLE

WEF: 19/09/2019

Class: III Semester **Lecture Hall: 205**

TIME / DAYS	8:30 A M To 9:25 A M	9:25 A M To 10:20 A M	10:20 A M To 10:40 A M	10:40 A M To 11:35 A M	11:35 A M To 12:30 P M	12:30 P M To 1:30 P M	1:30 P M To 2:25 P M	2:25 P M To 3:20 P M	3:20 P M To 4:15 P M	4:15 P M To 4:30 P M
MONDAY	TCFS&NT	FM	TEA BREAK	EG	BS (VR)	LUNCH BREAK	CABPAD B1 / BMT LAB B2			CONTACT SESSION
TUESDAY	BM&C (S)	BS (BCH)		SOM	TCFS&NT		BS (VR)	BM&C (G)	COUN B2	
WEDNESDAY	TCFS&NT	EG		BM&C (G)	FM		CABPAD B2 / BMT LAB B1			
THURSDAY	SOM	BS (BCH)		TCFS&NT	SOM		FM	EG	Vyavaharika Kannada	
FRIDAY	EG	SOM		FM	LIB /COUN B1		TCFS&NT	LIB	Aadalitha Kannada	
SATURDAY	SOM	BM&C (S)		MAT DIP						

Subject Code	Subject	Staff-in-charge
18MAT31	Transform Calculus, Fourier Series and Numerical Techniques (TCFS&NT)	Mathematics Dept.
18CV32	Strength of Materials (SOM)	Mr. C Nagaraja
18CV33	Fluid Mechanics (FM)	Mrs. Bhavya C H
18CV34	Building Materials and Construction (BM&C)	Dr. G Mahesh Kumar / Mrs. Sreelakshr
18CV35	Basic Surveying (BS)	Mr. Vinuthan V R / Mrs. Bhavya C H
18CV36	Engineering Geology (EG)	Mrs.Sreelakshmi S
18CVL37	Computer Aided Building Planning & Drawing (CABPAD)	Mr. Vinuthan V R / SS
18CVL38	Building Materials Testing Laboratory (BMT LAB)	Mrs. Bhavya C H / PJ
18KVK39 /18KAK39	Vyavaharika Kannada / Aadalitha Kannada /	MBA Dept.
18MATDIP31	Additional Mathematics - I	Mathematics Dept.

Counselors

B1 – Mr. Manogna H N

B2 – Mrs. Sreelakshmi S

Class Teacher

Mrs. Sreelakshmi S

(Mr. Manogna H N)
Time Table Coordinator

(Dr. G Mahesh Kumar)
Head of the Department

(Dr. Narendra Viswanath)
Principal

SHRIDEVI EDUCATION

SHRIDEVI INSTITUTE OF ENGINEERING AND TECHONOLOGY, TUMKUR-572106
(Recognised by Govt. of Karnataka, Affiliated to VTU, Belagavi and approved by AICTE, New Delhi)
Sira Road, Tumkuuru – 572 106. Karnataka.

Academic Year 2019– 2020 (Even Semester)

FS 543667
An ISO 9001:2015 Certified Institution

Department of Civil Engineering

PERSONAL TIME TABLE **WEF: 19/09/2019**

Name of the Staff: Mr. Nagaraja C

TIME / DAYS	8:30 A M To 9:25 A M	9:25 A M To 10:20 A M	10:20 A M To 10:40 A M	10:40 A M To 11:35 A M	11:35 A M To 12:30 P M	12:30 P M To 1:30 P M	1:30 P M To 2:25 P M	2:25 P M To 3:20 P M	3:20 P M To 4:15 P M	4:15 P M To 4:30 P M
MONDAY			TEA BREAK	CIV		LUNCH BREAK	CABPAD		CIV	CONTACT SESSION
TUESDAY	CIV			SOM			CABPAD B1			
WEDNESDAY		CIV		**ADSRS**				INTERNSHIP		
THURSDAY	SOM				SOM		CABPAD B2			
FRIDAY		SOM			CIV		COUN B1			
SATURDAY	SOM			**ADSRS**						

Theory	: 30	Units
Practical	: 06	Units
Project Work	: 03	Units
Internship	: 03	Units
Counselling	: 02	Units
Total	: 44	Units

C. Nagaraja

(Mr. C Nagaraja)
Staff

(Dr. G Mahesh Kumar)
Head of the Department

(Dr. Narendra Viswanath)
Principal

B.E., Semester: III Year: 2019 - 20

Course Title: **Strength of Materials**	*Course Code:* **18CV32**
Total lecture hours /week: **5**	*Duration of Exam*: **03 Hrs.**
SEE Marks: **60**	*CIE marks:* **40**
Credits:04	
Lesson plan author: **Mr. Nagaraja C**	*Date: 25/07/19*
Checked by: **Dr. G Mahesh Kumar**	*Date: 25/07/19*

Course Objectives:

The course will enable the students

1. To understand the basic concepts of the stresses and strains for different materials and strength of structural elements and solutions to problems under different conditions.
2. To know the development of internal forces and resistance mechanism for one dimensional and two dimensional structural elements.
3. To analyse and understand different internal forces and stresses induced due to representative loads on structural elements.
4. To determine the slope and deflection of beams
5. To evaluate the behaviour of torsion members, columns and struts.

Course Outcomes:

The students will be able to:

1. Evaluate the strength of various structural elements internal forces such as compression, tension, shear, bending and torsion.
2. Suggest suitable material from among the available in the field of construction and manufacturing.
3. Evaluate the behavior and strength of structural elements under the action of compound stresses and thus understand failure concepts.
4. Evaluate the basic concepts of slopes and deflections of structural elements.
5. Understand the basic concept of analysis and design of structural elements such as columns and struts.

Materials and resources required:

Presentation: Black board, Teaching charts, Models. / OHP/ LCD presentations

Text books:

Strength of Materials – B S Basavarajaiah and P Mahadevappa, Universities Press, 3rd Edition, 2010.

Mechanics of Materials – Ferdinand P Beer, E Russel Johnston and Jr. John T DeWolf, Tata Mc Graw Hill, Third Edition.

Reference Books:

Elements of Strength of Materials – D H Young and S P Timoshenko, EastWest Press Pvt ltd., 5th Edition(Reprint 2014).

A Text book of Strength of Materials – R K Bansal, 4th Edition, Laxmi Publications, 2010.

Strength of Materials – S S Rattan, McGraw Hill Education (India) Pvt. ltd. 2nd Edition (Sixth Reprint 2013).

Analysis of structures – Vazirani V N, Ratwani, M M and S K Duggal, Vol 1, 17th Edition, Khanna Publishers, New Delhi.

Scheme of Examination:

The question paper will have ten questions, each full question carrying 20 marks. There will be two full questions (with a maximum of three subdivisions, if necessary) from each module. Each full question shall cover the topics under a module. The students shall answer five full questions selecting one full question from each module. If more than one question is answered in modules, the best answer will be considered for the award of marks limiting one full question answer in each module. The marks scored for 100 marks will be reduced to 60 marks proportionately.

Evaluation:

Student Assessment: Through Internal Assessment Tests (30 Marks), Assignments (10 marks), University Examination (60 Marks)

Lesson Plan

17CV32 – Strength of Materials

Sl No	Date	Topics	Topics Covered	Remarks
		Module 1: Simple stresses and strains		
1	25/07/19	Introduction, Definition and concept and of stress and strain. Hooke's law		
2	26/07/19	Stress-Strain diagrams for ferrous materials		
3	30/07/19	Stress – strain diagrams for non ferrous materials,		
4	30/07/19	Elongation of tapering bars of circular and rectangular cross – sections,		
5	31/07/19	Elongation due to self weight		
6	01/08/19	Problems		
7	02/08/19	Problems		
8	06/08/19	Saint Venant's principle, Compound bars, Temperature stresses		
9	06/08/19	Compound section subjected to thermal stresses		
10	07/08/19	Problems		
11	08/08/19	state of simple shear		
12	09/08/19	Elastic constants and their relationship		
13	13/08/19	Problems		
14	13/08/19	Problems		
15	16/08/19	Problems		

		Module 2: Compound stresses		
16	20/08/19	Introduction, state of stress at a point,		
17	20/08/19	General two dimensional stress system,		
18	21/08/19	Principal stresses and principal planes.		
19	22/08/19	Mohr's circle of stresses		
20	23/08/19	problems		
21	27/08/19	Theories of failure: Maximum shear stress theory and maximum principal stress theory		
22	27/08/19	problems		
23	28/08/19	**Thick and thin cylinders**: Introduction, Thin cylinders subjected to internal pressure		
24	29/08/19	Hoop stresses, Longitudinal stress and change in volume.		
25	30/08/19	Thick cylinders subjected to both internal and external pressure;		
26	03/09/19	Lame's equation, radial and hoop stress distribution.		
27	03/09/19	Problems		
28	04/09/19	Problems		
29	11/09/19	Problems		
		Module 3: Shear Force and Bending Moment in Beams:		
30	12/09/19	Introduction to types of beams, supports and loadings.		
31	13/09/19	Definition of bending moment and shear force, Sign conventions,		
32	17/09/19	relationship between load intensity, bending moment and shear force.		
33	17/09/19	Shear force and bending moment diagrams for statically determinate beams subjected to point load		
34	18/09/19	Shear force and bending moment diagrams for statically determinate beams subjected to uniformly distributed loads		
35	19/09/19	Shear force and bending moment diagrams for beams subjected to uniformly varying loads		
36	20/09/19	Shear force and bending moment diagrams for statically determinate beams subjected to couple and their combinations.		
37	24/09/19	problems		
38	24/09/19	problems		
39	25/09/19	problems		
40	26/09/19	problems		
41	27/09/19	problems		
42	01/10/19	Problems		
43	01/10/19	Problems		
		Module 4: Bending and shear stresses in beams		
44	03/10/19	Introduction, pure bending theory, Assumptions, derivation of bending equation		

45	04/10/19	modulus of rupture, section modulus, flexural rigidity, Problems		
46	09/10/19	Expression for transverse shear stress in beams,		
47	10/10/19	Bending and shear stress distribution diagrams for circular, rectangular sections		
48	11/10/19	Bending and shear stress distribution diagrams for circular, rectangular sections. Problems		
49	17/10/19	problems		
50	18/10/19	Bending and shear stress distribution diagrams for 'I', and 'T' sections Problems. Shear centre(only concept)		
51	22/10/19	**Torsion in Circular Shafts**		
52	22/10/19	Introduction, pure torsion, Assumptions, derivation of torsion equation for circular shafts,		
53	23/10/19	Torsional rigidity and polar modulus Power transmitted by a shaft,		
54	24/10/19	Problems		
55	25/10/19	problems		
56	30/10/19	problems		
57	31/10/19	problems		
58	05/11/19	problems		
59	05/11/19	problems		
60	06/11/19	problems		
		Module 5: Deflection of Beams		
61	07/11/19	Definition of slope, deflection and curvature, Sign conventions		
62	08/11/19	Derivation of moment – curvature equation		
63	12/11/19	Double integration and Macaulay's method		
64	12/11/19	Slope and deflection for standard loading cases		
65	13/11/19	Slope and deflection for determinate prismatic beams subjected to point loads, Udl, Uvl and couple		
66	14/11/19	problems		
67	19/11/19	problems		
68	19/11/19	problems		
69	20/11/19	**Columns and Struts:** Introduction, short and long columns. Euler's theory		
70	26/11/19	Assumptions, Derivation for Euler's Buckling load for different end conditions, Limitations of Euler's theory.		
71	26/11/19	Rankine - Gordon's formula for columns.		
72	27/11/19	Rankine - Gordon's formula for columns		
73	28/11/19	problems		
74	29/11/19	problems		

C.Nagaraj
Mr. C Nagaraja
Staff Incharge

Gmaheshkumar 24/7/19
Dr. G Mahesh Kumar
HOD

03-08-19
Dr Hemadri Naidu T
Principal

Sri Shridevi Charitable Trust (R.)
SHRIDEVI INSTITUTE OF ENGINEERING & TECHNOLOGY
(Recognised by Govt. of Karnataka, Affiliated to VTU, Belagavi and Approved by AICTE, New Delhi)
Sira Road, Tumakuru - 572 106. Karnataka.
Phone: 0816-2212629 | Fax: 0816-2212628 | Email: info@shrideviengineering.org | Web: http://www.shrideviengineering.org

DEPARTMENT OF CIVIL ENGINEERING

STUDENTS LIST FOR THE ACADEMIC YEAR 2019-20

SEM: III & IV (II YEAR)

Roll No	USN NO	Name of the Student
1	1SV17CV009	Kiran Kumar M T
2	1SV18CV003	Apoorva A
3	1SV18CV004	B M Meghashree
4	1SV18CV007	Chandan Gowda P
5	1SV18CV008	Chandrahasa Patel K A
6	1SV18CV011	Deepa R
7	1SV18CV013	Doddanagouda Policepatil
8	1SV18CV014	Habib Ulla Khan
9	1SV18CV015	Hanamesh
10	1SV18CV017	Hruthvik P
11	1SV18CV018	Jayashree P
12	1SV18CV019	Karthik G
13	1SV18CV023	Nagalakshmi
14	1SV18CV026	Pavan Nag M A
15	1SV18CV027	Pooja M
16	1SV18CV028	Priyanka M D
17	1SV18CV029	Roshan Mahato Singh
18	1SV18CV030	Sandeep Kumar C
19	1SV18CV036	Vishwanath H P
20	1SV19CV400	Aasima Sultana
21	1SV19CV401	Bharath M
22	1SV19CV402	Bhavana G
23	1SV19CV403	Chandan A S
24	1SV19CV404	Chinthana B S
25	1SV19CV405	Darshan R
26	1SV19CV406	Deepika V Jain
27	1SV19CV407	Dhanushree M N
28	1SV19CV408	Gayithri S N
29	1SV19CV409	Guru H M
30	1SV19CV410	Harshitha M P
31	1SV19CV411	Meghana B U
32	1SV19CV412	Onkaraswamy C M
33	1SV19CV413	Rakesh H M
34	1SV19CV414	Ravikumar G R
35	1SV19CV415	Ruchithashri K
36	1SV19CV416	S A Sai Prakash
37	1SV19CV417	Shimsha I S
38	1SV19CV418	Shivakumar G
39	1SV19CV419	Veda B G
40	1SV19CV420	Vinay C K

(Dr. G. Mahesh Kumar)
HOD

HOD
Dept. of Civil Engineering
SIET, TUMKUR - 6.

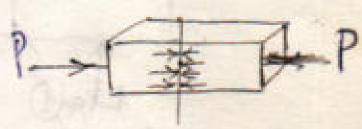
Simple Stresses and Strains

Introduction:

The subject of "Strength of materials" deals with the Capability of structures like beams, Columns, Shafts. Cylinders, Springs, etc. to Carry loads. Any Component needs to be designed so that the loads on the structure should not exceed the Capability of the material out of which they are made of. For the efficient designs, it is very necessary the materials have to be analysed for their strengths and hence the strength of materials is to be studied. SOM is an interdisciplinary subject and all the engineers need a basic course on SOM to be studied.

Stress:

Stress is the resistance offered by the body against externally applied force.
The external force acting on the body is known as "load"
When a body (member) of cross sectional area 'A' is subjected to load (P) as shown in fig1, then Stress 'σ' is given by



$$\text{Stress}: \sigma = \frac{\text{Load}}{\text{Cross sectional area}} = \frac{P}{A}.$$

$$\boxed{\sigma = \frac{P}{A}}$$

Units of Stress:

The basic unit of stress is Pascal.

01 pascal = 01 N/m² $\left[\frac{01}{1000 \times 1000} = 1\times10^{-6}\ N/mm^2\right]$

01 MPa = 01 Pascal × 10^6 = $\left[\frac{N}{1000\times1000}\right]\times10^6$
(01 MN/m²)
= 1 N/mm²

01 MPa = 01 N/mm² = 1×10^6 N/m²

01 GPa = 01×10^3 N/mm²

10N = 981 gms.
~~1kg = 981gms~~
98.1gm = 1N
kN/m²
1kg = 10.19N

01 MPa = 1 MN/m²
01 MPa = 1 N/mm²
1 GPa = 1000 N/mm²
1 GPa = 1000 MPa

The Basic Stresses:

The basic Stresses are as follows:

a) Normal Stress
   i) Tensile stress
   ii) Compressive stress.

b) Shearing Stress

NORMAL STRESS: (AXIAL STRESS) When external force is acting on a member with its line of action parallel to the axis of the member, the resulting internal stresses are also parallel to the axis. Such stresses are called normal stress or axial stresses.

OR

When the external force is acting on a member with its line of action normal to the cross sectional area of the member, the resulting internal stresses are called normal stresses or axial stresses.

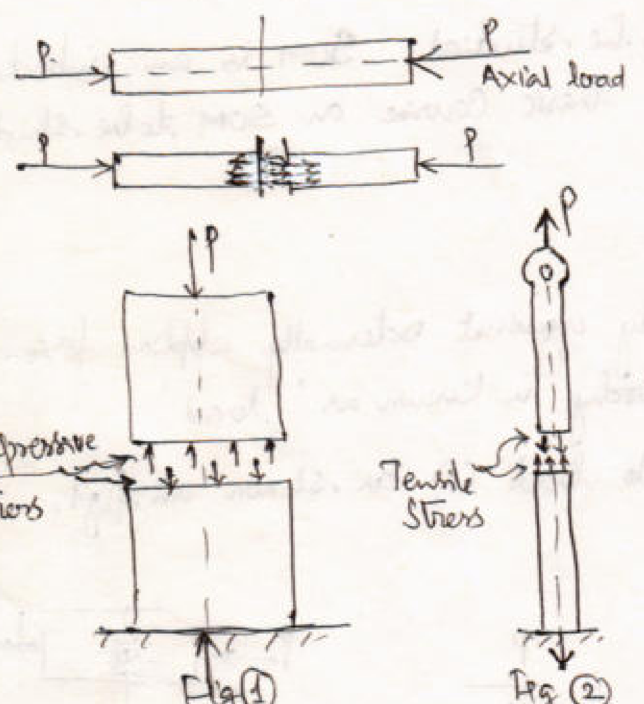
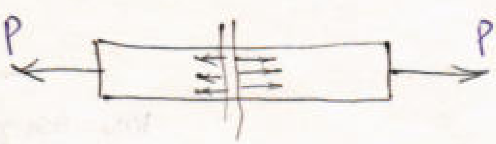


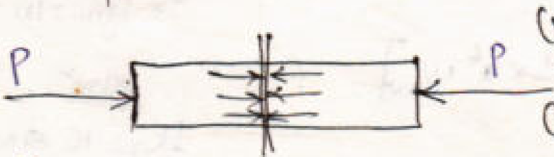
Fig ①   Fig ②

Tensile Stress:



When a member is subjected to tensile forces (pull), the stresses induced in the body are called Tensile stresses. (σ)

Compressive Stress:

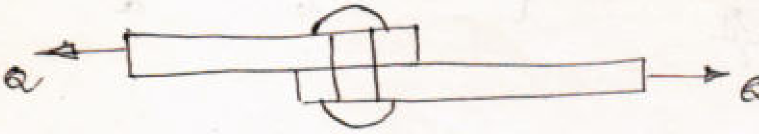


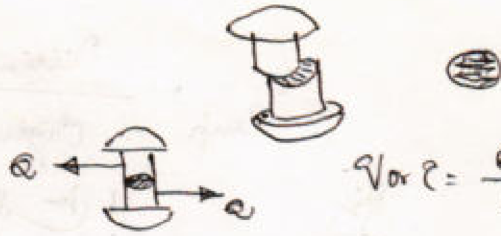
When a member is subjected to compressive forces (push), the stresses induced in the body are called compressive stresses. (σ)

Shear Stress:

When a member is subjected to (Shear) forces acting tangentially to the area of cross section, the stresses developed against the shearing force are called Shear Stress.

$$\text{Shear Stress} = \tau \text{ or } V = \frac{Q}{A} = \frac{\text{Shear force}}{\text{Tangential area of cross section}}$$





V or $\tau = \frac{Q}{A}$

Strain:

The ratio of change in dimension to its original dimension is called as Strain. Strain is a dimensionless quantity.

Mathematically,

$$\text{Strain} = e = \frac{\text{Change in dimension}}{\text{Original dimension}}$$

i) Normal Strain: [Axial strain]

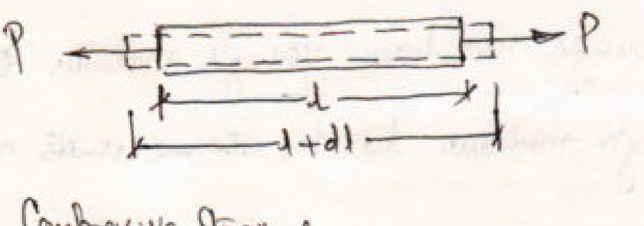
When the members are subjected to normal stresses, normal strain occurs. Normal strain is always parallel to the line of action of axial force.

a) Compressive Strain
b) Tensile Strain.

Tensile Strain:

When a member is subjected to tensile forces, the increase in the length of member occurs. [as shown in fig]. The ratio of increase in length to original length is called Tensile strain.

$$\text{Tensile strain} = e_{\text{Tensile}} = \frac{\text{Increase in length}}{\text{Original length}}$$

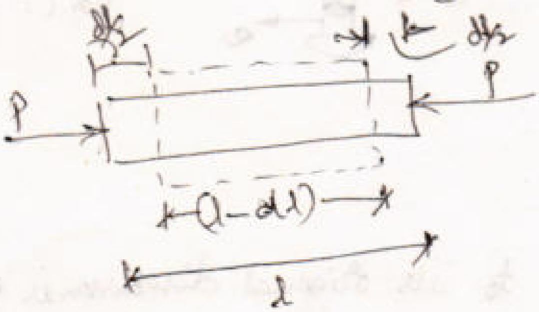


$$= \frac{\delta l}{l}$$

Compressive Strain:

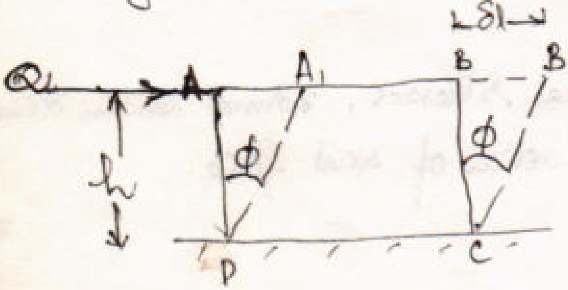
When a member is subjected to compressive forces, the decrease in the length of member occurs [as shown in fig]. The ratio of decrease in length to original length is called Compressive strain.

$$e_{Comp} = \frac{\text{Decrease in length}}{\text{Original length}} = \frac{\delta l}{l} \text{ or } \frac{\delta b}{b} \text{ or } \frac{\delta d}{d}$$



Shear Strain:

Consider a block ABCD subjected to tangential force Q as shown in fig. ~~Due to~~ The deformation occurs and AB is deformed to A_1B_1.



Then Shear strain is given by $\tan\phi$

$$\text{Shear Strain} = \tan\phi = \frac{BB_1}{AD} \quad \frac{\text{(Shear deformation)}}{\text{(original height/length)}}$$

Since ϕ is very small

$$\phi = \frac{BB_1}{h} = \frac{\delta l}{h} \text{ or } \frac{AA_1}{h}$$

Shear strain is the ratio of shear deformation by original length or height.
Shear strain is the change in the right angle of the element measured in radians. It is dimensionless.

* Hooke's Law:

Hooke's law states that " the stress is proportional to strain within the elastic limit".

Mathematically, $\sigma \propto e$

or $\sigma = E.e$ where E is called Modulus of Elasticity

The proportionality Constant 'E' is also young's modulus. young's modulus 'E' is given by $E = \frac{\sigma}{e}$. young's modulus has the same units as that of stress[MPa or GPa].

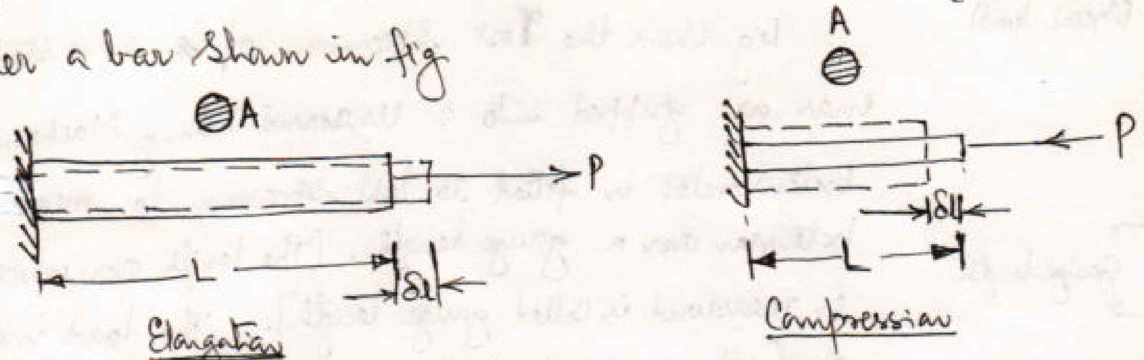
Young's Modulus or Modulus of elasticity:

It is defined as the ratio of Stress to Strain provided the stresses are within the elastic limits. It is denoted by the letter E. Mathematically, $E = \frac{\text{Stress}}{\text{Strain}} = \frac{\sigma}{e}$
It has the same units as that of stress i.e. MPa or GPa or N/mm^2.

Elongation or Shortening of a bar:

March 2001
Aug 1999

Consider a bar shown in fig



It is loaded axially as shown. Let 'A' be the area of cross section and L be its length. The stress in the bar is uniform and within the elastic limit.

The longitudinal strain $= e = \frac{\delta l}{L}$ and Stress $= \sigma = \frac{P}{A}$

As per Hooke's law,

$$E = \frac{\sigma}{e}$$

$$E = \frac{P/A}{\frac{\delta l}{L}} = \frac{P.L}{A.\delta l}$$

$$\boxed{\delta l = \frac{P.L}{A.E}}$$

Assumptions:

1) The stress and strains in the bar are uniformly distributed.
2) The stresses remain within the elastic limit and Hooke's law is valid.
3) The materials are homogeneous and isotropic
4) The stresses are average and considered at sections not in the immediate neighbourhood of the point of application of load.

Stress – Strain Curve for mild steel [Structural Steel]

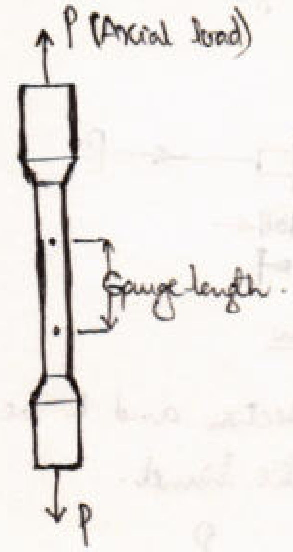
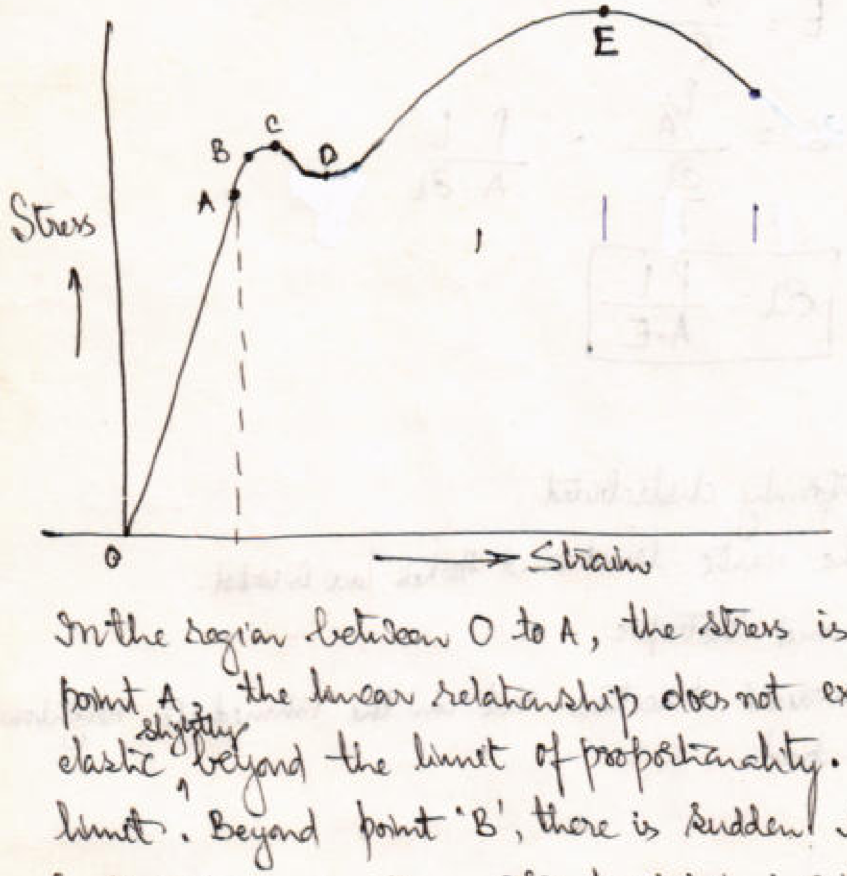
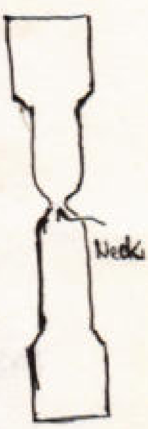


Fig shows the Test Specimen of ~~a~~ mild steel. Its ends are gripped into a Universal Testing Machine (UTM). Extensometer is fitted to test specimen to measure the extension over a gauge length. [The length over which the extension is measured is called gauge length]. The load is applied gradually and at regular intervals of loads, extension is measured. When the extension increases at a faster rate, extensometer is removed and is measured from scale on UTM. The typical stress – strain diagram for mild steel is as shown below: The salient points observed on the curve are as follows.



A = Proportionality limit
B = Elastic limit
C = upper yield point
D = Lower yield point
E = Ultimate point
F = Breaking point



In the region between O to A, the stress is proportional to strain. The Point 'A' is proportional limit. Beyond point A, the linear relationship does not exist. However, the material remains elastic slightly beyond the limit of proportionality. This point 'B' is called Elastic limit. Beyond point 'B', there is sudden increase in strain ~~without~~ with very small increase in stress. This point 'C' is called upper yield point. Then with no increase in stress, the strain increases upto point 'D'. This point 'D' is called lower yield point. After 'D', the material again increases resistance and ~~slightly~~ strain occurs with increase in stress upto point 'E'. This point 'E' is called Ultimate point and this is maximum stress, the material can resist. ~~At this stage the~~ ~~area~~ Further loading, causes the reduction in cross sectional area (necking of the specimen) and fracture or breaking occurs finally. This point 'F' is called Breaking point.

Definition of some of the terms:

Limit of proportionality: It is the limiting value of stress upto which stress is proportional to strain.

Elastic limit:

This is the limiting value of stress upto which the material is loaded and then unloaded, the strain disappears completely and original length is regained. This point is slightly beyond the limit of proportionality.

Upper yield point:

This is the stress at which the ~~stress~~ strain increases suddenly with ~~small~~ increase in stress. This phenomenon is called yielding of material. This occurs after the elastic limit.

Lower yield point:

This is the stress ~~at~~ upto which the strain continues with no increase in stress. This occurs after upper yield point

Ultimate Stress:

This is the maximum stress, that the material can resist. For mild steel, this stress is about 370–400 N/mm². At this stage normally, the cross sectional area at a particular section starts reducing. This is called neck formation. Hence the stress developed ~~also~~ starts reducing.

Breaking point:

The stress at which the specimen fails or breaks is called breaking stress.

Percentage Elongation and Percentage reduction in area:

These are the terms used to ~~define~~ measure the ductility of the material.

a) Percentage Elongation:

It is the ratio of final extension at rupture to original length expressed as percentage. ie

$$\text{Percentage Elongation} = \frac{\text{Length of specimen at Fracture} - \text{Length original}}{\text{original length}} \times 100$$

$$= \left[\frac{L' - L}{L}\right] \times 100$$

Percentage reduction in Area

It is defined as the ratio of maximum decrease in cross sectional area to original cross sectional area expressed as percentage.

i.e.

$$\text{Percentage reduction in Area} = \left[\frac{A - A'}{A}\right] \times 100$$

Where A = original cross sectional area

A' = minimum cross sectional area (at fracture)

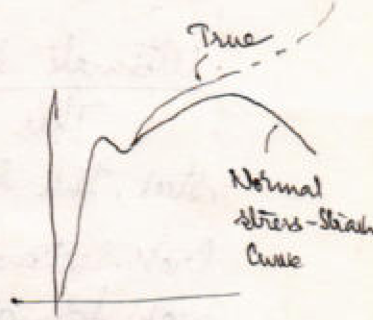
Nominal Stress and True stress:

In plotting stress-strain diagram, the original area of cross section is considered for computing all stress values.

Nominal Stress:

The ratio of load to original area of cross section is called nominal stress.

$$\text{Nominal Stress} = \frac{\text{Load}}{\text{Original area of cross section}}$$



$$\text{True Stress} = \frac{\text{Load}}{\text{Actual area of cross section}}$$

Note: If true stress is considered, it is increasing continuously as strain increases.

Factor of safety:

The ratio of ultimate stress to working stress is called factor of safety.

[For ductile materials, it is the ratio of yield stress to working stress]

$$\text{Factor of Safety} = \frac{\text{Ultimate Stress}}{\text{Working Stress}}$$

[Note: The maximum stress to which any member is designed is much less than the ultimate stress and this stress is called working stress.]

The value of factor of safety ~~vary~~ varies between 2 to 5

Factor of safety is considered for following reasons:

a) Reliability of material may not be 100 percent in designs.

b) The resulting deflections or deformations create problems in functional performance.

c) The loads taken for designs are only estimated but there can be excess loading. Unexpected impact and temperature loading may act on the member.

d) Assumptions are made in designs and calculated stresses will not be 100 percent real stresses.

Poisson's ratio:

When a prismatic bar undergoes change in length, it also undergoes changes in lateral directions. For example, if a bar is subjected to tension the length elongates and ~~[illegible]~~ cross sectional area decreases. Now there is a relationship between the strains in both dimensions.

The ratio of the lateral strain to longitudinal strain is known as Poisson's ratio.

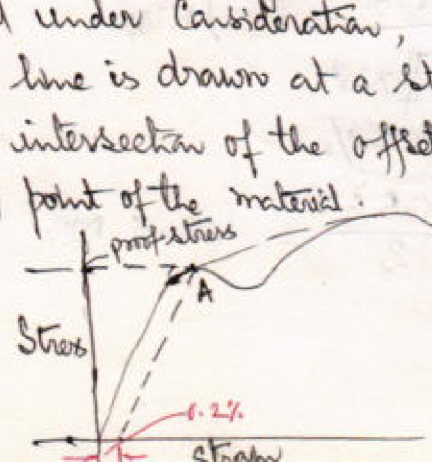
$$\text{Poisson's ratio} = \frac{\text{Lateral Strain}}{\text{Longitudinal Strain}} = \frac{e_{lateral}}{e_{longitudinal}}.$$

It is denoted by $\frac{1}{m}$ or μ. For most metals, its value is between 0.25 to 0.3

Proof stress:

The stress at which a non-proportional elongation equal to 0.2 percent of the original gauge length takes place.

Most of the metals except steel do not show well defined yield point but large strains are observed after the proportional limit is exceeded. ~~[illegible]~~ The proof stress can be found out by offset method. On the stress-strain diagram of the metal under consideration, a line is drawn parallel to initial linear curve. This line is drawn at a standard offset of strain value i.e. 0.002 [0.2%]. The intersection of the offset line and the stress strain curve ~~defines~~ gives the yield point of the material.



Some alloys of steel and light alloys of aluminium and magnesium do not indicate well defined yield points. In such cases, the stress-strain diagram may be curved even at origin. The yield point for such material is determined as the stress inducing a residual strain of 0.2%. In the material which is known as proof stress

Q1 The following data refer to a mild steel specimen tested in a laboratory. Dia of specimen = 25 mm. Gauge length of the specimen = 300 mm. Length of specimen after failure = 360 mm. extension observed under a load of 20 KN = 0.060 mm. load at yield point = 150 KN and load at failure = 252 KN. Neck diameter at failure point = 18.25 mm.

Determine a) young's modulus b) yield stress c) ultimate stress d) percentage elongation e) percentage reduction in cross sectional area. f) Safe stress adopting a factor of safety 2

Soln:-

a) Young's modulus:

$$E = \frac{\sigma}{e} = \frac{P/A}{\frac{\delta l}{l}} = \frac{\frac{20\times10^3}{\pi\times25^2/4}}{\frac{0.060}{300}} = 2\times10^5 \text{ N/mm}^2$$

b) Yield stress:

$$\sigma_y = \frac{\text{load at yield point}}{\text{Area of cross section}} = \frac{150\times10^3}{\frac{\pi\times25^2}{4}} = 305.6 \text{ N/mm}^2$$

c) Ultimate stress:

$$\sigma_u = \frac{\text{Ultimate load}}{\text{Area of cross section}} = \frac{252\times10^3}{\frac{\pi}{4}\times25^2} = 513.6 \text{ N/mm}^2$$

d) percentage elongation:

$$\%\ \text{elongation} = \frac{l_f - l}{l}\times100$$

$$= \frac{360-300}{300}\times100 = 20\%$$

e) percentage reduction in area $= \frac{A - A_f}{A}\times100$

$$= \left(\frac{\frac{\pi}{4}(25^2 - 18.25^2)}{\frac{\pi}{4}\times25^2}\right)\times100$$

$$= 46.7\%$$

f) Safe stress $= \frac{\text{Yield stress}}{\text{Factor of safety}} = \frac{305.6}{2} = 152.8$ MPa

02. A Circular rod of dia 20mm and 500 mm long is subjected to a tensile force of 45 KN. $E = 200\ kN/mm^2$. Find the stress, strain and elongation of the bar due to applied load.

Soln: Load $= 45 \times 10^3$ N

$E = 200 \times 10^3\ N/mm^2$

$L = 500$ mm

$d = 20$ mm.

$A = \frac{\pi \times 20^2}{4} = 314.159\ mm^2$

$$\text{Stress} = \frac{P}{A} = \frac{45\times10^3}{314.159} = 143.24\ N/mm^2$$

$$\text{Strain} = \frac{\sigma}{E} = \frac{143.24}{200\times10^3} = 0.0007162$$

$$\text{Elongation} = \delta l = \frac{P \cdot l}{AE} = \frac{45\times10^3 \times 500}{314.159 \times 200\times10^3} = 0.358\ mm$$

*03. A tension test was conducted on a specimen and the following readings recorded.

Dia = 22 mm, Gauge length = 200mm. LC of extensometer = 0.001mm

At a load of 22 KN, Extensometer reading = 60. (0.06mm)

36 KN, — ,, ——— = 94. (0.094mm)

Yield load = 95 KN, Max load = 157 KN.

Dia of neck = 15 mm, Final extension over 110mm original length = 132mm

Find young's modulus, yield stress, ultimate stress, percentage elongation and percentage reduction in area.

$$\text{Area} = \frac{\pi \times 22^2}{4} = 380.132\ mm^2$$

At a load of 22 KN, $\sigma = \frac{22\times10^3}{380.132} = 57.874\ N/mm^2$.

$e = \frac{0.06}{200} = 3\times10^{-4}$ or 0.0003

$E = \frac{\sigma}{e} = 192913.33\ N/mm^2$

At a load of 36 KN, $\sigma = \frac{36\times10^3}{380.132} = 94.70\ N/mm^2$

$e = \frac{0.094}{200} = 4.7\times10^{-4}$ or 0.00047

$E = \frac{\sigma}{e} = 2.0148935\ N/mm^2$

For 14 KN, extension observed $0.094 - 0.060 = 0.034$ mm

$$\sigma = \frac{14\times10^3}{380.132} = 36.83 \text{ N/mm}^2$$

$$E = \frac{\sigma}{e} = 216647.05 \text{ N/mm}^2$$

$$e = \frac{0.034}{200} = 1.7\times10^{-4}$$

$$\text{Yield Stress} = \frac{95\times10^3}{380.132} = 249.91 \text{ N/mm}^2$$

$$\text{ultimate stress} = \frac{157\times10^3}{380.132} = 413.01 \text{ N/mm}^2$$

$$\%\ \text{Elongation} = \left[\frac{132-110}{110}\right]\times100 = 20\%$$

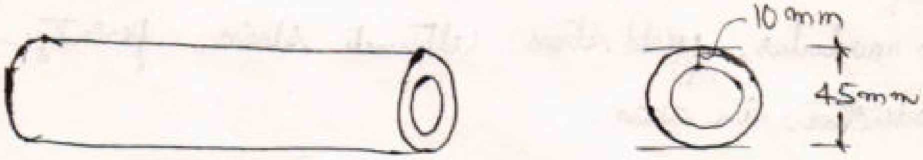
$$\%\ \text{Reduction in area} = \left[\frac{380.132 - \frac{\pi\times15^2}{4}}{380.132}\right]\times100 = 53.51\%$$

04. A steel pipe of length 6m, outside dia 45mm and wall thickness 10mm is subjected to an axial compressive load of 400kN. Assuming that $E = 200$ GPa and Poisson's ratio of 0.3. Find

a) Shortening of the pipe

b) The increase in outside dia

c) The increase in wall thickness.



Soln

$$\text{c/s Area of the pipe} = \frac{\pi}{4}(45^2 - 25^2)$$

$$= 1099.56 \text{ mm}^2$$

$L = 6000$ mm

$E = 200\times10^3$ N/mm²

$\mu = 0.3$

$$\delta l = \frac{P \cdot l}{A \cdot E}$$

$$= \frac{400 \times 10^3 \times 6000}{1099.56 \times 200 \times 10^3}$$

$$= 10.91 \text{ mm.}$$

From poisson's ratio

$$\mu = \frac{e_{lateral}}{e_{longitudinal}} \qquad e_{lng} = 1.818 \times 10^{-3}$$

$$e_{lateral} = \mu \times e_{lng} = \frac{10.91}{6000} \times 0.3 = 5.455 \times 10^{-4}$$

$$e_{lateral} = \frac{\delta d}{d} = 5.455 \times 10^{-4}$$

$$\therefore \delta d = 5.455 \times 10^{-4} \times 45 = 0.024 \text{ mm.}$$

IIIly $e_{lateral} = \frac{\delta t}{t} = 5.455 \times 10^{-4}$

$$\delta t = 5.455 \times 10^{-4} \times 10 = 0.00545 \text{ mm.}$$

A Compression member constructed from steel pipe has an outside dia of 90 mm and a cross sectional area of 1580 mm². What axial force P will cause the outside diameter to increase by 0.0094 mm? Given E = 200 GPa and 1/m = 0.3

Solution:

$$e_{lateral} = \frac{0.0094}{90} = 1.04 \times 10^{-4}$$

$$\mu = \frac{1}{m} = 0.3 = \frac{e_{lat}}{e_{lng}}$$

$$e_{lng} = \frac{1.04 \times 10^{-4}}{0.3} = 3.466 \times 10^{-4}$$

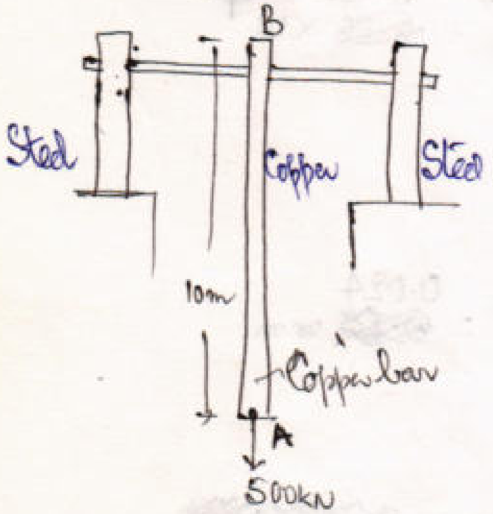
$$\sigma = e_{lng} \times E = 3.466 \times 10^{-4} \times 200 \times 10^3$$

$$= 69.32 \text{ N/mm}^2$$

$$\sigma = \frac{P}{A}$$

$$P = \sigma \cdot A$$
$$= 69.32 \times 1580$$
$$= 1.09 \times 10^5 \text{ N}$$

A Copper bar AB under a tensile load P = 500 kN hangs from a pin supported by two steel pillars. The Copper bar has length of 10 m, Cross sectional area 8100 mm² and $E_c = 103$ GPa. Each steel pillar has height 1 m, Cross sectional area 7500 mm², and $E_s = 200$ GPa. Determine the displacement of point A.



Soln:

$L_c = 10$ mtrs $\qquad L_s = 1$ m

$A_c = 8100$ mm² $\qquad A_s = 7500$ mm²

$E_c = 103 \times 10^3$ N/mm². $\qquad E_s = 200 \times 10^3$ N/mm²

The load acting on Copper bar is distributed equally on both steel pillars i.e. 250 kN on each steel pillars.
The Copper bar elongates and steel pillars Compress.

Elongation of Copper bar $= \delta l = \dfrac{P l_c}{A_c E_c}$

$$= \frac{500 \times 10^3 \times 10 \times 1000}{8100 \times 103 \times 10^3}$$
$$= 5.99 \text{ mm.}$$

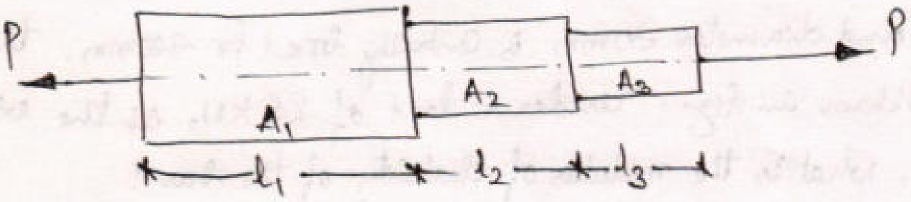
Compression of steel bar $= \delta l = \dfrac{P l_s}{A_s E_s}$

$$= \frac{250 \times 10^3 \times 1 \times 1000}{7500 \times 200 \times 10^3}$$
$$= 0.166 \text{ mm.}$$

Displacement of Point A = 5.99 + 0.166 = 6.156 mm

Bars with Cross sections varying in steps :

A bar with varying cross-sections and subjected to axial load is shown in fig. Let the length of three portions be l_1, l_2 and l_3 respectively and areas of Cross sections be A_1, A_2 and A_3 respectively.



The extension or elongation in the length of bar $= \delta l = \delta l_1 + \delta l_2 + \delta l_3$

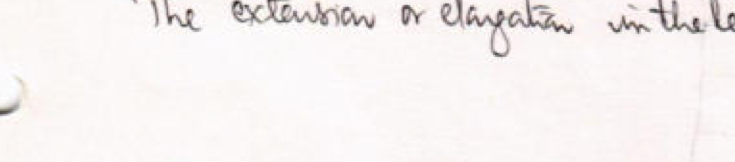
$$= \frac{P l_1}{A_1 E} + \frac{P l_2}{A_2 E} + \frac{P l_3}{A_3 E}.$$

A Steel bar 3m long has a circular section of diameter 1.5cm over one half of its length and diameter 1cm over the other half.

a) How much will be the extension under a tensile load of 50kN?

b) If the same volume of a material is rolled into a bar of constant diameter 'd' and length 3m, what will be elongation under the same load? Take $E = 200 GPa$.

Soln:



$$A_1 = \frac{\pi \times 15^2}{4} = 176.41 \; mm^2$$

$$A_2 = \frac{\pi \times 10^2}{4} = 78.53 \; mm^2$$

a) The total extension $= \delta l = \frac{P l_1}{A_1 E} + \frac{P l_2}{A_2 E}$

$$= \frac{50 \times 10^3 \times 1500}{176.71 \times 200 \times 10^3} + \frac{50 \times 10^3 \times 1500}{78.53 \times 200 \times 10^3}$$

$$= 6.895 \; mm$$

b)

Total volume of bar $176.71 \times 1500 + 78.53 \times 1500$

$$= 1500(255.24)$$

$$= 382860 \; mm^3$$

If the whole volume is rolled into a bar of constant dia 'd' and length 3m

$$\frac{\pi d^2}{4} \times 3000 = 382860$$

$$d = 12.75 \; mm.$$

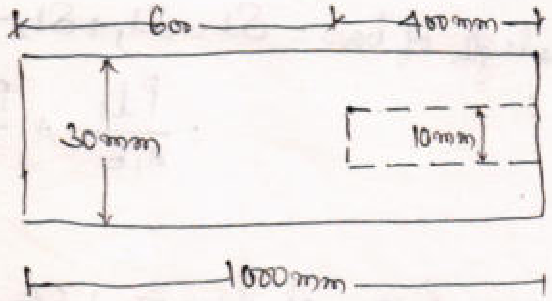
$$\delta l = \frac{PL}{AE}$$

$$= \frac{50\times10^3 \times 3000}{\frac{\pi\times(12.75)^2}{4}\times 2.00\times10^5}$$

$$= 5.874 \text{ mm.}$$

A bar of length 1000mm and diameter 30mm is centrally bored for 400mm, the bore diameter being 10mm as shown in fig. Under a load of 25 kN, if the extension of the bar is 0.185 mm, what is the modulus of elasticity of the bar?

Soln:



$L_1 = 600mm$ $\quad A_1 = \frac{\pi d_1^2}{4} = \frac{\pi\times30^2}{4} = 706.85 \text{ mm}^2$

$d_1 = 30mm$

$L_2 = 400mm$ $\quad A_2 = \frac{\pi(d_1^2 - d_2^2)}{4} = 628.31$

$d_1 = 30, \; d_2 = 10mm$

The total extension $= \delta l = \delta l_1 + \delta l_2$

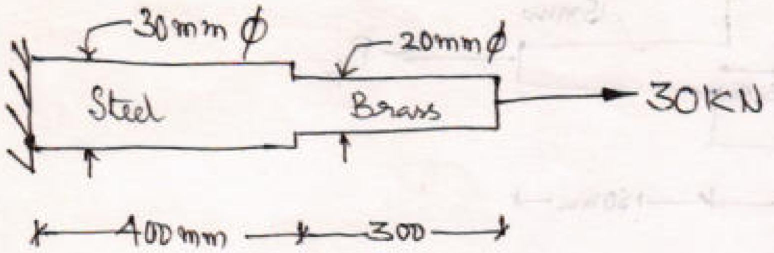
$$= \frac{Pl_1}{A_1E} + \frac{Pl_2}{A_2E}$$

$$0.185 = \frac{25\times10^3}{E}\left[\frac{600}{706.85} + \frac{400}{628.31}\right]$$

$$E = \frac{25\times10^3}{0.185}$$

$$= 2.007\times10^5 \text{ N/mm}^2 = 200739 \text{ N/mm}^2$$

The composite bar shown in fig is subjected to a tensile force of 30KN. The extension observed is 0.186 mm. Find E_{brass} if $E_{steel} = 2\times10^5$ N/mm²



Soln.

$$\delta l = \delta l_{brass} + \delta l_{steel}$$

$$= \frac{P \cdot l_b}{A_b E_b} + \frac{P l_s}{A_s E_s}$$

$$0.186 = \frac{30\times10^3 \times 300}{314.159 \times E_b} + \frac{30\times10^3 \times 400}{706.86 \times 2\times10^5}$$

$$E_b = \frac{30\times10^3 \times 300}{314.159\times 0.101} = 283313.49 \text{ N/mm}^2$$

$$= 2.83\times10^5 \text{ N/mm}^2.$$

Principle of Superposition:

It states that "the net deformation caused due to several loads acting simultaneously is equal to the algebraic sum of deformations due to each load acting separately"

or

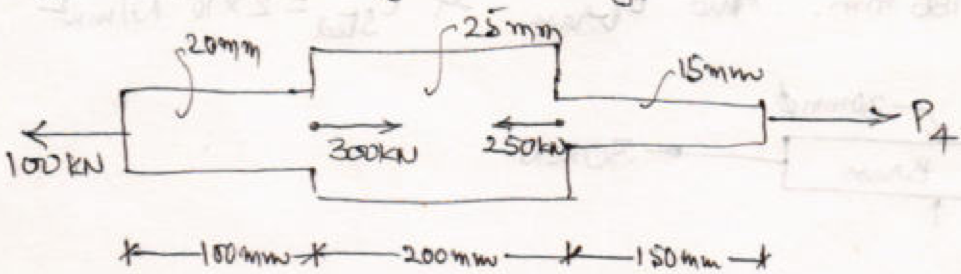
When a number of loads are acting on a member, the resultant strain will be the algebraic sum of individual strains caused by each load separately.

Prerequisites or Conditions:

1) Axial loads and the displacements they produce are linearly related.

2) Stresses do not exceed the proportional limit for the material.

Determine the stresses in various segments of the circular bar shown in fig. Compute the elongation taking $E = 195$ GPa. Aug 2000

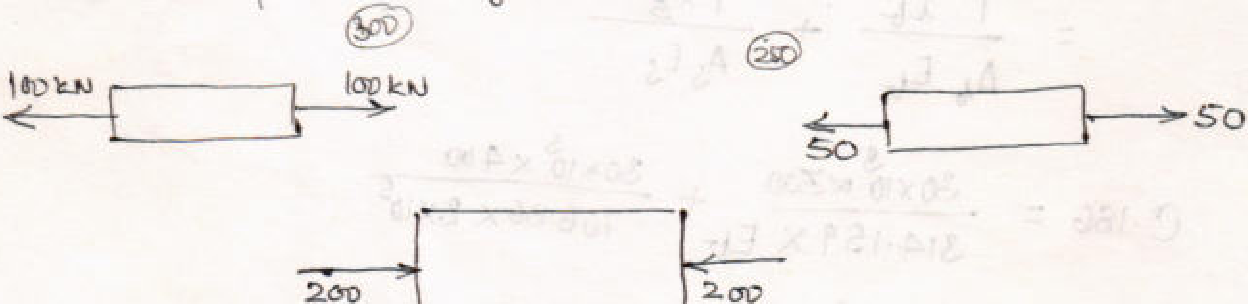


For the bar to be in equilibrium

$$-100 + 300 - 250 + P_4 = 0$$

$$P_4 = 50 \text{ kN}$$

The FBD of the each segments are as below:



$$\delta l = \delta l_1 - \delta l_2 + \delta l_3$$

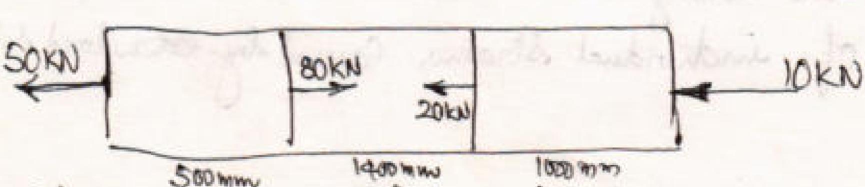
$$= \frac{1}{E}\left[\frac{P_1 l_1}{A_1} - \frac{P_2 l_2}{A_2} + \frac{P_3 l_3}{A_3}\right]$$

$$= \frac{1}{195\times10^3}\left[\frac{100\times10^3\times100}{314.159} - \frac{200\times10^3\times200}{490.87} + \frac{50\times10^3\times150}{176.71}\right]$$

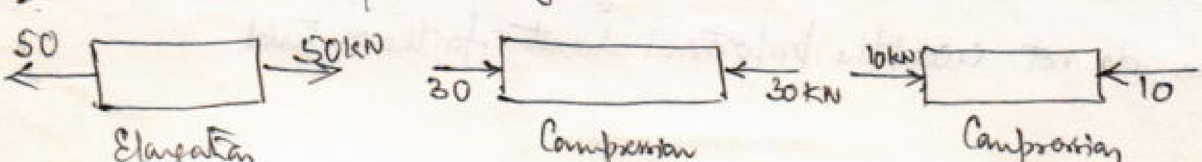
$$= -0.037 \text{ mm (Compression)}$$

A brass bar having cross sectional area 300 mm^2 is subjected to axial forces as shown in fig 1. Find the total elongation of the bar. Take $E = 84$ GPa

March 2001



Soln The FBD's of each segments are as shown below:



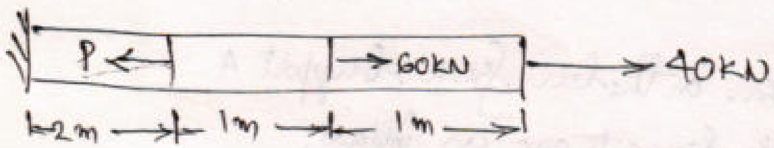
$$\delta l = \delta l_1 - \delta l_2 - \delta l_3$$

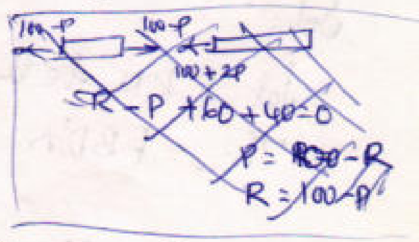
$$= \frac{50\times10^3 \times 500}{300 \times 84\times10^3} - \frac{30\times10^3\times1400}{300\times84\times10^3} - \frac{10\times10^3 \times 1000}{300\times84\times10^3}$$

$$= \frac{10^3}{300\times84\times10^3}\left[-27000\right] = -1.071\,mm$$

Determine the magnitude of load P necessary to produce zero net change in the length of the straight steel bar shown in fig $A = 400\,mm^2$

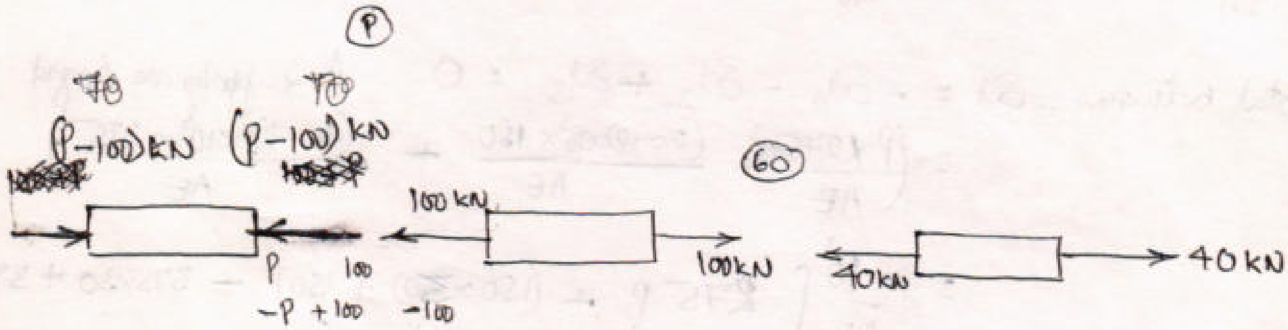
Aug 2001 10 marks.





Soln:

The FBD's of each of the segments are as shown in fig.



The total deformation $\delta l = -\delta l_1 + \delta l_2 + \delta l_3$

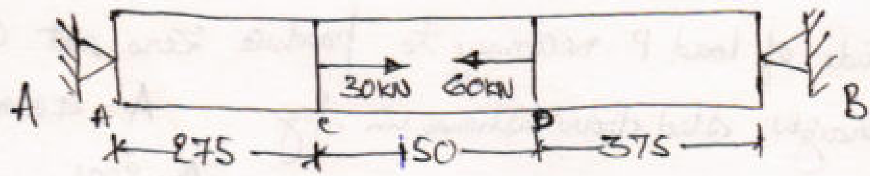
$$0 = -\frac{(P-100)\times 2000\times10^3}{400\times E} + \frac{100\times10^3\times1000}{400\times E} + \frac{40\times10^3\times1000}{400\times E}$$

$$0 = +(100-P)\times 2\times10^6 + 1\times10^8 + 40\times10^6$$

$$P = \frac{2\times10^8 + 1\times10^8 + 0.4\times10^8}{2\times10^6} = \frac{10^8(2+1+0.4)}{10^6\times2} = 170\,kN$$

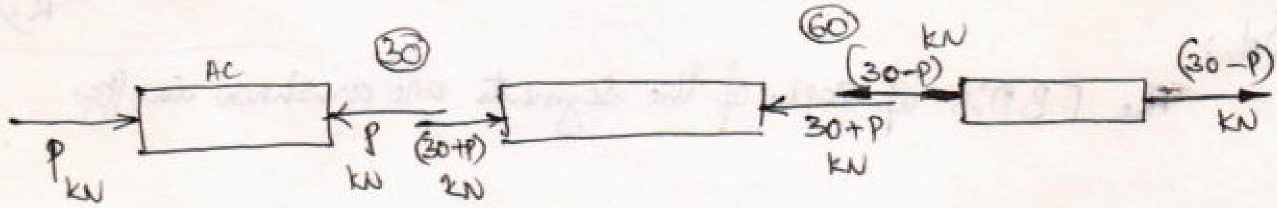
A bar 800 mm length is attached rigidly at A and B as shown in fig. Forces 30 kN and 60 kN act as shown on the bar. If E = 200 GPa, determine the reactions at the two ends. If the bar diameter is 25 mm, find the stresses and change in the length of each portion.

Unyielding supports.



Soln:

Let P kN be the reaction on the bar from support A.
Then FBD's of each segments are as follows:



Total extension $= \delta l = -\delta l_1 - \delta l_2 + \delta l_3 = 0$ As supports are rigid

$$= -\left(\frac{P \times 275 \times 10^3}{AE} + \frac{(30+P)\times 10^3 \times 150}{AE} - \frac{(30-P)\times 10^3 \times 375}{AE}\right)$$

$-(30+P)$
$-(30-P)$

$$= -\frac{10^3}{AE}\left[275P + (150\times 30) + 150P - 375\times 30 + 375P\right)$$

$-30-P$
$-30+P$

$$0 = -10^3\left[275P + 150P + 375P + 4500 - 11250\right]$$

$$800\times 10^3 P = +10^3 \times 6750$$

$$P = 8.4375 \text{ kN}$$

Reaction of Support at A = 8.4375 kN. Reaction at B = 30 − P = 21.5625 kN

Stresses : Portion AC $\sigma_{AC} = \frac{8.4375}{490.87384} = 0.017 \text{ kN/mm}^2$
$= 17.18 \text{ N/mm}^2$ (Comp)

$A = \frac{\pi \times 25^2}{4}$

$\sigma_{CD} = \frac{38.4375\times 10^3}{490.87} = 78.30 \text{ N/mm}^2$ (Comp)

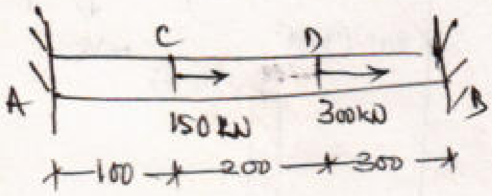
$\sigma_{DB} = \frac{21.56\times 10^3}{490.87} = 43.92$ (Tensile)

Shortening of AC $= \delta l_1$ or $\delta l_{AC} = \dfrac{8.43 \times 10^3 \times 275}{490.87 \times 200 \times 10^3} = 0.02363$ mm

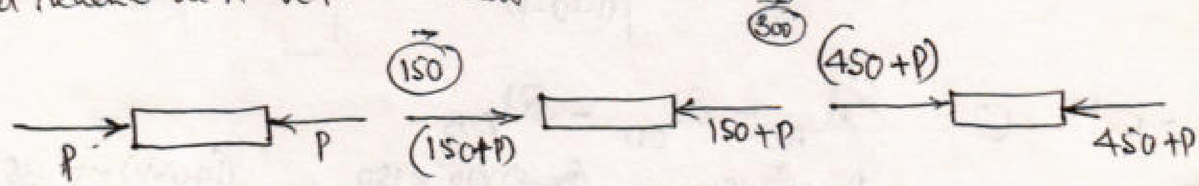
$$\delta l_{CD} = \frac{38.43 \times 10^3 \times 150}{490.87 \times 200 \times 10^3} = 0.05873 \text{ mm}$$

$$\delta l_{DB} = \frac{21.56 \times 10^3 \times 375}{490.87 \times 2 \times 10^5} = 0.08236 \text{ mm}$$

A horizontal spacer bar of uniform cross section is fastened at its two ends to ~~an~~ unyielding walls. Determine the reaction exerted by the wall at end B, if loads are applied at intermediate points.



Soln. Let reaction at A be P. Then FBD are as follows



$$\delta l = 0 = \delta l_{AC} + \delta l_{CD} + \delta l_{DB}.$$

$$0 = -\frac{P \times 100 \times 10^3}{AE} - \frac{(150+P) \times 10^3 \times 200}{AE} - \frac{(450+P) \times 10^3 \times 300}{AE}$$

$$-100P \times 10^3 - 150 \times 10^3 \times 200 - P \times 10^3 \times 200 - 450 \times 10^3 \times 300 - P \times 10^3 \times 300 = 0$$

$$-\left(P \times 10^5 + P \times 2 \times 10^5 + 3 \times P \times 10^5 + 150 \times 10^5 \times 2 + 450 \times 3 \times 10^5\right) = 0$$

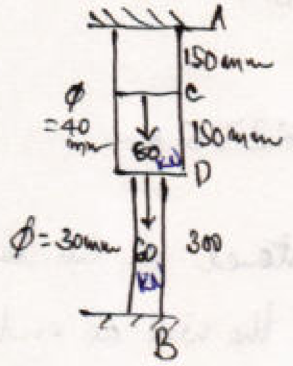
$$6P \times 10^5 = -10^5(300 + 1350)$$

$$P = \frac{-1650}{6}$$

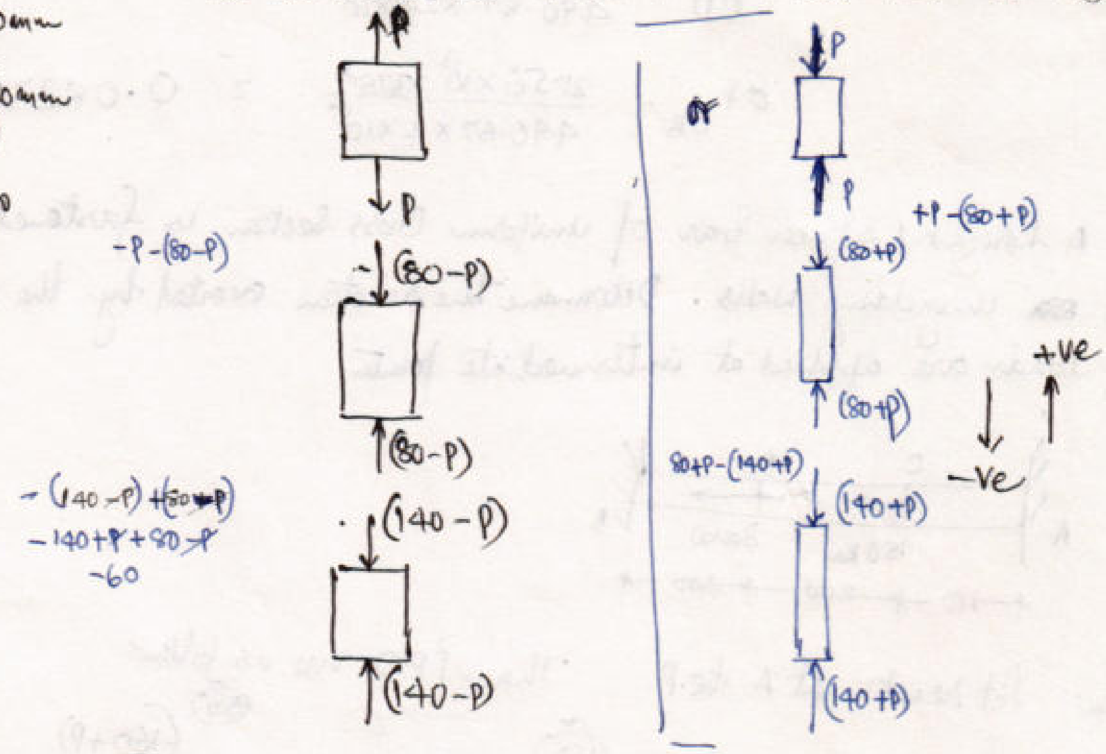
$$P = R_A = -275 \text{ kN} \quad (\text{Tensile})$$

$$R_B = 450 - 275 = 175 \text{ kN} \quad (\text{Comp})$$

A stepped bar of steel is held between two unyielding supports as shown. Find the reactions at A and B.



Soln Let the reaction at A be P.. Then FBD are as follows



Total extension $= \delta l = 0 = \delta l_{AC} - \delta l_{CD} - \delta l_{DB}$

$$= \frac{P \cdot \times 10^3 \times 150}{E \times 1256.64} - \frac{(80-P) \times 10^3 \times 150}{E \times 1256.64} - \frac{(140-P) \times 10^3 \times 300}{E \times 706.86}$$

$$0 = \frac{1}{E}\left[119.365P - 9549.26 + 119.365P - 59417.4 + 424.41P\right]$$

$$R_A = P = \frac{68966.66}{663.14} = 104 \text{ kN (Tensile)}$$

$$R_B = 140 - 104 = 36 \text{ kN (Compressive)}$$

II method.

If there is no wall at B, then load in DB is zero, in CD, it would be 60 kN and in AC, it would be 140 kN.

$\therefore$ Elongation of AB = Elongation of AC + $\delta l_{CD} + \delta l_{DB}$ (δl_{AC})

$$= \frac{140 \times 10^3 \times 150}{1256.6E} + \frac{60 \times 10^3 \times 150}{1256.63E} + 0$$

$$= \frac{1}{E}(23.87 \times 10^3) \text{ mm} \quad -(1)$$

Due to R_B developed at B, Contraction occurs.

Contraction due to $R_B = \dfrac{R_B \times 1000}{E}\left[\dfrac{300}{706.85} + \dfrac{150}{1256.63} + \dfrac{150}{1256.63}\right]$

$$= \frac{R_B}{E} \times 663.15 \quad -(2)$$

Equating (1) and (2)

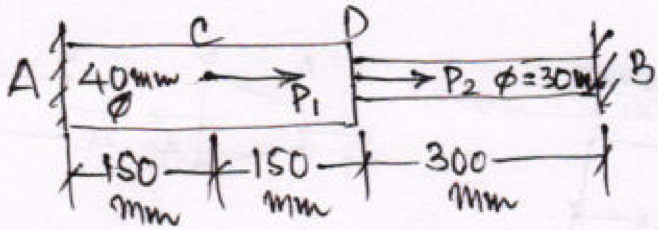
$$R_B = \frac{23.87 \times 10^3}{663.15} = 36 \text{ kN (Comp)}$$

Then $\Sigma F_x = 0$

$R_A + 36 - 80 - 60 = 0$

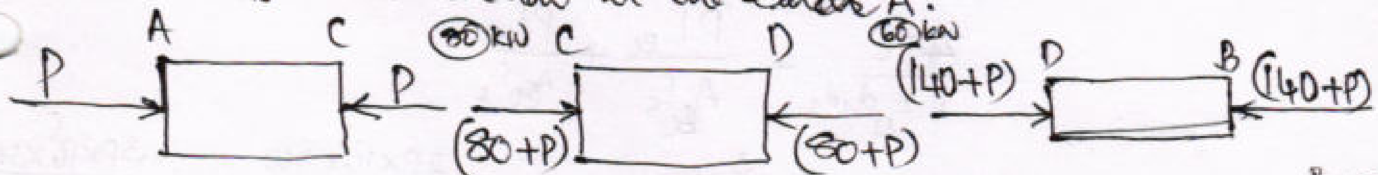
$R_A = 104$ kN (Tens)

A stepped bar of steel held between two supports as shown in fig, is subjected to loads $P_1 = 80kN$ and $P_2 = 60kN$. Find the reactions developed at the ends A and B.



Soln:

FBD of each segment of bar are as follows:
Let P be the reaction at the end A.



$-P+F=80$
$F=80+P$

$-(80+P)+P_2=60$
$P=60+80+P=(140+P)$

$+P+80+60$
$-140-P=0$

Due to rigidity, the displacements are zero.

$$dl = 0 = -dl_{ac} - dl_{cd} - dl_{db}$$

$$= \frac{-1}{E}\left[\frac{P\times10^3\times150\times4}{\pi\times40^2} + \frac{(80+P)\times150\times4\times10^3}{\pi\times40^2} + \frac{(140+P)\times300\times4\times10^3}{\pi\times30^2}\right]$$

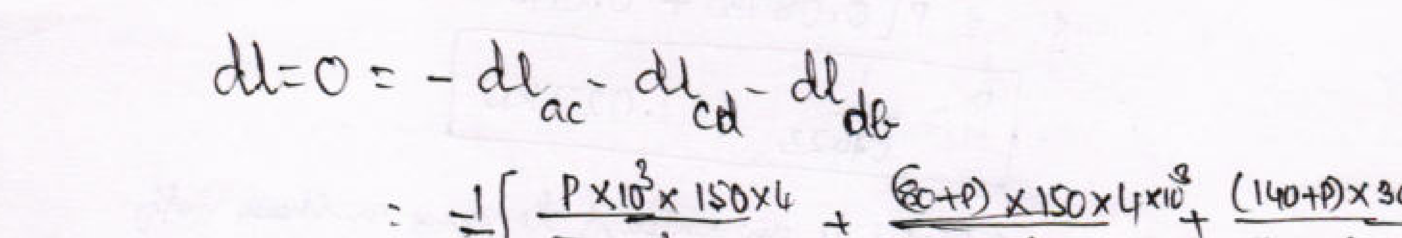
$$0 = -\left[P\times119.366 + 9.549\times10^3 + 0.1193P\times10^3 + 59.417\times10^3 + 0.4244P\times10^3\right]$$

663.066 $P = -68966$

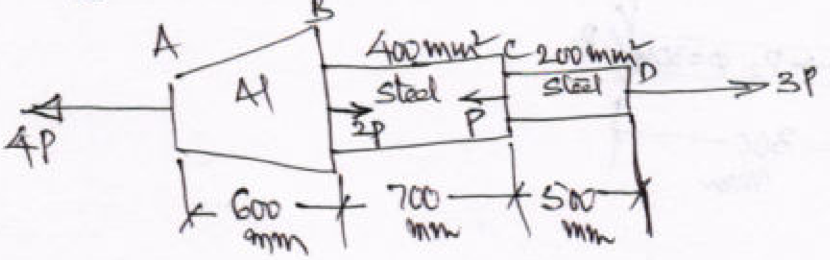
$\boxed{P = 104.01kN}$

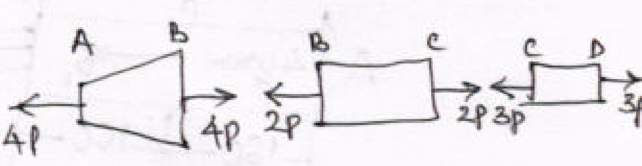
Tensile

Compressive



A round bar with stepped portion is subjected to these forces as shown in fig. Determine the magnitude of force P such that the net deformation in the bar does not exceed 1mm. E for steel is 200 GPa and that for aluminium is 70GPa. Big end diameter and small end diameter are 40mm and 12.5mm respectively.





$$dl = 1 = dl_{AB} + dl_{BC} + dl_{CD}$$

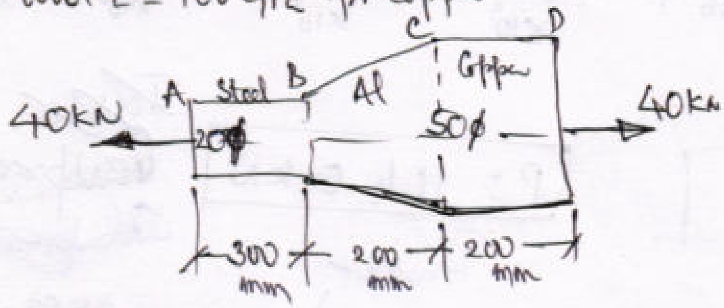
$$= \frac{4PL}{\pi E_{Al} d_1 d_2} + \frac{PL_{BC}}{A_{BC} E_s} + \frac{PL_{CD}}{A_{CD} E_s}$$

$$= \frac{4 \times 4P \times 10^3 \times 600}{\pi \times 70 \times 10^3 \times 40 \times 12.5} + \frac{2P \times 10^3 \times 700}{400 \times 200 \times 10^3} + \frac{3P \times 10^3 \times 500}{200 \times 200 \times 10^3}$$

$$1 = P[0.08730 + 0.0175 + 0.0375]$$

$$\boxed{P = \frac{1}{0.1423} = 7.027\,kN}$$

A stepped bar is subjected to an external loading as shown in fig. Calculate the change in the length of bar. Take E = 200 GPa for steel, E = 70 GPa Al and E = 100 GPa for Copper



$$dl = dl_{AB} + dl_{BC} + dl_{CD}$$

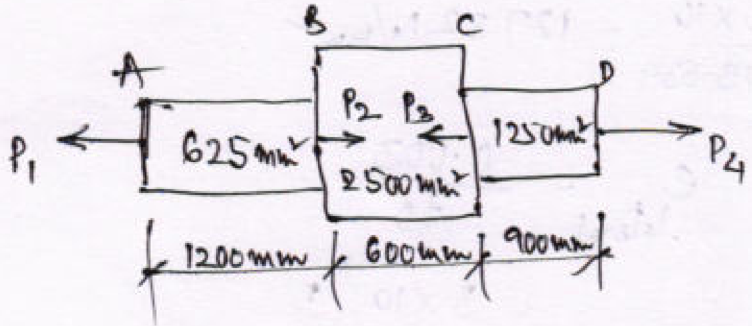
$$dl = \frac{40 \times 10^3 \times 300}{\frac{\pi \times 20^2}{4} \times 200 \times 10^3} + \frac{4 \times 40 \times 10^3 \times 200}{\pi \times 70 \times 10^3 \times 20 \times 50} + \frac{40 \times 10^3 \times 200}{\frac{\pi \times 50^2}{4} \times 200 \times 10^3}$$

$$dl = 0.19098 + 0.1455 + 0.02037$$

$$\boxed{dl = 0.35685\ mm}$$

A member ABCD is subjected to Point loads P_1, P_2, P_3 and P_4 as shown in fig. Calculate the force P_2 necessary for equilibrium, if $P_1 = 45kN$, $P_3 = 450kN$ and $P_4 = 130kN$. Determine the total elongation of the member assuming the modulus of elasticity to be $2.1 \times 10^5 N/mm^2$

June/July 2013



Soln

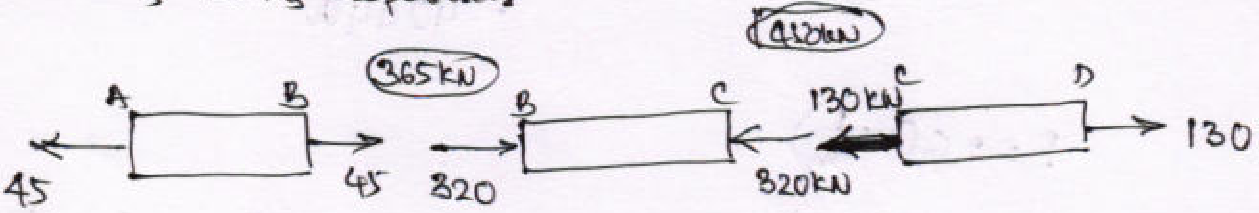
For Equilibrium,

$$P_1 + P_2 + P_3 + P_4 = 0$$

$$-45 + P_2 - 450 + 130 = 0$$

$$\boxed{P_2 = 365kN}$$

Considering portions separately



$$\delta l = \delta l_1 + \delta l_2 + \delta l_3$$

$$= \frac{45 \times 10^3 \times 1200}{625 \times 2.1 \times 10^5} - \frac{320 \times 10^3 \times 600}{2500 \times 2.1 \times 10^5} + \frac{130 \times 10^3 \times 900}{1250 \times 2.1 \times 10^5}$$

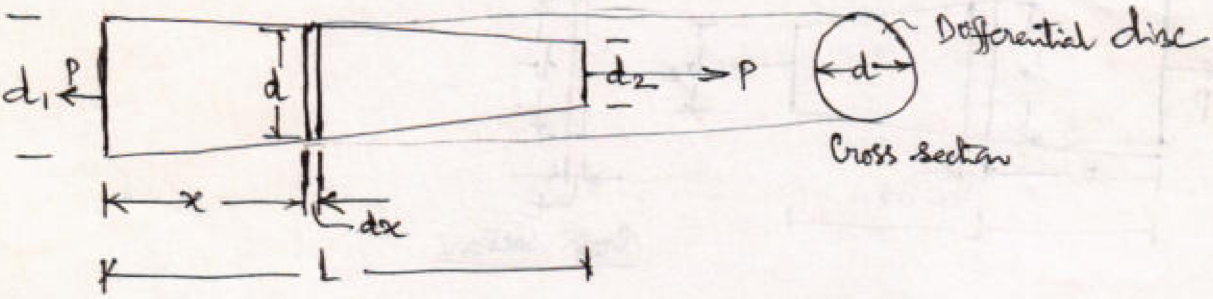
$$= 0.41142 - 0.36571 + 0.44571$$

$$\boxed{\delta l = 0.49142\ mm}$$

Derivation for ~~stress~~ strains in bars of Varying Cross section: Tapering circular bar

A tapering rod has a diameter d_1 at one end and it tapers uniformly to a diameter d_2 at the other end in a length 'L' as shown in fig. If modulus of elasticity of the material is E, find the Change in length when Subjected to an axial force 'P'.

Aug 2001



Change in diameter in length 'L' is $= d_1 - d_2$

$\therefore$ Rate of change of diameter or decrease in diameter per unit length $= \dfrac{d_1 - d_2}{L} = k$

Consider an elemental length dx at a distance 'x' from larger end.

The diameter of the elemental bar at this section $= d = d_1 - k \cdot x$

$$\text{Cross sectional area} = \frac{\pi}{4}(d_1 - k \cdot x)^2$$

$$\therefore \text{Extension of the elemental bar} = \frac{P \cdot dx}{\frac{\pi}{4}(d_1 - k \cdot x)^2 \cdot E}$$

$$\text{Extension of entire bar} = \delta l = \int_0^L \frac{P \cdot dx}{\frac{\pi}{4}(d_1 - k \cdot x)^2 \cdot E}$$

$$= \frac{4P}{\pi E}\int_0^L \frac{dx}{(d_1 - kx)^2}$$

$$= \frac{4P}{\pi E}\int_0^L \frac{t^{-2}}{-k}\, dt$$

$$= \frac{4P}{\pi E k}\left[\frac{t^{-1}}{(-2+1)}\right]_0^L$$

when $x = 0$, $t = d_1$

$x = L$, $t = d_1 - kL$

$d_1 - kx = t$

$-k\,dx = dt$

$dx = -\frac{dt}{k}$

We know that $d_1 - d_2 = kL$

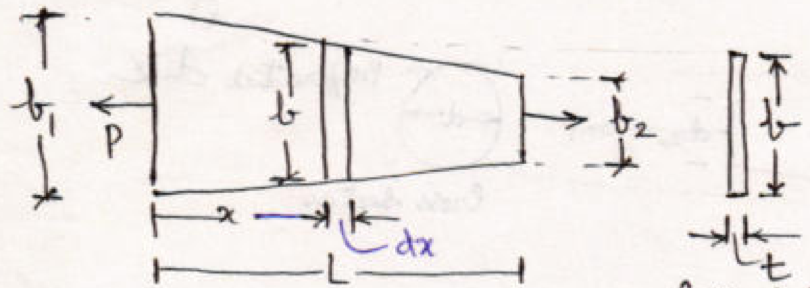
$d_1 - kL = d_2$

$$= \frac{4P}{\pi E k}\left[\frac{1}{[d_1 - kL]} - \frac{1}{d_1}\right]$$

$$= \frac{4P}{\pi E k}\left[\frac{1}{d_2} - \frac{1}{d_1}\right] = \frac{4PL}{\pi E(d_1 - d_2)}\left[\frac{d_1 - d_2}{d_1 d_2}\right]$$

$$\boxed{\delta l = \frac{4PL}{\pi E d_1 d_2}}$$

A bar of uniform thickness 't' tapers uniformly from a width of b_1 at one end to b_2 at other end in a length 'L' as shown in fig. Find the expression for the change in length of the bar when subjected to an axial force 'P'.



Cross-section

Soln: Consider a elemental length dx at a distance 'x' from larger end.

Rate of Change of breadth or Decrease in breadth per unit length $= \frac{b_1 - b_2}{L} = K$

Width at section 'x' $= b = b_1 - \frac{b_1 - b_2}{L} \cdot x$

$= b_1 - Kx.$

Crosssectional area of the element $= A = t\,(b_1 - kx)$

Extension of element $= \frac{P \cdot dx}{A \cdot E}$

$= \frac{P \cdot dx}{t(b_1 - kx) \cdot E}$

Total extension of bar $= \delta l = \int_0^L \frac{P \cdot dx}{tE(b_1 - kx)}$

$= \frac{P}{tE} \int_0^L \frac{dx}{(b_1 - kx)}$ put $b_1 - kx = u$

$-k\,dx = du$

$= \frac{P}{tE(-k)} \int_0^L \frac{du}{u}$ $= \frac{P}{-tEK}\left[\log u\right]_{b_1}^{b_1 - kL}$ $dx = \frac{du}{-k}$

when $x = 0$, $u = b_1$; $x = L$, $u = b_1 - kL$

$= \frac{P}{-tEK}\left[\log_e (b_1 - kx)\right]_0^L$

$= \frac{P}{-tEK}\left[\log_e \left(b_1 - \frac{(b_1 - b_2)}{L} \cdot x\right)\right]_0^L$

$= \frac{P}{-tEK}\left[\log_e b_1 - \log_e b_1 + \log_e b_2 - \log_e b_1\right]$

$\frac{P}{-tEK}\left[\log_e b_2 - \log_e b_1\right]$

$\delta l = \frac{P}{tEK}\left[\log_e b_1 - \log_e b_2\right]$

$= \frac{P}{tEK} \log_e \frac{b_1}{b_2}$

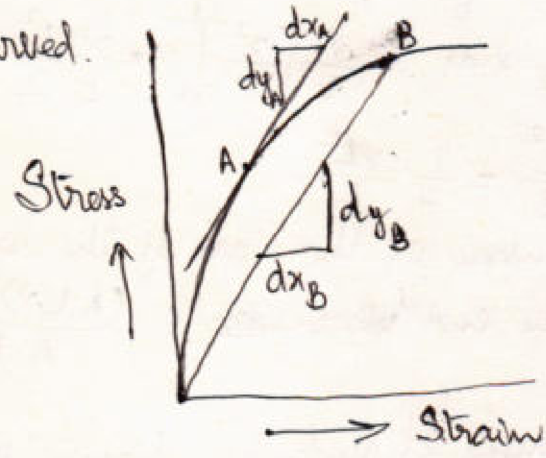
$\boxed{\delta l = \frac{PL}{tE(b_1 - b_2)} \log_e \frac{b_1}{b_2}}$

Strain Hardening:

After undergoing the large strains in the region of upper and lower yield points, the steel begins to strain harden. Strain hardening is a process where material undergoes changes in its atomic & crystalline structure. This process brings in a new lease of life for the material and it regroups its resistance to further loading. Thus to elongate the specimen further, the load must be increased until the ultimate stress point is reached.

Secant modulus & Tangent Modulus:

There are materials such as cast iron, concrete, etc which are imperfectly elastic. The stress-strain diagram for such materials are continuously curved.



$$\text{Tangent Modulus} = \frac{dy_A}{dx_A}$$

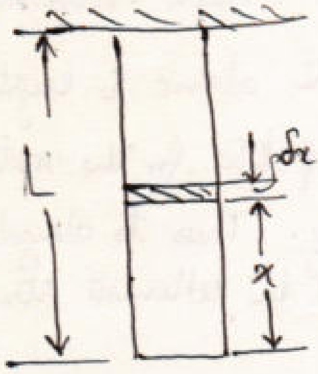
$$\text{Secant Modulus} = \frac{dy_B}{dx_B}$$

The slope of the tangent drawn at a point of interest, say point 'A' is known as Tangent Modulus.

The slope of the line joining the origin and a point of interest on the curve, say point 'B' is known as Secant Modulus.

These moduli are defined for imperfectly elastic materials.

* P.T the extension of uniform bar due to self weight is half of the extension when the load equal to its self weight is applied at the end of the suspended bar.



Consider a bar hanging freely under its own weight as shown in the fig.
Consider an elemental length δx of the bar at a distance 'x' from the free end.

Let A = Area of cross section
γ = ~~Weight per unit length of the bar~~ (unit weight) of the material
W = Weight of the whole bar

The load acting on the section considered $= P = A.x.\gamma$

Extension of elemental length $= \frac{P.dx}{A.E} = \frac{A.x.\gamma.dx}{A.E}$

$= \frac{\gamma\, x\, dx}{E}$

$\gamma = \frac{W}{AL}$

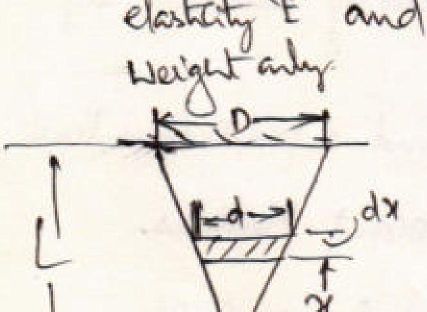
W = Total weight of bar

Extension of entire bar $= \int_0^L \frac{\gamma}{E} x.dx = \frac{\gamma}{E}\left[\frac{x^2}{2}\right]_0^L = \frac{\gamma}{E}\frac{L^2}{2}$

$= \frac{\gamma L^2}{2E} = \frac{1}{2}\frac{\gamma L^2}{E}$

Thus this extension is half the extension of the bar if the load equal to self weight is applied at the end ~~which~~ ie. $\frac{(A.L.\gamma)L}{A.E} = \frac{\gamma L^2}{E}$

* A solid conical bar of uniformly varying diameter has diameter 'D' at one end and zero at the other end. If the length of bar is 'L', modulus of elasticity 'E' and unit weight 'γ'. find the extension of bar due to self weight only.



Consider an elemental length 'dx' at a distance 'x' from the free end. Let the diameter be 'd' at this section

Self weight acting of the element $= \gamma . \frac{1}{3}\frac{\pi d^2}{4} x$

Extension of the element $= \frac{\left[\frac{\gamma}{3}\frac{\pi d^2}{4}.x\right][dx]}{\frac{\pi d^2}{4}.E} = \frac{1}{3}\frac{\gamma . x}{E}.dx$

Total extension of the bar $= \int_0^L \frac{1}{3}\frac{\gamma x}{E}.dx = \frac{\gamma}{3E}\left[\frac{x^2}{2}\right]_0^L$

$\boxed{\text{Total extension of conical bar due to self weight} = \frac{\gamma L^2}{6E}}$

* A Signal is being worked by a Steel wire 750m long and 6mm in diameter. Find the movement which must be given to the signal box end of the wire at a pull of 1.6kN, if the movement at the signal end is to be 250mm. Take $E = 2\times10^5$ N/mm². 07 marks.

June-July14.

Soln:

$$\text{Extension of wire} = \frac{PL}{AE} = \frac{1.6\times10^3 \times 750\times10^3}{28.27 \times 2\times10^5}$$

$$A = \frac{\pi\times6^2}{4} = 28.27\,mm^2 \qquad = 212.23\,mm$$

The movement to be given to Signal box end of wire = 250 + 212.23

= ~~[illegible]~~

= 462.23 mm

* A member is of total length 2m. It's diameter is 40mm for the first 1m length. In the next 0.5m length, its diameter gradually reduces from 40mm to 'd' mm. For the remaining length of the member, the diameter 'd' mm remains uniform. When this member is subjected to axial tensile force of 150kN, the total elongation observed is 2.39mm. Determine the diameter 'd'. $E = 2\times10^5$ N/mm²

Dec 13 - Jan - 2014 12 marks

$$\delta l = \delta l_1 + \delta l_2 + \delta l_3$$

d = 19.97 ≈ 20 mm

$$2.39 = \frac{150\times10^3\times1000}{1256.63\times2\times10^5} + \frac{4\times150\times10^3\times500}{\pi\times2\times10^5\times40\times d} + \frac{150\times10^3\times500\times4}{\pi\times d^2\times2\times10^5}$$

d = 13.37 mm

$$\frac{\pi\times40^2}{4} = 1256.63\,mm^2$$

$$2.39 = \frac{150\times10^3}{2\times10^5}\left[\frac{1000}{1256.63} + \frac{4\times500}{\pi\times40\times d} + \frac{500\times4}{\pi d^2}\right]$$

$$2.39 = \frac{150\times10^3}{2\times10^5}\left[0.7958 + \frac{15.91}{d} + \frac{636.61}{d^2}\right]$$

$$0.7958d^2 + 15.91d + 636.61 = 3.1866d^2$$

$$2.3908d^2 - 15.91d - 636.61 = 0$$

$$d^2 - 6.654d - 266.27 = 0$$

$$\boxed{d = 19.97\,mm}$$

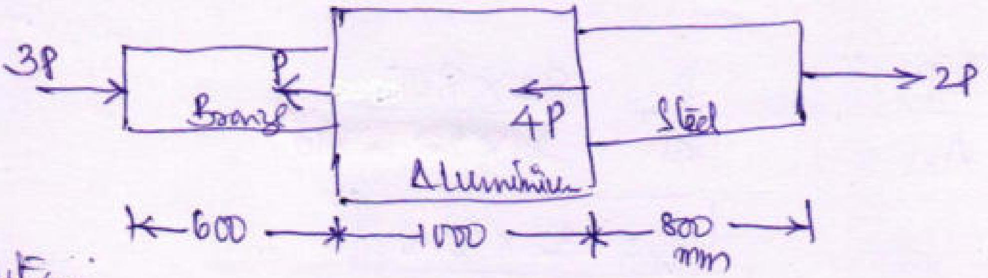
Determine the value of 'P' that will not accept a maximum deformation of 2 mm or a stress of 120 MPa in steel, 80 MPa in Aluminium and 115 MPa in Bronze. Given the following data:

Aug-Sept 2020

$A_B = 600\,mm^2$ $\quad E_B = 0.84 \times 10^5\ N/mm^2$

$A_{al} = 800\,mm^2$ $\quad E_{al} = 0.70 \times 10^5\ N/mm^2$

$A_s = 400\,mm^2$ $\quad E_{steel} = 2.10 \times 10^5\ N/mm^2$

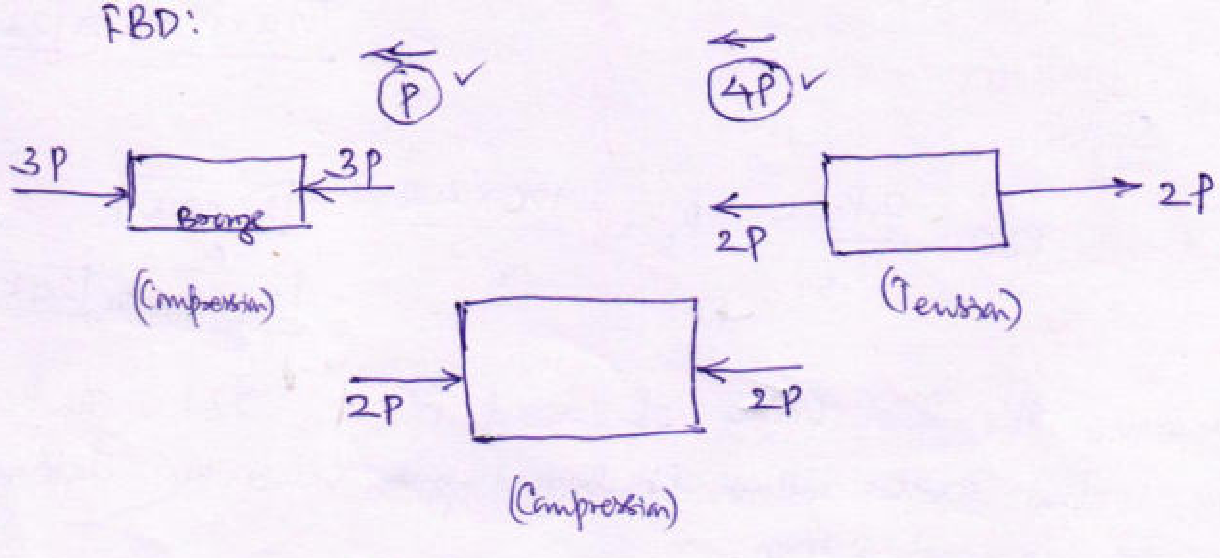


Solution:

Equilibrium Condition

$+3P - P - 4P + 2P = 0$ ✓

FBD:



$$\delta l = -\delta l_1 - \delta l_2 + \delta l_3$$

$$2 = -\left[\frac{3P \times 600}{600 \times 0.84 \times 10^5}\right] - \left[\frac{2P \times 1000}{800 \times 0.7 \times 10^5}\right] + \left[\frac{2P \times 800}{400 \times 2.10 \times 10^5}\right]$$

$$2 = -P\left[35.71 \times 10^{-6}\right] - P\left[3.571 \times 10^{-5}\right] + P\left[1.90 \times 10^{-5}\right]$$

$$2 = P\left[-5.23 \times 10^{-5}\right]$$

$\boxed{P = -38.15 \times 10^3\ N}$ Compressive load.

$$\sigma_{Bronze} = \frac{3P}{A_B} = \left(\frac{38.15\times10^3}{600}\right)\times3 = 190.75 \text{ N/mm}^2 > 115 \text{ N/mm}^2$$

$$\sigma_{Al} = \frac{2P}{A_{al}} = \left(\frac{38.15\times10^3}{800}\right)\times2 = 95.375 \text{ N/mm}^2 > 80 \text{ N/mm}^2$$

$$\sigma_{steel} = \frac{2P}{A_S} = \left(\frac{38.15\times10^3}{400}\right)\times2 = 190.75 \text{ N/mm}^2 > 120 \text{ N/mm}^2$$

Considering the σ in aluminium,

$$80 = \frac{2P}{A_{al}} \qquad P_{al} = \frac{80\times800}{2}$$

$$\boxed{P_{al} = 32\times10^3 \text{ N}}$$

IIIly: Bronze

$$115 = \frac{3P_B}{A_B} \qquad P_B = \frac{115\times600}{3} = \boxed{23000\text{N}} \text{ or } \boxed{23\times10^3 \text{ N}} \text{ or } \boxed{23 \text{ kN}}$$

Steel

$$120 = \frac{2P_S}{A_S} \qquad P_S = \frac{120\times400}{2} = \boxed{24000\text{N}} \text{ or } \boxed{24\times10^3 \text{ N}} \text{ or } \boxed{24 \text{ kN}}$$

Considering the least value of load P, i.e. 23 kN can be applied so that stresses will not be beyond given values and deformation will not be beyond 2 mm.

A steel flat of thickness 16 mm tapers uniformly from 80 mm at one end to 50 mm at the other end in a length of 800 mm. If the flat is subjected to 120 kN load, find the extension of the flat. Take $E = 2\times10^5$ MPa. Also calculate % error if average area is used for calculating extension. 10 Marks

Aug 2000

Given

$t = 16$ mm

$b_1 = 80$ mm

$b_2 = 50$ mm

$L = 800$ mm

$P = 120$ kN

$$\delta l = \frac{P.L}{tE(b_1-b_2)} \log\frac{b_1}{b_2}$$

$$= \frac{120\times10^3\times800}{16\times2\times10^5\times30} \log\frac{80}{50} \times 2.303$$

$$= 0.2041 \text{ mm.} \times 2.303 = 0.47 \text{ mm}$$

If average area is used, then.

$$A_{average} = \frac{80\times16 + 50\times16}{2}$$

$$= 1040$$

$$\delta l = \frac{P.L}{AE}$$

$$= \frac{120\times10^3\times800}{1040\times2\times10^5}$$

$$= 0.4615$$

$$\% \text{ error} = \left[\frac{0.4615 - 0.47}{0.47}\right]\times100$$

$$= \cancel{55.77}\ 1.81\ \%$$

A steel flat of thickness 15 mm tapers uniformly from 70 mm at one end to 40 mm at the other end in a length of 800 mm. If the flat is subjected to a load of 80 kN, find its extension. $E = 2\times10^5$ MPa Aug 1999. Also calculate the percentage error if average area is used for calculating extension.

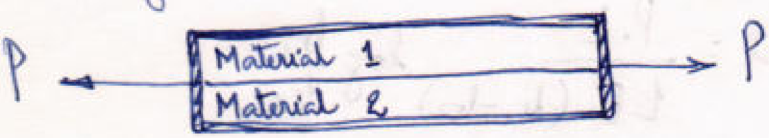
$\delta l = 0.39$ mm

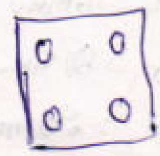
% error = 2.53

Compound bars or Composite bars:

Members Consisting of two or more materials (may be a bar or tube) in parallel is called Compound or Composite bar.

Consider a member with two materials. Let the forces developed due to applied loads in material 1 and 2 be P_1 and P_2 respectively.





According to Static equilibrium Condition,

Total applied load = Load taken by material 1 + Load taken by material 2

$$P = P_1 + P_2 \quad \text{——— (1)}$$

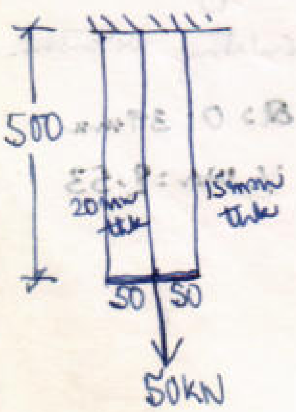
Composite materials are so designed that the strains in both the materials are same $(dl_1 = dl_2)$ Compatibility Condition

$$e_1 = e_2$$

$$\frac{P_1 l_1}{A_1 E_1} = \frac{P_2 l_2}{A_2 E_2} \quad \text{——— (2)}$$

From the above two eqns P_1 and P_2 can be found uniquely.

A Compound bar of length 500 mm Consists of a strip of aluminium 50 mm wide × 20 mm thick and a strip of steel 50 mm wide × 15 mm thk rigidly joined at ends. If the bar is subjected to a load of 50 kN, find the stresses developed in each material and the extension of the bar. Take $E_{Al} = 1\times10^5$ MPa and $E_S = 2\times10^5$ N/mm² respectively.



$L = 500$ mm

$P = 50$ kN

$A_{al} = 20\times50 = 1000\ mm^2$

$A_s = 15\times50 = 750\ mm^2$

$E_a = 1\times10^5\ N/mm^2$

$E_s = 2\times10^5\ N/mm^2$.

Now

$$P = P_{al} + P_s = 50\times10^3 \quad — (1)$$

also $\delta l_1 = \delta l_2$

$$\frac{P_{al}\, l_{al}}{A_{al} E_{al}} = \frac{P_s\, l_s}{A_s E_s}$$

$$P_{al} = \frac{P_s \cancel{l_s} \cdot A_{al} E_{al}}{\cancel{l_{al}}\; A_s E_s}$$

$$P_{al} = \frac{1000\times1\times10^5}{750\times2\times10^5} \cdot P_s$$

$$P_{al} = 0.666 P_s$$

From eqn (1)

$$1.666\, P_s = 50\times10^3$$

$$\boxed{P_s = 30\times10^3 \text{ N}}$$

$$\boxed{P_a = 20\times10^3 \text{ N}}$$

✓ $\sigma_s = \frac{30\times10^3}{750} = 40 \text{N/mm}^2$

✓ $\sigma_{al} = \frac{20\times10^3}{1000} = 20 \text{N/mm}^2$

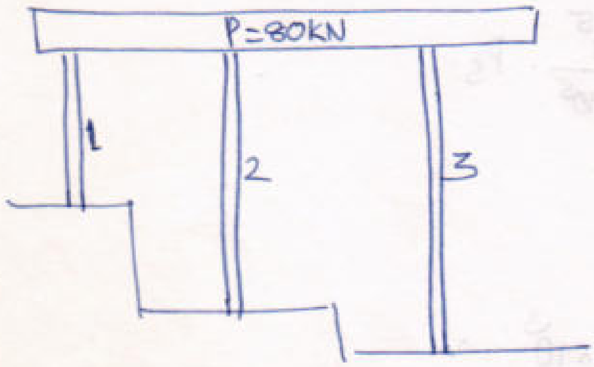
Extension of the bar

$$\delta l_1 = \frac{\sigma_{al} \cdot l_{al}}{E_{al}}$$

$$= \frac{20\times500}{1\times10^5} = 0.1 \text{ mm.}$$

The Compound bar shown in fig Consists of three materials and Supports a rigid platform weighing 80KN. Find the stresses developed in each bar if the platform remains horizontal even after the loading. Given

	Bar 1	Bar 2	Bar 3
Length	1000 mm	1500mm	2000mm
C/S Area	600 mm^2	800mm^2	1000mm^2
E	2×10^5 MPa	1.2×10^5 MPa	1×10^5 MPa



Now

$$P = 80KN = P_1 + P_2 + P_3 \quad \text{—(1)}$$

also

$$\delta l_1 = \delta l_2 = \delta l_3$$

$$\frac{P_1 l_1}{A_1 E_1} = \frac{P_2 l_2}{A_2 E_2} = \frac{P_3 l_3}{A_3 E_3}$$

$$P_1 = \frac{P_2 l_2 A_1 E_1}{A_2 E_2 \cdot l_1} = \frac{P_2\, 1500 \times 600 \times 2\times10^5}{800 \times 1.2\times10^5 \times 1000} = 1.875\, P_2$$

$$\boxed{P_2 = 0.5333 P_1} \quad \text{—(2)}$$

$$P_1 = \frac{P_3 l_3 A_1 E_1}{A_3 E_3 l_1} = \frac{P_3\, 2000 \times 600 \times 2\times10^5}{1000 \times \cancel{1\times10^5} \times 1000} = 2.4 P_3$$

$$\boxed{P_3 = 0.417\, P_1} \quad \text{—(3)}$$

From eqn (1)

$$P_1 + 0.5333P_1 + 0.417 P_1 = 80\times10^3$$

$$P_1 = \frac{80\times10^3}{1.9503} = 41019.328N \qquad \sigma_1 = \frac{P_1}{A_1}$$

$$\sigma_1 = 68.365\ N/mm^2$$

$$P_2 = 0.5333 \times 41019.328 = 21875.608\ N \qquad \sigma_2 = \frac{P_2}{A_2}$$

$$\sigma_2 = 27.345\ N/mm^2$$

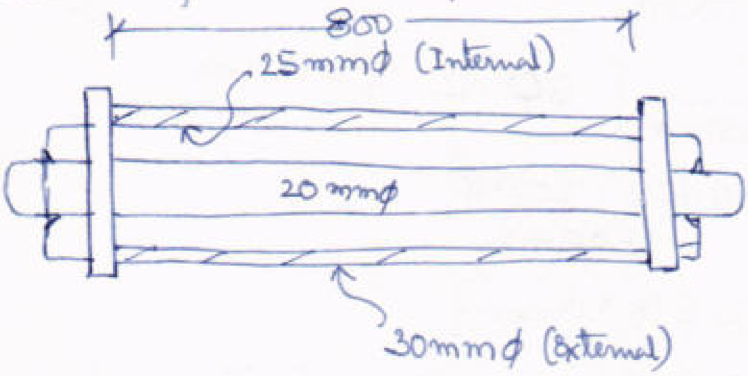
$$P_3 = 0.417 \times 41019.328 = 17105.059\ N \qquad \sigma_3 = \frac{P_3}{A_3}$$

$$\sigma_3 = 17.10\ N/mm^2$$

A steel rod 20 mm in dia passes centrally through a steel tube of 25 mm internal diameter and 30mm external diameter. The tube is 800mm long and is closed by rigid washer of negligible weight and thickness which are fastened by nuts threaded on the rod. The nuts are tightened until the compressive load on the tube is 20 kN. Calculate the stresses in the rod and tube. Also find the increase in stresses when one nut is tightened by one quarter of a turn relative to the other. There are four threads per 10 mm. Take $E_s = 200 kN/mm^2$.

March 2000

10 Marks



When nuts are tightened rod is elongated and tube is compressed. But as no external forces have been applied, the compressive load must be equal to tensile load.

A_T = Area of tube $= \frac{\pi}{4}\left[30^2 - 25^2\right] = 215.98\ mm^2$

A_R = Area of rod $= \frac{\pi}{4} \times 20^2 = 314.159\ mm^2$

Tensile load on rod = Compressive load on tube

$$P_R = P_T$$

$$\sigma_R A_R = \sigma_T A_T$$

$$\sigma_R = \sigma_T \times \frac{215.98}{314.159} = 0.6874\,\sigma_T$$

When compressive load on tube is 20 kN,

$$\text{Stress in tube} = \sigma_T = \frac{20 \times 10^3}{215.98} = 92.601\ N/mm^2$$

$$\sigma_R = 0.6874 \times 92.601 = 63.65\ N/mm^2$$

$$\boxed{\sigma_T = 92.601\ N/mm^2}$$

$$\boxed{\sigma_R = 63.65\ N/mm^2}$$

When the nut is further tightened,

the movement of the nut $= \frac{10}{4} \times \frac{1}{4} = \boxed{0.625\,mm}$

$\delta l = \delta l_{tube} + \delta l_{rod}$

Here total elongation = Elongation of rod + Shortening of tube

$$= \frac{P_R\, l_R}{A_R E_R} + \frac{P_T\, l_T}{A_T E_T}$$

$$= \frac{1}{E}\left[\sigma_R l_R + \sigma_T l_T\right]$$

$$0.625 = \frac{1}{200\times10^3}\left[\sigma_R\, 800 + \sigma_T\, 800\right]$$

$\sigma_R = 0.6894\,\sigma_T$

$$= \frac{1}{200\times10^3}\left[549.92\,\sigma_T + \sigma_T\, 800\right]$$

$$\sigma_T = \frac{0.625\times200\times10^3}{1349.92}$$

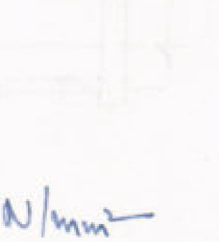
$$\boxed{\sigma_T = 92.598\ N/mm^2}$$

$$\sigma_r = 0.6874 \times 92.598 = 63.65\ N/mm^2$$

$$\boxed{\sigma_{rod} = 63.65\ N/mm^2}$$

A Compound bar consists of a circular rod of steel of diameter 20mm rigidly fitted into a copper tube of internal dia 20mm and thickness 5mm as shown in fig. If the bar is subjected to a load of 100 kN, find the stresses developed in two materials.

$E_S = 2\times10^5\ N/mm^2 \qquad E_C = 1.2\times10^5\ N/mm^2$



Soln $A_S = \frac{\pi\times20^2}{4} = 314.159\,mm^2$

$A_C = \frac{\pi}{4}(30^2 - 20^2) = 392.69\,mm^2$

From Static Equilibrium Condition,

$$P = P_S + P_C = 100\,kN \quad \text{—— (1)}$$

Also

$$\delta l_S = \delta l_C$$

$$\frac{P_S\, l_S}{A_S E_S} = \frac{P_C\, l_C}{A_C E_C} \qquad l_S = l_C$$

Here $P_S = \frac{P_C A_S E_S}{A_C E_C}$

$= \frac{P_C \, 314.159 \times 2 \times 10^5}{392.69 \times 1.2 \times 10^5}$

$\boxed{P_S = 1.34\, P_C}$ —— ①

$P_C + 1.34 P_C = 100$

$\boxed{P_C = 42.73 \text{ kN}}$

$\boxed{P_S = 57.25 \text{ kN}}$

Stress in steel $= \sigma_S = \frac{57.25 \times 10^3}{314.159} = 182.23 \text{ N/mm}^2$

$\sigma_c = \frac{42.73 \times 10^3}{392.69} = 108.81 \text{ N/mm}^2$

A RCC Column is 230 mm x 400 mm in Cross Section. The Column Consists of 6 bars of 20 mm dia. The Column Carries a load of 400 kN. Find the Stresses in Concrete and Steel bars. Given $E_S = 210$ GPa and $E_C = 15$ GPa.

$A_S = 6 \times \frac{\pi \times 20^2}{4} = 1884.95 \text{ mm}^2$

$A_C = (230 \times 400) - 1884.95 = 90115.05 \text{ mm}^2$.

We know that

$\delta l_c = \delta l_s$

~~Strain~~ in Concrete = ~~Strain~~ in steel

Change of length   Change of length

$\frac{P_S l}{A_S E_S} = \frac{P_c l}{A_c E_c}$ —— ①

Here $P_S + P_C = P$ —— ②

From ①

$P_S = \frac{P_c}{A_c E_c} \cdot A_S E_S$

$= \frac{P_c \; 1884.95 \times 210 \times 10^3}{90115.05 \times 10^3 \times 15}$

$P_S = 0.293 P_c$

From ②

$0.293 P_c + P_c = 400$

$P_c = 309.35$ kN

$P_S = 90.64$ kN

$\sigma_c = \frac{P_c}{A_c} = \frac{309.35 \times 10^3}{90115.05} = 3.432 \text{ N/mm}^2$

$\sigma_S = \frac{P_S}{A_S} = \frac{90.64 \times 10^3}{1884.95} = 48.08 \text{ N/mm}^2$

A Concrete Column of 300 mm × 300 mm supports a load of 350 kN. The column is reinforced with steel to cover an area of 2515 mm². If modular ratio (E_s/E_c) is 15 for steel and concrete, estimate stress in steel and concrete. If the stress in concrete should not exceed 4.5 N/mm², find the area of steel required in order that column carries a load of 600 kN.

Solution:

$A_S = 2515\ mm^2$

$A_C = 90000 - 2515 = 87485\ mm^2$

I Case: When Column supports a load of 350 kN

$$P_S + P_C = 350\,kN \quad \text{— (1)}$$

Change in length in steel = Change in length in Concrete

$$\frac{P_S\, l}{A_S E_S} = \frac{P_C\, l}{A_C E_C}$$

$$P_S = P_C \frac{A_S}{A_C} \cdot \frac{E_S}{E_C}$$

$$P_S = P_C \frac{2515}{87485} \times 15$$

$$\boxed{P_S = 0.4312\, P_C} \quad \text{— (2)}$$

$$P_S + P_C = 350$$

$$0.4312 P_C + P_C = 350$$

$$\boxed{P_C = 244.55\ kN} \qquad \boxed{\sigma_C = 2.795\ N/mm^2}$$

$$P_S = 0.4312 \times 244.55$$

$$\boxed{P_S = 105.45\ kN} \qquad \boxed{\sigma_S = 41.92\ N/mm^2}$$

II Case When Column supports a load of 600 kN and $\sigma_c = 4.5\ N/mm^2$

$$P_S + P_C = 600\,kN$$

$$\sigma_S A_S + 4.5(90000 - A_S) = 600\,kN \quad \text{— (1)}$$

$$A_S(\sigma_S - 4.5) = 600 \times 10^3 - 90000 \times 4.5 = 195000$$

$$A_S = \frac{19500}{(\sigma_S - 4.5)}\ mm^2 \quad \text{— (3a)}$$

$$\delta l_1 = \delta l_2$$

$$P_S = 4.5 \cdot \frac{A_S}{(90000 - A_S)} \times 15$$

$$\sigma_S A_S = 4.5 \times 15 \times \frac{A_S}{90000 - A_S}$$

Strain in Steel = Strain in Concrete

$$\frac{\sigma_s}{E_s} = \frac{\sigma_c}{E_c} \quad \Bigg| \quad \frac{P_s L_s}{A_s E_s} = \frac{P_c L_c}{A_c E_c}$$

$$\sigma_s = \frac{E_s}{E_c} \sigma_c$$

$$\boxed{\sigma_s = 15 \sigma_c} \text{ ——— } (3)$$

$$P_s + P_c = 600 \times 10^3 \text{ N}$$

$$\sigma_s A_s + \sigma_c A_c = 600 \times 10^3$$

$$15 \sigma_c A_s + \sigma_c (90000 - A_s) = 600 \times 10^3$$

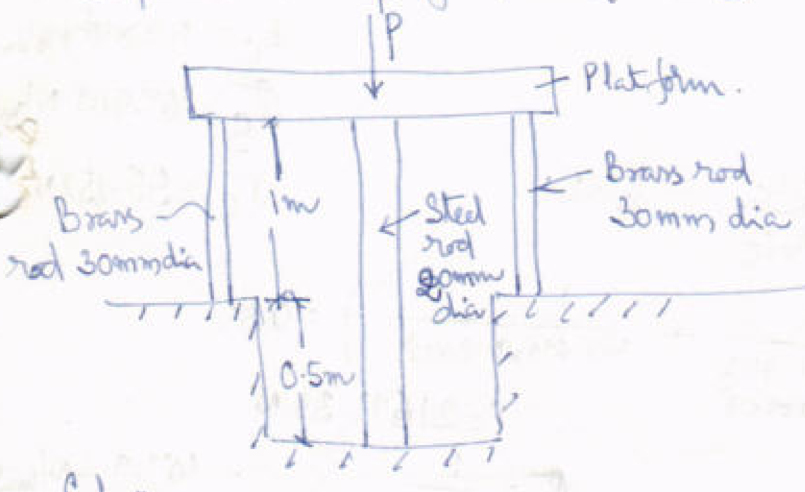
When $\sigma_c = 4.5 \text{ N/mm}^2$

$$67.5 A_s + 4.5 (90000 - A_s) = 600 \times 10^3$$

$$63 A_s = 600 \times 10^3 - 405000$$

$$\boxed{A_s = 3095.238 \text{ mm}^2}$$

Two brass rods and one steel rod together have to support load as shown in fig 2.46. The stress in steel and brass are not supposed to exceed 140 MPa and 80 MPa respectively. Find the safe load that can be placed on the platform. Given $E_s = 200$ GPa and $E_b = 110$ GPa.



Soln:

$$2 P_b + P_s = P \text{ ——— } (1)$$

Area of brass

$= 2 \times \frac{\pi}{4} \times 30^2 = 1413.72 \text{ mm}^2$

Area of steel

$= \frac{\pi}{4} \times 20^2$

$= \cancel{706.85} \text{ mm}^2$

314.15

Change in brass = Change in length of steel

$$\frac{P_b \cdot L_b}{A_b E_b} = \frac{P_s L_s}{A_s E_s}$$

$$P_b = P_s \cdot \frac{1500}{1000} \cdot \frac{A_b}{A_s} \cdot \frac{E_b}{E_s} = P_s \times 1.5 \times \cancel{4}^{2.25} \times \frac{110}{200}$$

$$\boxed{P_b = 1.856 P_s} \text{ — } (1)$$

Here ~~$\sigma_b A_b = 1.856\,\sigma_s A_s$~~ Here $A_s/A_b =$

~~$\sigma_b = 1.856\,\sigma_s$~~ $\sigma_b \cdot A_b = 1.856\,\sigma_s A_s$

$\sigma_b = 1.856\,\sigma_s \frac{A_s}{A_b}$ i.e. $\boxed{\sigma_b = 0.825\,\sigma_s}$

when $\sigma_s = 140\ N/mm^2$, $\sigma_b = 115.5\ N/mm^2 > 80 N/mm^2$

Hence σ_s should not be allowed to reach its permissible value.

When $\sigma_b = 80\ N/mm^2$, $\sigma_s = 96.96 N/mm^2 < 140\ N/mm^2$.

Hence, this value is permitted.

Now

Safe load $P = P_s + 2 P_b$

$= 96.96 \times 314.15 + 2 \times 80 \times 706.85$

$\boxed{P = 143.556\ kN.}$

A Steel bolt of 16mm dia passes centrally through a Copper tube of internal dia 20mm and external dia 30mm. The length of the whole assembly is 500mm. After tight fitting of the assembly, the nut is overtightened by quarter of a turn. What are the stresses introduced in bolt and tube, if pitch of nut is 2mm. Take $E_s = 200 GPa$

$E_c = 1.2 \times 10^5 MPa$.

Soln. $P_s = P_c = P$

$l_s = l_c = 500mm$

$\sigma_s = 107.914\ N/mm^2$

$\sigma_c = 55.252 N/mm^2$

$A_s = \frac{\pi}{4} \times 16^2 = 201.062 mm^2$

$A_c = \frac{\pi}{4}(30^2 - 20^2) = 392.699 mm^2$

$$\frac{P_s L_s}{A_s E_s} + \frac{P_c L_c}{A_c E_c} = \frac{1}{4} \times 2$$

$$P\,500\left[\frac{1}{201.062 \times 200 \times 10^3} + \frac{1}{392.699 \times 1.2 \times 10^5}\right] = 0.5$$

$P = 21697.33\ N$

$\sigma_s = \frac{P}{A_s} = 107.914 N/mm^2$

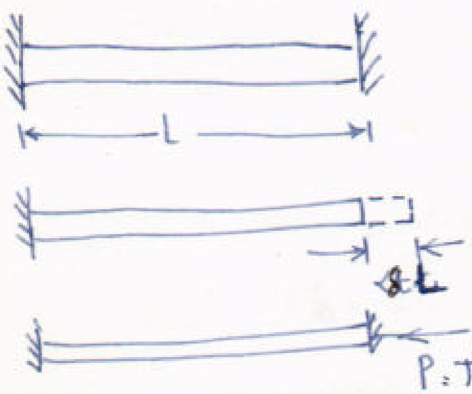
$\sigma_c = \frac{P}{A_c} = 55.252 N/mm^2$

Temperature Stresses :

Every material expands when temperature rises and Contracts when temperature falls. If the expansion or Contraction is restricted, thermal stresses are developed. If bar is allowed to expand or Contract freely, thermal stresses do not develop.

Consider a bar of length 'L'. If the bar is free to expand, when the temperature Changes by 'δt' degrees, then, free expansion will be $\alpha \delta t L$. But this extension is Completely prevented by the forces that develop at the supports i.e. Say 'P'. Here 'P' Causes shortening of the bar by $\alpha \delta t L$

α = Coefficient of thermal expansion.



$$\delta l = \alpha \delta t L = \frac{P.L}{A.E}$$

$$\frac{P}{A} = \frac{\alpha \delta t L . E}{L}$$

$$\boxed{\sigma_{Thermal} = \alpha \delta t E} \quad \text{—— (1)}$$

Note : Here 'α' = Coefficient of thermal expansion and is defined as Change in Unit length due to unit Change in temperature. units mm/°c

A Steel rail is 12.6 m long and is laid at a temperature of 24°c. The maximum temperature expected is 44°c

i) Estimate the minimum gap between two rails to be left so that temp stresses do not develop.

ii) Calculate the thermal stress developed in rails if
  a) no Expansion joint is provided
  b) If a 2mm gap is provided for expansion.

iii) If the stress developed is 20 MN/m², What is the gap left between the rails. Take $E = 2 \times 10^5$ MN/m², and $\alpha = 12 \times 10^{-6}$ /°c

Soln: (i) For no stresses to develop, the free extension is to be allowed

Free extension $\delta l = \alpha \delta t L$

$= 12 \times 10^{-6} \times 20 \times 12600$

$\delta t = 24 - 44 = +20°c$

$$\boxed{\delta l = 3.024 \text{ mm}}$$

II) a) If no expansion joint is provided,

$$E = \frac{2\times10^5 \times 10^6\,N}{1000\times1000} = 2\times10^5\,N/mm^2$$

$$\sigma_t = E\alpha\delta t$$

$$\sigma_{Thermal} = 2\times10^5 \times 12\times10^{-6} \times 20 = 48\,N/mm^2$$

or

$$\boxed{\begin{aligned}\frac{PL}{AE} &= \alpha\delta t L \\ \frac{P}{A} &= \frac{\alpha\delta t E.L}{L}\end{aligned}}$$

b) If a 2mm gap is provided for expansion,

then free expansion prevented is $\delta_1 = \alpha\delta t L - 2$

$$= 3.024 - 2 = 1.024\ mm.$$

$$\therefore \frac{PL}{AE} = 1.024$$

$$\frac{P}{A} = 1.024 \times \frac{E}{L} = 1.024 \times \frac{2\times10^5}{12600}$$

$$= 16.254\ N/mm^2$$

III) If stress developed is $= 20MN/m^2 = \frac{20\times10^6}{10^6} = 20N/mm^2$

$$\frac{P}{A} = 20\ N/mm^2$$

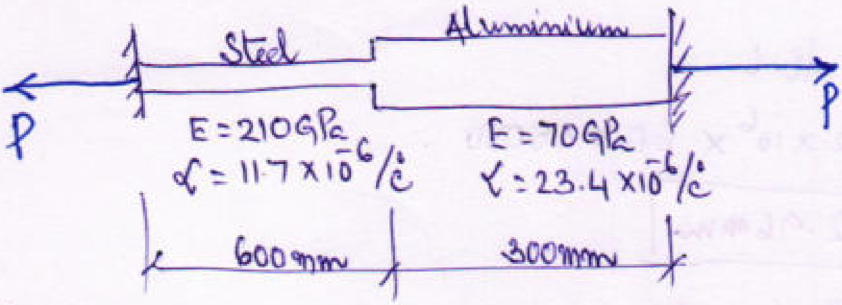
$$\frac{PL}{AE} = \alpha\delta t L - \delta$$

$$20 = (3.024 - \delta)\cdot\frac{2\times10^5}{12600}$$

$$3.024 - \delta = 1.26$$

$$\boxed{\delta = 1.764\ mm}$$

A Composite bar made of aluminium and steel is held between two supports as shown in fig 2(c). What will be the stress in bars when the temperature falls by 20°c, given that the bars were initially stress free. The supports are unyielding. c/s area of steel bar = 200 mm². c/s area of aluminium = 300 mm².



Soln

Free expansion = $\alpha_a \delta t\, l_a + \alpha_s \delta t\, l_s$
(without any supports)

$= 23.4 \times 10^{-6} \times 20 \times 300 + 11.7 \times 10^{-6} \times 20 \times 600$

$= 0.2808$ mm

Since free ~~expansion~~ contraction is prevented, thermal stress develop.
If 'P' is the support reaction, then $dl = \frac{P L_a}{A_a E_a} + \frac{P L_s}{A_s E_s}$

$$0.2808 = P\left[\left[\frac{300}{300 \times 70 \times 10^3}\right] + \left[\frac{600}{200 \times 210 \times 10^3}\right]\right]$$

$$\boxed{P = \frac{0.2808}{2.857 \times 10^{-5}} = 9828\,N}$$

A steel rod is 18m long at a temperature of 25°C. Find the free expansion when the temperature is raised to 85°C. Also find the temperature stresses produced when i) the expansion is fully prevented
ii) the rod is permitted to expand by 4.5mm.

$E = 200 kN/mm^2$ and $\alpha = 12 \times 10^{-6} /°C$. July 2014 (06M)

Free expansion $= \alpha \, dt \, L$

$\delta t = 85 - 25 = 60°$ $\quad = 12 \times 10^{-6} \times 60 \times 18000$

$$\boxed{\delta l = 12.96 mm}$$

1) When the expansion is fully prevented

$$\sigma_{thermal} = \alpha \, dt \, E$$
$$= 12 \times 10^{-6} \times 60 \times 200 \times 10^{3}$$
$$\sigma_{thermal} = 144 N/mm^2$$

2) Expansion prevented $= 12.96 - 4.5mm = 8.46mm$.

$$\frac{PL}{AE} = 8.46$$

$$\frac{P}{A} = \frac{8.46 \times 200 \times 10^{3}}{18000}$$

$$\boxed{\sigma_{thermal} = 94 N/mm^2}$$

A metallic rod is 3m long at a temperature of 20°C. Find the extension of the rod when the temperature is raised to 90°C. If this expansion is prevented, find the stresses induced in the material of the rod.
Take $E = 1\times10^5$ MN/m² and $\alpha = 0.000012$ per degree Centigrade

Aug/ Sept 1999.

Soln:

Free expansion $= \alpha \delta t L$

$t = 90-20 = 70°C$

$= 0.000012 \times 70 \times 3\times10^3$

$= 2.52$ mm.

$E = \dfrac{10^5 \times 1\times10^6}{1000\times1000}$

$= 1\times10^5$ N/mm²

$\sigma_{Thermal} = E\alpha\delta t$

$= 1\times10^5 \times 0.000012 \times 70$

$= 84$ N/mm²

Explain the reason for development of stress in bars when their temperature rises or falls. Accordingly, Calculate the nature of stress and its magnitude for a rod of 2m length and dia 2cm when its temperature is raised by 70°c with both ends constrained $E = 10^5$ N/mm² and $\alpha = 0.000012$ per °c.

Aug 1999

Soln:

$\sigma_{Thermal} = E\alpha\delta t$

$\delta f_{Thermal} = E\alpha\Delta t$

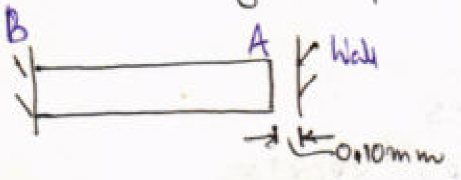
$= 1\times10^5 \times 0.000012 \times 70^0$

$= 84$ N/mm².

Free expansion is prevented. Hence Compressive stresses develop in the bar

A Copper bar AB of length 1m is placed in position at room temperature with a gap of 0.10mm between end A and a rigid wall. Calculate the stress in the bar if temperature rises 40°C. For Copper $\alpha = 17\times10^{-6}$/°c and $E = 110$ GPa. What is the nature of stress.

Free Elongation of bar $= \delta l = \alpha \delta t L$



$= 17\times10^{-6} \times 40 \times 1000$

$= 0.68$ mm.

Free expansion prevented $= 0.68 - 0.1 = 0.58$mm

$\dfrac{PL}{AE} = [0.58]$

$\dfrac{P}{A} = \sigma_{Thermal} = 0.58 \times \dfrac{110\times10^3}{1000} = 63.8$ N/mm².

An aluminium rod 2m long is secured between two walls. If the stress in the rod is zero when temperature is 10°c, Compute the stress when temperature drops to −10°c. Given $\alpha = 23\times10^{-6}/°c$ and $E = 60\,GPa$.

Solve assuming that a) walls are rigid

b) walls yield by 0.5mm together as temperature drops.

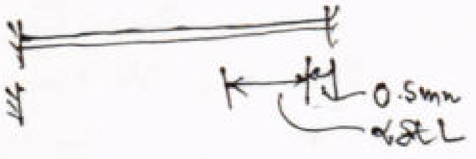
a) when the walls are rigid

$\Delta t = +10$ to -10
$= 20°c$.

Stress in the bar $= E\alpha\delta t = 60\times10^{3}\times23\times10^{-6}\times20°$
for not allowing free Contraction. $= 27.6\ N/mm^2$.

b) when the walls yield by 0.5mm together as temperature drops.

when the walls yield, the Contraction along with yielding ~~amount~~ distance is to be considered.



~~Free temperature expansion~~.

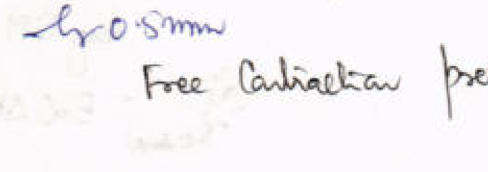
Free Contraction $= \alpha\,\delta t\,L$
$= 23\times10^{-6}\times20\times2000$
$= 0.92\,mm$

The walls are allowed to move freely by 0.5mm

Free Contraction prevented $= 0.92 - 0.5mm$
$= 0.42\,mm$.

Stress in the bar $\frac{P}{A} = \sigma_{thermal} = \frac{0.42\times60\times10^{3}}{2000}$
$= 12.6\ N/mm^2$

A Composite bar is rigidly fitted at the supports A and B as shown in fig. Determine the reactions at the supports when temp rises by 20°c. Take $E_{al} = 70\ GN/m^2$, $E_{steel} = 200\ GN/m^2$. $\alpha_{al} = 11\times10^{-6}/°c$ and $\alpha_s = 12\times10^{-6}/°c$.



Find also stresses

Soln The free expansion $= \alpha_a\,\delta t\,L_a + \alpha_s\,\delta t\,L_s$
(when supports are released) $= 11\times10^{-6}\times20\times1000 + 12\times10^{-6}\times20\times3000$
$= 0.94\,mm$.

Since the free expansion is prevented, thermal stresses develop.

If 'P' is support reaction, then $\delta l = \frac{P L_{al}}{A_{al} E_{al}} + \frac{P L_s}{A_s E_s}$

$E_{al} = 70\ GN/m^2$

$= \frac{70 \times 10^9}{10^3 \times 10^3} = 70 \times 10^3\ N/mm^2$

$E_s = 200\ GN/m^2$

$= \frac{200 \times 10^9}{10^6} = 200 \times 10^3\ N/mm^2$

$0.94 = P\left[\frac{1000}{600 \times 70 \times 10^3} + \frac{3000}{300 \times 200 \times 1000}\right]$

$\boxed{P = 12735.48\ N}$

A Steel wire AB is stretched between rigid supports as shown in fig. The initial prestress in the wire is 30 MPa when the temperature is 20°C.

a) what is the stress in the wire when temperature drops to 0°C?

b) At what temperature T will the stress in the wire becomes zero?

$\alpha = 14 \times 10^{-6}/°C$. $E = 210\ GPa$.

Soln a) The wire is in tension with stress 30 MPa at a temperature of 20°C. The wire should be cooled to 0°C.

$\sigma_{Thermal} = \alpha\, \delta t\, E$

$= 14 \times 10^{-6} \times 20^\circ \times 210 \times 10^3 = 58.8\ N/mm^2$.

$\therefore$ Magnitude of Stress in wire = Prestress + $\sigma_{thermal}$

$= 30 + 58.8 = 88.8\ N/mm^2$

b) In order that the stress in the wire be zero, the temperature should be increased such that stresses becomes compressive in nature with magnitude being $30\ N/mm^2$.

$\sigma = \alpha\, \delta t\, E$

$30 = 14 \times 10^{-6} \times \delta t \times 210 \times 10^3$

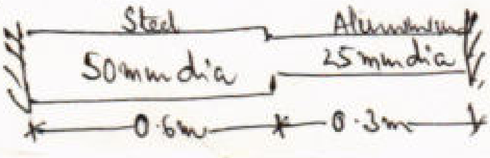
$\delta t = 10.20^\circ$

$\therefore$ Temperature should be $20 + 10.20 = 30.20°C$

A Composite bar made of steel and aluminium is held between two supports as shown in fig. The Composite member is stress-free at a temperature of 38°C. what will be the stresses in the two materials when the temperature is 21°C if the supports are unyielding?

$E_s = 210\ GPa$ $E_a = 74\ GPa$. $\alpha_s = 11.7 \times 10^{-6}$ per °c. $\alpha_a = 23.4 \times 10^{-6}$ per °c.

Aug 2001



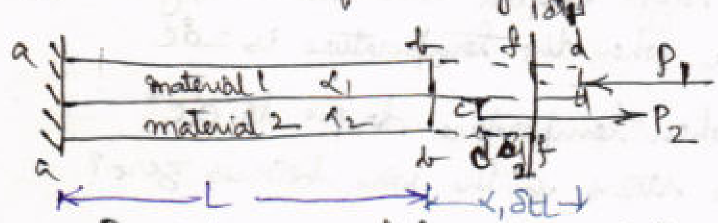
$\sigma_s = 12.5137\ N$

$\sigma_{al} = 50.05\ N$

Temperature stresses in Compound bars:

When temperature changes, the two materials of Compound bar experience different free expansions or contractions. Since they are connected rigidly, they have to move together which leads to development of thermal stresses.

Consider a Compound bar of length 'L' as shown in fig. Let the rise in temp be δt. Let α_1 and α_2 be the coefficients of thermal expansion and E_1 and E_2 be the modulii of elasticity of two materials. Let α_1 be greater than α_2



$bd = \alpha_1 \delta t L$

$bc = \alpha_2 \delta t L$

$cf = \delta l_2$

$fd = \delta l_1$

Free expansion of bar 1 = $\alpha_1 \delta t L$ [bd]

Free expansion of bar 2 = $\alpha_2 \delta t L$ [bc]

Since $\alpha_1 > \alpha_2$, material 1 will expand to dd freely and material 2 will expand to cc freely. But since the two bars are rigidly connected, they will attain position somewhere at ff. Here a compressive force P_1 will act on bar 1 to causing shortening and a tensile force P_2 will act on bar 2 to cause expansion. For equilibrium $P_1 = P_2 = P$.

$bf = bf$

$bd - fd = bc + cf$

Now,

$$\alpha_1 \delta t L - \delta l_1 = \alpha_2 \delta t L + \delta l_2$$

$$\delta l_1 + \delta l_2 = \alpha_1 \delta t L - \alpha_2 \delta t L$$

$$\delta l_1 + \delta l_2 = \delta t L [\alpha_1 - \alpha_2]$$

$$\frac{P.L}{A_1 E_1} + \frac{PL}{A_2 E_2} = L \delta t [\alpha_1 - \alpha_2]$$

$$PL\left[\frac{1}{A_1E_1} + \frac{1}{A_2E_2}\right] = L \delta t [\alpha_1 - \alpha_2] \quad \text{—— (1)}$$

Hence 'P' can be calculated and also $\sigma_1 = \frac{P}{A_1}$ and $\sigma_2 = \frac{P}{A_2}$

$\sigma_1 = \frac{P}{A_1}$ and $\sigma_2 = \frac{P}{A_2}$

Free expansion when supports are released $= \alpha_s \delta t L_s + \alpha_a \delta t L_a$

$\delta t = 38-21 = 17°c$ $= 11.7 \times 10^{-6} \times 17 \times 600 + 23.4 \times 10^{-6} \times 17 \times 300$

$L_s = 600mm$ $= 0.23868mm$

$L_a = 300mm$

If P is the support reaction, then

$A_s = 1963.495 mm^2$ $\delta l = \frac{P.L_s}{A_s E_s} + \frac{P L_a}{A_a E_a} = 0.23868\ mm$

$A_a = 490.87 mm^2$

$$P\left[\frac{600}{1963.498 \times 210 \times 10^3} + \frac{300}{490.87 \times 74 \times 10^3}\right] = 0.23868\ mm$$

$$P = 24570.609\ N$$

$$\sigma_s = \frac{P}{A_s} = \frac{24570.60}{1963.495} = 12.5137\ N/mm^2$$

$$\sigma_a = \frac{P}{A_a} = 50.05\ N/mm^2$$

Two parallel walls 6m apart are stayed together by a steel rod of 25mm dia at a temperature of 80°c. The steel rod passes through washers and nuts at each end. Calculate the pull exerted by the rod when it has cooled to 20°c a) if the wall don't yield b) the total yield together at two ends is 1.5mm. Given $E = 2 \times 10^5\ N/mm^2$ $\alpha = 11 \times 10^{-6}/°c$.

Ans. a) Pull $= 64.795$ kN

b) pull $= 40.25$ kN

Soln: a) when the walls are rigid:

$\delta t = 80-20° = 60°$

$d = 25mm$

$l = 6mts$

Stress developed for not allowing free ~~expansion~~ $= E\alpha\delta t = 2 \times 10^5 \times 11 \times 10^{-6} \times 60°$

$\sigma_{th} = 132\ N/mm^2$

Pull $= 132 \times \frac{\pi \times 25^2}{4} = 64.79$ kN

b) when the walls yield by 1.5mm.

Free Contraction: $\alpha \delta t L = 11 \times 10^{-6} \times 60 \times 6000 = 3.96$ mm.

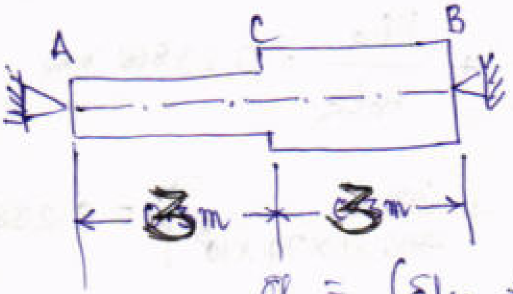
Free Contraction prevented $= 3.96 - 1.5 = 2.46$mm

$\delta l = \frac{P.L}{A.E} = 2.46$ $P = 2.46 \times \frac{\pi}{4} \times 25^2 \times \frac{2 \times 10^5}{6000}$

$= 40.251$ kN

Calculate the values of the stress and strain in portion AC and CB of the steel bar shown in fig. A close fit exists at both the rigid supports at room temperature and the temperature is raised by 75°c. Take E = 200 GPa and $\alpha = 12\times10^{-6}$ per °c for steel. Area of cross-sections of AC is 400 mm² and of BC is 800 mm².

Soln:



$$\delta l = (\delta l_{AC} + \delta l_{CB})$$

Free expansion: $\delta l = \alpha \,\delta t\, L_{AC} + \alpha\,\delta t\, L_{CB}$

$$= 12\times10^{-6}\times 75\,(3000+3000)$$

$$\boxed{\delta l = 5.4\,mm}$$

If P is the support reaction, then $\delta l = \dfrac{P\,l_{AC}}{A_{AC}E_{AC}} + \dfrac{P\,L_{CB}}{A_{CB}E_{CB}}$

$$5.4 = \frac{P}{200\times10^3}\left[\frac{3000}{400} + \frac{3000}{800}\right]$$

$$\boxed{P = 96000\,N}$$

$\sigma_{AC} = \dfrac{96000}{400}$ $\quad\sigma_{CB} = 120\,N/mm^2$

$= 240\,N/mm^2$

$$\delta l_{AC} = \frac{96000\times 300}{400\times 200\times10^3}$$

$= 0.36\,mm$ or δl_{CB} 1.8 m

$$\delta l_{CB} = \frac{96000\times 300}{800\times 200\times10^3}$$

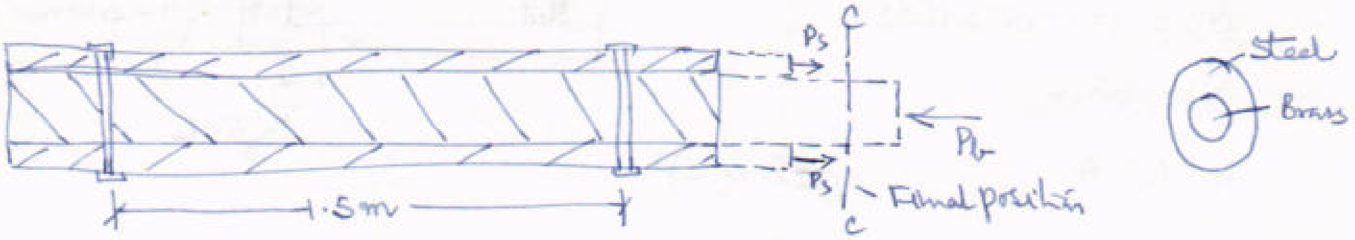
$= 0.18\,mm$

A bar of brass 25 mm dia is enclosed in a steel tube 50 mm external dia and 25 mm internal diameter. The bar and the tube are both initially 1.5 m long and are rigidly fastened at both ends using 20 mm dia pins. Find the stresses in the two materials when temperature rises from 30°C to 100°C.

$E_s = 200\,\text{kN/mm}^2$, $E_b = 100\,\text{kN/mm}^2$ $\quad \alpha$ for steel $= 11.6 \times 10^{-6}/°C$

α for brass $= 18.7 \times 10^{-6}/°C$

Find also shear stress induced in pins.



$\delta t = 100 - 30 = 70°C$

$A_b = \frac{\pi}{4} \times 25^2 = 490.87\ \text{mm}^2$

$A_s = \frac{\pi}{4} \times (50^2 - 25^2) = 1472.62\ \text{mm}^2$

Since the free expansion of brass is more than than that of steel, compressive force P_b develops in the brass and tensile force P_s develops in steel to keep them in same position C-C.

Now $\quad P_s = P_b = P.$

also $\quad \delta l_s + \delta l_b = \alpha_b \delta t L - \alpha_s \delta t L$

$P_s = P_b$ $\quad \frac{P_s L_s}{A_s E_s} + \frac{P_b L_b}{A_b E_b} = \alpha_b \delta t L - \alpha_s \delta t L$

$$P \times 1500 \left[\frac{1}{1427.62 \times 2 \times 10^5} + \frac{1}{490.87 \times 1 \times 10^5} \right] = 70 \times 1500 (7.1 \times 10^{-6})$$

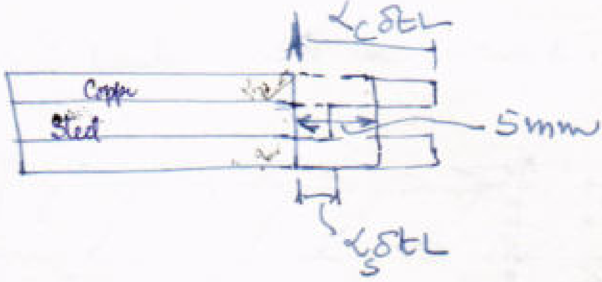
$$\boxed{P = 20817.45\ \text{N}}$$

Stress in steel $P_s = \frac{20817.45}{1427.62} = 14.58\ \text{N/mm}^2.$

$P_b = \frac{20817.45}{490.87} = 42.40\ \text{N/mm}^2$

Shear stress in pins $= \frac{20817.45}{2 \times \frac{\pi}{4} \times 20^2} = 33.13\ \text{N/mm}^2$

A Steel bar is sandwitched between two Copper bars of same area and same length, with their ends rigidly Connected. The temperature of the assembly is maintained at 30°c. on raising the temperature to 150°c, it was found that length of the unit increased by 5mm. Determine the original length and stress in bars. Given $E_S = 200$ GPa, $E_C = 100$ GPa $\alpha_S = 12\times10^{-6}/°c$ and $\alpha_C = 17\times10^{-6}/°c$.



$\delta t = 150-30 = 120°c.$

$\delta l = 5mm.$

$A_C = A_S = A$ $\qquad P_S = 2P_C = P$

$$\delta_S + \delta_C = \delta t L(17\times10^{-6} - 12\times10^{-6})$$

$L_S = L_C$

$A_S = A_C$

$$\frac{P_S L_S}{A_S E_S} + \frac{P_C L_C}{A_C E_C} = 120L(5\times10^{-6})$$

$$\frac{L}{A}\left[\frac{2P_C}{E_S} + \frac{P_C}{E_C}\right] = 120L\times5\times10^{-6}$$

$$\frac{P_C}{A}\left[\frac{2}{200\times10^3} + \frac{1}{100\times10^3}\right] = 120\times5\times10^{-6}$$

$$\frac{P_C}{A} = \frac{120\times5\times10^{-6}\cancel{\times A}}{2\times10^{-5}} = 30\,N/mm^2$$

$$P_S = 2P_C = 60\,N/mm^2.$$

$$\frac{P_S L_S}{A_S E_S} + \frac{P_C L_C}{A_C E_C} = 5$$

$$\frac{L}{A}\left[\frac{2P_C}{E_S} + \frac{P_C}{E_C}\right] = 5$$

$$L\frac{P_C}{A}\left[\frac{2}{E_S} + \frac{1}{E_C}\right] = 5$$

$$L\left[\frac{2}{E_S} + \frac{1}{E_C}\right] = \frac{5}{30}$$

$$L = 8333.33\text{ mm}$$

Check:-

$\delta l = \alpha_C \delta t L - \delta l_C$ of Comp[d] bar

$$5 = 17\times10^{-6}\times120\,L - \frac{30\cdot L}{100\times10^3}$$

$= 2873.56$ mm

$$\delta l = \alpha_S \delta t L + \delta l_S$$

$$5 = 12\times10^{-6}\times120\times L + \frac{P_S L_S}{A_S E_S}$$

$$5 = 12\times10^{-6}\times120\times L + \frac{60\cdot L}{200\times10^3}$$

$$L = \frac{5}{\cancel{3.48\times10^{-4}}\ 1.74\times10^{-3}}$$

$= 2873.56$ mm

A Copper rod of dia 30mm passes centrally through a tight fitting steel tube of external dia 50mm. The tube is closed by rigid washers of negligible thickness and nuts threaded on the rod. The nuts are tightened till the compressive load on the tube is 60kN. Find the stresses in the rod and the tube when the temperature of the assembly falls by 60°c below room temperature. Given $E_S = 200\,GPa$, $E_C = 100\,GPa$. $\alpha_S = 12\times10^{-6}/^\circ c$

$\alpha_C = 18\times10^{-6}/^\circ c$

$A_C = \frac{\pi}{4}\times 30^2 = 706.85\ mm^2$

$A_S = \frac{\pi}{4}\times(50^2-30^2) = 1256.63\,mm^2$

Stress due to 60kN load acting on the tube:

Steel tube is in compression and Copper rod is in tension.

$$\sigma_{Copper} = \frac{60\times10^3}{706.85} = 84.88\ N/mm^2\ \text{(Tension)}$$

Check:

$$\sigma_{Steel} = \frac{60\times10^3}{1256.63} = 47.75\ N/mm^2\ \text{(Compression)}$$

When temperature drops, Copper will have more free contraction than steel. The steel tube will prevent the free contraction of the copper rod. Hence Copper rod will be in tensile stress while steel will be compressive in nature.

Now $P_S = P_C = P$.

$$\delta_S + \delta_C = \delta t\cdot L\cdot(\alpha_C - \alpha_S)$$

$\delta t = 60^\circ$

$$\frac{P_S L_S}{A_S E_S} + \frac{P_C L_C}{A_C E_C} = 60\times L\times(6\times10^{-6})$$

$L_S = L_C$

$P_S = P_C$

$$P\cancel{L}\left[\frac{1}{1256.63\times200\times10^3} + \frac{1}{706.85\times100\times10^3}\right] = 60\cancel{L}(6\times10^{-6})$$

$P = 19860.78\,N$. $\quad \sigma_{S\,thermal} = \frac{P}{A_S} = 15.80\ N/mm^2$

$\sigma_{C\,thermal} = \frac{P}{A_C} = 28.09\ N/mm^2$

Stress Copper rod $\quad 28.09 + 84.88 = 112.97\ N/mm^2$ (Tensile)

Steel $\quad 15.80 + 47.75 = 63.55\ N/mm^2$ (Compressive)

A load of 15kN is suspended by two outer Copper wires and one steel wire each of 100 mm^2 cross sectional area. The length of the wires are so adjusted to share equal load at 20°c i) Find the stresses in wires at 45°c ; ii) If the length of the wires are same at 20°c before applying the load, Find the stresses at 45°c when 15 kN load is applied.

$E_s = 200$ GPa ; $E_C = 100$ GPa; $\alpha_s = 12 \times 10^{-6}$ /°c $\alpha_c = 18 \times 10^{-6}$ /°c.

March 2001

i) $P_c = P_s = P$

σ_{wire} = Temp stress + Stress due to load

$2P_c + P_s = 15000$ N

$P_c = P_s = 5000$ N

$\delta t = 45 - 20 = 25°$

$$\sigma_{st} = \alpha_s \delta t E + \frac{5000}{100}$$
$$= 12 \times 10^{-6} \times 25 \times 200 \times 10^3 + \frac{5000}{100}$$
$$= 60 + 50 = 110 \text{ MPa}.$$
$$\sigma_c = 18 \times 10^{-6} \times 25 \times 100 \times 10^3 + 50 = 95 \text{ MPa}.$$

(ii) $l_c = l_s$, $\delta_s = \delta_c$, $2P_c + P_s = 15000$ N

$$\delta_s = \delta_c$$
$$\frac{P_s}{A_s}\frac{L_s}{E_s} = \frac{P_c}{A_c}\frac{L_c}{E_c}$$

$A_s = A_c$

$$\frac{P_s}{P_c} = \frac{E_s}{E_c} = \frac{200 \times 10^3}{100 \times 10^3} = 2$$ $P_s = 2P_c$

Since $2P_c + P_s = 15000$ N
$4P_c = 15000$ N

$P_c = 3750$ N
$P_s = 7500$ N

$$\sigma_s = \alpha_s \delta t E + \frac{7500}{100}$$

σ_{wire} = Temp stress + Stress due to load

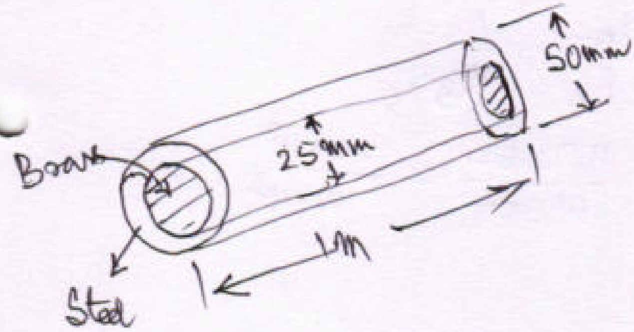
$$= 60 + 75 = 135 \text{ MPa}$$

$$\sigma_c = 45 + 37.5 = 82.5 \text{ MPa}$$

A bar of brass 25 mm dia is enclosed in a steel tube of 50 mm external dia and 25 mm internal diameter. The bar and tube are both initially 1 m long and rigidly fastened at both the ends. Find the stresses in two materials when the temperature rises from 10°c to 90°c.

If the composite bar is then subjected to an axial load tensile of 60 KN, find the resulting stresses given that $E_s = 200 \times 10^3$ MPa, $E_b = 100 \times 10^3$ MPa, $\alpha_s = 11.6 \times 10^{-6}/°c$, $\alpha_b = 18.7 \times 10^{-6}/°c$. (10 Marks)

Aug-Sept 2020



$\delta t = 90 - 10° = 80°c$

$A_{brass} = \frac{\pi \times 25^2}{4} = 490.87$ mm²

$A_{steel} = \frac{\pi}{4}(50^2 - 25^2) = 1472.62$ mm²

Now $P_s = P_b = P$

$$\delta l_s + \delta l_b = \alpha_b \cdot \delta t \cdot L - \alpha_s \, \delta t \cdot L$$

$L_s = L_b = 1000$ mm

$$\frac{P_s L_s}{A_s E_s} + \frac{P_b L_b}{A_b E_b} = \delta t \cdot L \left[\alpha_b - \alpha_s\right]$$

$$P \times 1000 \left[\frac{1}{1472.62 \times 200 \times 10^3} + \frac{1}{490.87 \times 100 \times 10^3}\right]$$

$$= 80 \times 1000 \left[18.7 \times 10^{-6} - 11.6 \times 10^{-6}\right]$$

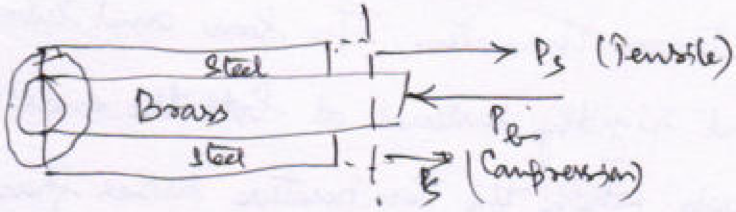
$$\left[3.395 \times 10^{-9} + 2.037 \times 10^{-8}\right] P = 5.68 \times 10^{-4}$$

$$\boxed{P = 23900.69 \text{ N}}$$

Stress in steel $= \sigma_s = \frac{23900.69}{1472.62} = 16.23$ N/mm² (Tension)

$\sigma_b = \frac{23900.69}{490.87} = 48.69$ N/mm² (Compression)

II Part: Composite bar is subjected to an axial tensile load of 60 kN.



$$P_s + P_b = 60\,kN = 60 \times 10^3\,N$$

$$\delta l_s = \delta l_b$$

$$\frac{P_s\, l_s}{A_s E_s} = \frac{P_b\, l_b}{A_b E_b}$$

$$P_s = \frac{P_b \cdot \cancel{l_b}}{A_b \cdot E_b} \cdot \frac{A_s \cdot E_s}{\cancel{l_s}}$$

$$P_s = P_b \times \frac{1472.62}{490.87} \times \frac{200 \times 10^3}{100 \times 10^3}$$

$$\boxed{P_s = 6\,P_b}$$

Now

$$6P_b + P_b = 60 \times 10^3$$

$$\boxed{P_b = 8.57 \times 10^3}$$

$$\boxed{P_s = 51.43 \times 10^3}$$

$$\sigma_s = \frac{51.43 \times 10^3}{1472.62} = 34.92\ N/mm^2$$

$$\sigma_b = \frac{8.57 \times 10^3}{490.87} = 17.45\ N/mm^2$$

Total stress:

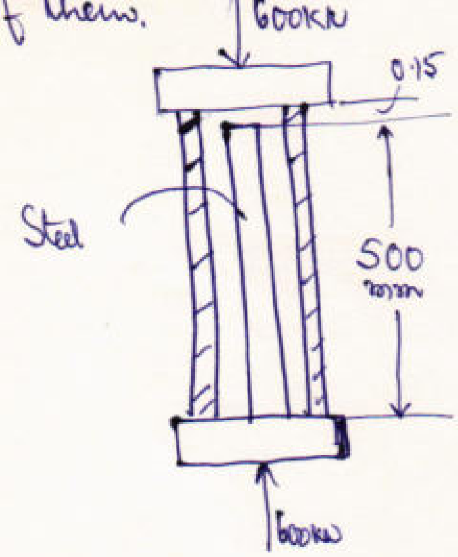
$$\sigma_s = (\sigma_s)_{thermal} + \sigma_s = 16.23 + 34.92 = 51.15\ N/mm^2$$

(Both stresses are tensile) (Tensile)

$$\sigma_b = (\sigma_b)_{thermal} + \sigma_b = -48.69 + 17.45 = -31.24\ N/mm^2$$

(Compression)

A Solid steel bar 500 mm long and 70 mm diameter is placed inside an aluminium tube having 75 mm inside diameter and 100 mm outside diameter. The aluminium Cylinder is 0.15 mm larger than the steel bar. An axial load of 600 kN is applied to the bar and the cylinder through rigid Cover plates as shown in fig. Find the stresses developed in the steel bar and aluminium tube and the deformations in each of them. $E_s = 210\,kN/mm^2$ and $E_{al} = 70\,kN/mm^2$



$A_s = \frac{\pi \times 70^2}{4} = 3848.45\,mm^2$

$A_L = \frac{\pi}{4}(100^2 - 75^2) = 3436.11\,mm^2$

To have a deformation of 0.15 mm, the load required is

$dl = \frac{PL}{AE}$ $\quad P = \frac{dl.A.E}{L}$

$P = \frac{0.15 \times 3436.11 \times 70 \times 10^3}{500.15}$

$P = 72136.669\,N = 72.136\,kN$

The load to be shared by steel and alumin = 600 − 72.136 = 527.86 kN.

Now deformations are Equal

$dl_s = dl_{Al}$ $\quad P_{AL} + P_s = 527.86\,kN$

$\frac{P_s L_s}{A_s E_s} = \frac{P_{AL} L_{AL}}{A_{AL} E_{AL}}$ &

$P_s = \frac{P_{AL} \times 500 \times 3848.45 \times 210 \times 10^3}{3436.11 \times 70 \times 10^3 \times 500}$

$\boxed{P_s = 3.36\,P_{AL}}$

Then $4.36\,P_{AL} = 527.86$

$\boxed{P_{AL} = 121.06\,kN}$

$\boxed{P_s = 406.79\,kN}$

$\boxed{\sigma_s = \frac{P_s}{A_s} = 105.70\,N/mm^2}$

$dl_s = \frac{105.70 \times 500}{210 \times 10^3}$

$\boxed{dl_s = 0.2516\,mm}$

$dl_{al} = \frac{56.22 \times 500.15}{70 \times 10^3}$

$\boxed{dl_{Al} = 0.4016\,mm}$

Total load by Aluminium = 121.06 + 72.136 = 193.196 kN

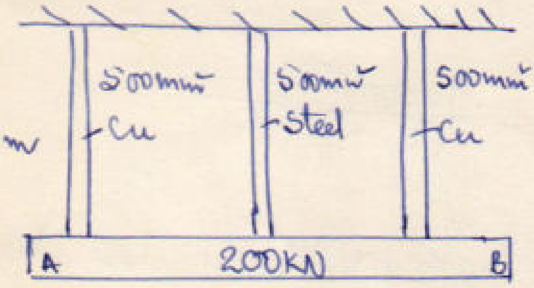
$\boxed{\sigma_{Al} = \frac{193.196 \times 10^3}{3436.11} = 56.22\,N/mm^2}$

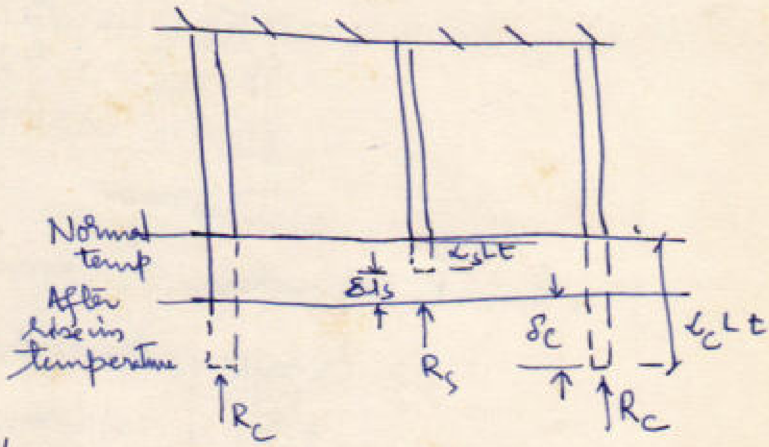
A reinforced Concrete Column 300mm x 300mm has 4 reinforcement bars of steel each 20mm in diameter. Calculate the safe load the Column Can take if the permissible stress in Concrete 5.2N/mm² and $\frac{E_{steel}}{E_{Concrete}} = 18$. (08 marks)

July 2015

A horizontal rigid bar AB weighing 200KN is hung by three vertical rods each of 1m length and 500mm² in cross section is shown in fig. The central rod is of steel and the outer rods are copper. If the temperature rise is 40°c, estimate the load carried by each rod and by how much the load will descend. Take $E_S = 200 GN/m^2$ $\alpha_s = 1.2\times10^{-5}/°c$. $E_c = 100GN/m^2$ $\alpha_c = 1.8\times10^{-5}/°c$

What should be the temperature rise if the entire load of 200KN is to be carried by steel rod alone?





Soln:- $E_S = \frac{200\times10^9}{1000\times1000} = 200\times10^3 N/mm^2$

$E_C = 100\times10^3 N/mm^2$.

i) When only 200KN acts on bars:

$P_s + 2P_c = 200KN$ — ①

also $\delta_s = \delta_c$

$\frac{P_s L}{A_s E_s} = \frac{P_c L}{A_c E_c}$ $P_s = 2P_c$ — ②

Applying ② in 1, We get $P_c = 50KN$

$P_s = 100$ KN.

$\delta_s = \delta_c = \frac{50\times10^3\times1000}{500\times100\times10^3} = 1mm$

ii) when there is a rise of 40°c

when $\alpha_c > \alpha_s$, free expansion of copper is more than steel. As the bars are rigid, compressive forces develop in copper and tensile forces in steel.

$R_s = 2R_c$

and $\delta_s + \delta_c = t L[\alpha_c - \alpha_s]$

$\frac{R_s L}{A_s E_s} + \frac{R_c L}{A_c E_c} = 40°\times100[0.6\times10^{-5}]$

$R_S = 2R_C$

$$R_C\left[\frac{2}{500\times2\times10^5}+\frac{1}{500\times1\times10^5}\right]=0.6\times10^{-5}\times40$$

$$\boxed{R_C = 6000N}$$

$$\boxed{R_S = 12\,000N}$$

Extension of the bar due to temperature rise

$$= \alpha_S \delta t L + \frac{R_S L}{A_S E_S}$$

$$= 1.2\times10^{-5}\times40\times1000 + \frac{12000\times1000}{500\times2\times10^5}$$

$$= 0.60mm$$

The ~~total~~ expansion or amount by which the bar will descend

= Expansion due to load + Expansion due to temp

$= 1 + 0.6 = 1.6$ mm

Load Carried by Steel $= P_S + R_S$

$$= 100\times10^3 + 12\times10^3$$

$$= 112\times10^3 \text{ kN (Tensile)}$$

Copper $= P_C - R_C$

$$= 50\times10^3 - 6\times10^3$$

$$= 44 \text{ kN (Tensile)}$$

ii) Rise in temperature if entire load is to be carried by steel rod alone:

$P_C = R_C$
(Tension) (Compression)

$P_C = R_C = 50,000$ ~~k~~N

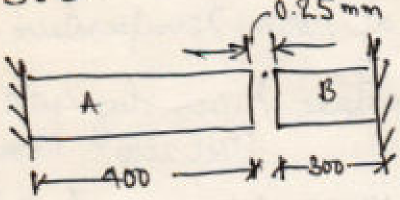
$$R_C\left[\frac{2}{500\times2\times10^5}+\frac{1}{500\times1\times10^5}\right]=0.6\times10^{-5}\times t$$

ie

$$50000\left[\frac{2}{500\times2\times10^5}+\frac{1}{500\times1\times10^5}\right]=0.6\times10^{-5}t$$

$$t = 333.33^\circ C$$

At room temperature, the gap between bar A and bar B as shown in fig is 0.25 mm. What are the stresses induced in the bars, if temperature rise is 35°C.



Given: $A_a = 1000\,mm^2$ $\quad A_b = 800\,mm^2$

$E_a = 2\times10^5\,mm^2$ $\quad E_b = 1\times10^5\,mm^2$

$\alpha_a = 12\times10^{-6}/°c$ $\quad \alpha_b = 23\times10^{-6}/°c$

$L_a = 400\,mm$ $\quad L_b = 300\,mm$

$\delta t = 35°$

$P_a = 27.73\,N/mm^2$

$P_b = 34.67\,N/mm^2$

Soln

Change in the length of bar (free expansion) $= \delta l_a + \delta l_b = \delta t L_b \alpha_b + \delta t L_a \alpha_a$

$= 35\left(300\times23\times10^{-6} + 400\times12\times10^{-6}\right)$

$= \cancel{0.0117200}\ 0.4095\,mm$

Expansion prevented $= 0.4095 - 0.25 = 0.1595\,mm.$

Due to prevention of expansion, compressive force 'P' is produced at the ends.

Now

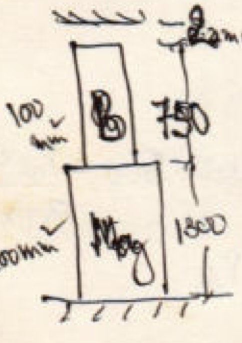
$$\frac{PL_a}{A_aE_a} + \frac{PL_b}{A_bE_b} = 0.1595$$

$$P\left[\frac{400}{1000\times2\times10^5} + \frac{300}{800\times1\times10^5}\right] = 0.1595$$

$$P = 27739.13\,N \qquad \sigma_a = 27.73\,N/mm^2$$

$$\sigma_s = 34.67\,N/mm^2.$$

The composite bar shown in fig is 02 mm short of distance between the rigid supports at room temperature. What is the maximum temperature rise which will not produce stresses in the bar? Find the stresses induced when temperature rise in 300°c



$\alpha_b = 10\times10^{-6}/°c$

$\alpha_m = 14.5\times10^{-6}/°c$

Free expansion $= 02 = \delta t\, l_b \alpha_b + \delta t\, l_m \alpha_m$

$= \delta t\left[750\times10\times10^{-6} + 1300\times14.5\times10^{-6}\right]$

$\delta t = \dfrac{02}{[0.026]}$

$= 75.9°C.$

(Series*)

Free expansion prevented $=$ Free expansion of B + Free expansion of Mg $- 2$

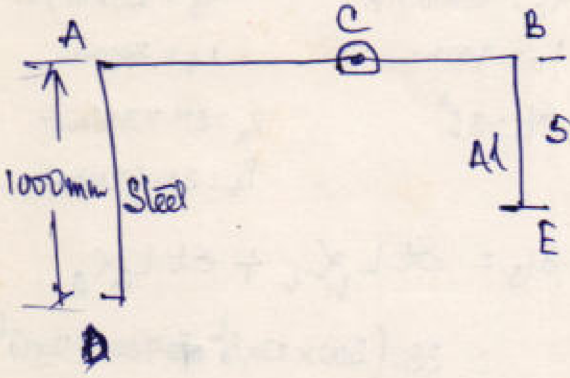
$= \alpha_b \delta t\, l_b + \alpha_m \delta t\, L_m - 2$

$= 10\times10^{-6}\times300\times750 + 14.5\times10^{-6}\times300\times1300 - 2$

$= 7.905 - 2 = 5.905\,mm$

AB is a rigid bar and has an hinged support at C as shown in fig. A steel and an aluminium bar support it at ends A and B respectively. The bars were stress free at room temperature. What are the stresses induced, when the temperature rises by 40°C

(Dec 2010 14 marks)

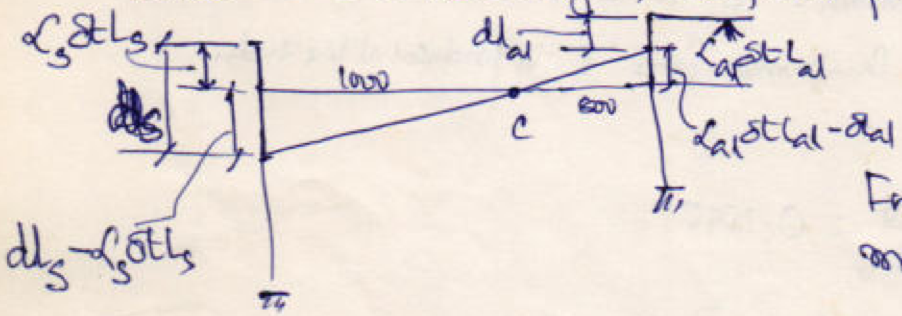


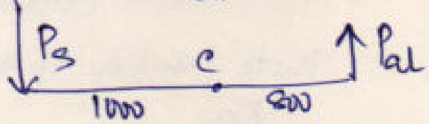
$A_s = 1200\,mm^2$ $\quad A_{al} = 800\,mm^2$

$\alpha_s = 12\times10^{-6}/°c$ $\quad \alpha_{al} = 23\times10^{-6}/°c$

$E_s = 2\times10^5\,N/mm^2$ $\quad E_{al} = 1\times10^5\,N/mm^2$

The free thermal expansion of aluminium is more than steel. Since the bar ABC is rigid, the final position after thermal rise is





For static equilibrium, taking moments about C

$$P_s \times 1000 = P_{al} \times 800$$

$$P_s = 0.8 P_{al}$$

From similar triangles,

$$\frac{dl_s - \alpha_s \delta t L_s}{1000} = \frac{\alpha_{al}\delta t L_{al} - \delta l_{al}}{800}$$

$$0.8\left(\frac{P_s L_s}{A_s E_s} - \alpha_s \delta t L_s\right) = \left(\alpha_{al}\delta t L_{al} - \frac{P_{al}L_{al}}{A_{al}E_{al}}\right)$$

$$0.8\left(\frac{0.8 P_{al}\times1000}{1200\times2\times10^5} - 12\times10^{-6}\times40\times1000\right) = \left(23\times10^{-6}\times40\times500 - \frac{P_{al}\times500}{800\times1\times10^5}\right)$$

$$P_{al}\,2.666\times10^{-6} - 0.384 = 0.46 - 6.25\times10^{-6} P_{al}$$

$$8.916\times10^{-6} P_{al} = 0.844$$

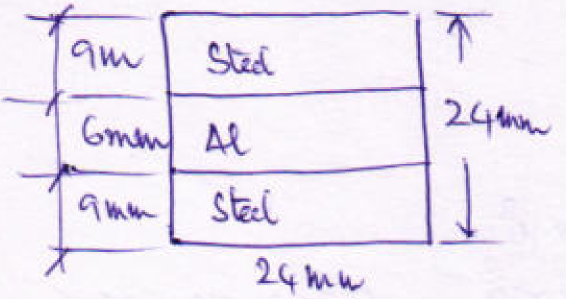
$$\boxed{P_{al} = 94661.28\,N}$$

$$\boxed{P_s = 75729.02\,N}$$

$\sigma_{al} = 118.32\,N/mm^2$

$\sigma_s = 63.10\,N/mm^2$

A flat bar of aluminium alloy 24 mm wide and 6 mm thick is placed between steel bars each 24 mm wide and 9 mm thick to form a Composite bar (24x24) mm as shown in fig. The three bars are fastened together at their ends when the temperature is 10°C. Find the stresses in each of the materials when the temperature of the whole assembly is raised to 50°C. If at the new temperature, a Compressive load of 20 kN is applied to the Composite bar, what are the final stresses in steel and aluminium? $E_s = 2\times10^5\ N/mm^2$ $E_a = 2.8 \times 10^5\ N/mm^2$, $\alpha_s = 1.2\times10^{-5}/°c$ $\alpha_a = 2.3\times10^{-5}/°c$.



Here

$$2P_s = P_{al} = P$$

$$\frac{2P_s\, l_s}{A_s E_s} + \frac{P_{al}\, l_{al}}{A_{al} E_{al}} = \delta t \cdot L(\alpha_a - \alpha_s)$$

Let $2P_s = P$

$l_{al} = l_s = l$

$$l P\left[\frac{1}{A_s E_s} + \frac{1}{A_{al} E_{al}}\right] = \delta t\, l(\alpha_a - \alpha_s)$$

$A_s = 2\times9\times24 = 432\ mm^2$

$A_l = 6\times24 = 144\ mm^2$

$\delta t = 50-10 = 40°c$

$$P\left[\frac{1}{432\times2\times10^5} + \frac{1}{144\times2\times10^5}\right] = 40(2.3\times10^{-5} - 1.2\times10^{-5})$$

$$P[2.3148\times10^{-8} + 3.472\times10^{-8}] = 40(1.1\times10^{-8})$$

$$\boxed{P = }$$

Here Al bar is in Compression
Steel bar is in tension.

$P = 2P_s$ $P_s = 3801.74\ N$

$P_{al} = 7603.49N$

$$(\sigma_s)_t = \frac{3801.74}{216} = 17.60\ N/mm^2\ T$$

$$(\sigma_{al})_t = \frac{7603.49}{144} = 52.80\ N/mm^2\ C$$

ii) When a compressive load of 20 kN is applied

$$2P_s + P_{al} = 20\times10^3\ N$$

$$dl_{al} = dl_s$$

$$\frac{P_{al}\,l_{al}}{A_{al}E_{al}} = \frac{P_s\,l_s}{A_sE_s}$$

$$P_{al} = \frac{P_s\,l_s}{A_sE_s}\cdot\frac{A_{al}E_{al}}{l_{al}}$$

$$= \frac{P_s\times l_s}{432\times2\times10^5}\times\frac{144\times2.3\times10^5}{l_{al}}$$

$$\boxed{P_{al} = 0.383P_s}$$

$$2P_s + 0.383P_s = 20\times10^3$$

$$P_s = \frac{20\times10^3}{2.383} = 8392.78\ N$$

$$P_{al} = 3214.43\ N$$

$$\sigma_s = \frac{8392.78}{216} = 38.84\ C\ N/mm^2$$

$$\sigma_{al} = \frac{3214.43}{144} = 22.32\ C\ N/mm^2.$$

Total stress,

$$\sigma_{s\,final} = (\sigma_s)_{thermal} + (\sigma_s)$$

$$= 38.84 - 19.42$$

$$\boxed{\sigma_s = 19.75\ N/mm^2\ \text{Compressive}}$$

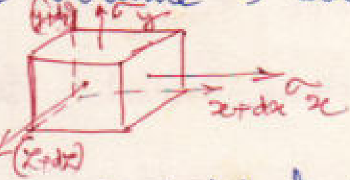
$$\sigma_{al} = 52.80 + 22.32$$

$$\boxed{\sigma_{al} = 75.12\ N/mm^2\ \text{(Compressive)}}$$

Volumetric Strain:

When a member is subjected to stresses, it undergoes deformation in all directions. Hence there will be a change in volume. The ratio of change in volume to original volume is called volumetric strain.

$$e_v = \frac{\delta v}{V}.$$



Consider a cube shown in fig. Let it be subjected to stresses σ_x, σ_y and σ_z along x, y and z directions respectively. Let the changed dimensions be $(x+dx)$, $(y+dy)$ and $(z+dz)$ along x, y and z directions respectively.

The changed volume is $(V+dV) = (x+dx)(y+dy)(z+dz)$

$$= (xy + ydx + \cancel{xdy} + dxdy)(z+dz)$$

$$V + dV = xyz + yzdx + \cancel{xzdy} + \cancel{zdxdy} + xydz + \cancel{ydxdz}$$

$$xyz + dV = \cancel{xyz} + xzdy + yzdx + xydz + \cancel{ydxdz} + \cancel{dxdydz}$$

$$dV = yzdx + xzdy + xydz$$

dividing throughout by xyz or V

$$\frac{dV}{V} = \frac{dx}{x} + \frac{dy}{y} + \frac{dz}{z}$$

$$\boxed{e_v = e_x + e_y + e_z} \quad \text{--- (1)}$$

Thus volumetric strain is the sum of strains along x, y and z directions respectively.

also
The net strain along x direction $e_1 = \frac{\sigma_x}{E} - \frac{\sigma_y}{mE} - \frac{\sigma_z}{mE}$

The net strain along y direction $e_2 = \frac{\sigma_y}{E} - \frac{\sigma_x}{mE} - \frac{\sigma_z}{mE}$

The net strain along z direction $e_3 = \frac{\sigma_z}{E} - \frac{\sigma_y}{mE} - \frac{\sigma_x}{mE}$

Then volumetric strain $e_v = \frac{1}{E}\left[\sigma_x + \sigma_y + \sigma_z\right] - \frac{2}{mE}\left[\sigma_x + \sigma_y + \sigma_z\right]$

$$= \frac{1}{E}\left[\sigma_x + \sigma_y + \sigma_z\right]\left[1 - \frac{2}{m}\right]$$

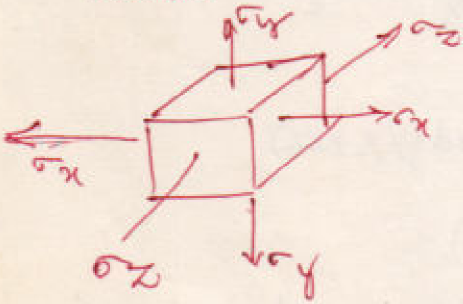
P.T Volumetric Strain is sum of strains in three mutually perpendicular directions Aug 1999 ✓

Establish the relationship between E and K. Aug 99. ✓

From I principles, establish the relationship between E and C. Aug 99 ✓

What are elastic Constants? Establish the Relationship between E and C Mar 2000 ✓

A test element is subjected to three mutually perpendicular unequal stresses: i) Find the change in the volume of the element ii) If the algebraic sum of these stresses is equal to zero, find the change in the volume of the element Mar 2001 ✓



We know that

The net strain $e_x = \frac{1}{E}\left[\sigma_x - \frac{1}{m}\sigma_y - \frac{1}{m}\sigma_z\right]$

$$e_y = \frac{1}{E}\left[\sigma_y - \frac{\sigma_z}{m} - \frac{\sigma_x}{m}\right]$$

$$e_z = \frac{1}{E}\left[\sigma_z - \frac{\sigma_x}{m} - \frac{\sigma_y}{m}\right]$$

$$e_v = e_x + e_y + e_z$$

$$\delta V = \frac{V}{E}\left[\sigma_x + \sigma_y + \sigma_z\right]\left[1 - 2/m\right]$$

If $\sigma_x + \sigma_y + \sigma_z = 0$ then $\boxed{\delta V = 0}$

Elastic Constants:

Modulus of elasticity, modulus of rigidity and bulk modulus are the three elastic constants

Modulus of elasticity: Ratio of stress to strain within elastic limit.

$E = \frac{\sigma}{e}$ where σ = Stress, e = Strain,

Modulus of rigidity:

It is defined as the ratio of shear stress to shear strain within elastic limit. It is usually denoted by letter G or C or N. Thus

$C = \frac{q}{\phi}$ where C = Modulus of rigidity.
q = Shear stress
ϕ = Shear strain.

Bulk modulus:

When a body is subjected to three equal and mutually perpendicular pairs of stresses, the body undergoes uniform changes in three directions. The ratio of normal stress to volumetric strain is called Bulk modulus.

It is denoted by the letter 'K'.

$$\text{Bulk modulus} = K = \frac{\sigma}{e_V}$$

σ = normal stress

$e_V = \frac{\delta V}{V}$ where V = original volume, δV = Change in volume

Here $\sigma = \sigma_x = \sigma_y = \sigma_z$

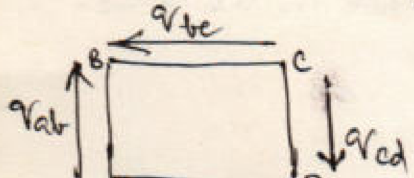
Simple Shear:

[P.T a material subjected to pure shear in two perpendicular planes has diagonal tension and compression of the same magnitude at 45° to the planes of shear.

March 2000]

A material is said to be in a state of simple shear if it is subjected to only shearing stress. Consider a bolt subjected to pure shear.

A rectangular element at this section is shown in fig.



Let the intensity of shear stress be q_{ab} and thickness of the element be 't'. Consider the equilibrium of the element.

The vertical force on AB = $q_{ab} \times t \times AB$.

This is balanced by vertical downward force on CD.

Then $q_{ab} \times t \times AB = q_{cd} \times t \times CD$

$$q_{ab} = q_{cd} = q.$$

The forces q_{ab} and q_{cd} being equal and opposite forces with moment arm AD form a couple $= q \times t \times AB \times AD$. —— ①

This can be balanced by another couple only. i.e. q_{bc} and q_{ad} should form equal and opposite couple. Hence their direction will be as shown in fig.

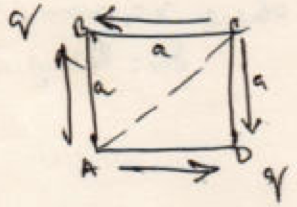
Let $q_{bc} = q_{ad} = q'$.

The couple formed by these forces $= q' \times t \times AD \times AB$. — ②

Equating ① and ② $\quad q = q'$.

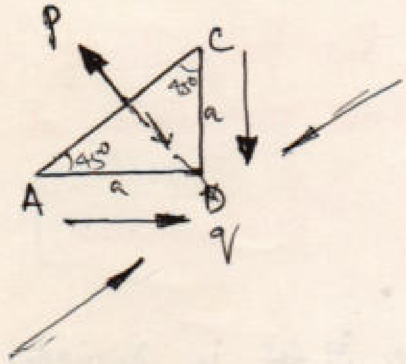
Thus if a section is subjected to pure shear, the state of stress in any element at that section is as shown in fig.

Consider a square element of side 'a' and thickness 't' under simple shear. Now the diagonal AC at 45° to shear direction is considered



$AC = \sqrt{2}\,a$. Consider a section along AC and let 'p' be the stress on this section.

Consider the direction normal to AC and for equilibrium, we get,



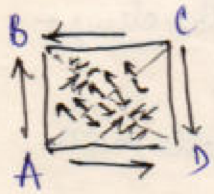
$$p \times AC \times t = q\,CD\,t\cos 45^\circ + q\,AD\,t\cos 45^\circ$$

$$p \times \sqrt{2}\,a \times t = q\,a\,t\,\frac{1}{\sqrt{2}} + q\,a\,t\,\frac{1}{\sqrt{2}}$$

$$p\sqrt{2}\,a\,t = q\,a\,t\sqrt{2}$$

$$\boxed{p = q}$$

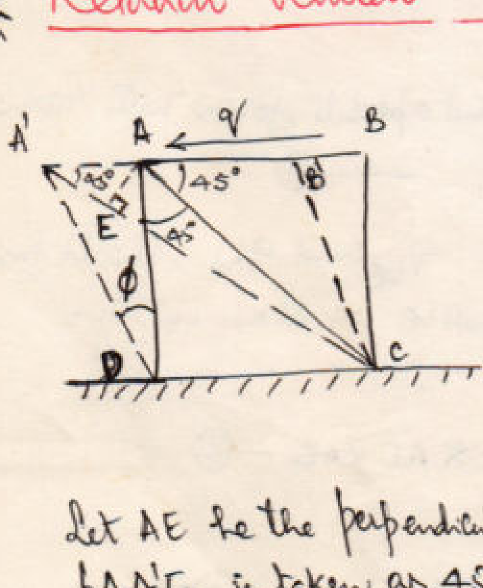
Thus a tensile stress of the same magnitude as shear stress develops at 45° to shear plane. This is called diagonal tension. By taking stresses on diagonal BD it can be shown that compressive stress of magnitude 'q' acts on this plane.



* Relation between E, C and K :

Relation between E and C

Consider an element ABCD subjected to shear stress 'q' as shown fig. The diagonal AC is elongated and the diagonal BD is compressed.



Shear strain $= \dfrac{AA'}{AD}$
$(\tan\phi)$

Since ϕ is very small, $\tan\phi = \phi = \dfrac{AA'}{AD}$.

Let AE be the perpendicular to A'C. Since AA' is very very small, the $\angle AA'E$ is taken as 45°. From triangle AA'E,

$$\cos 45^\circ = \frac{A'E}{A'A}$$

$$A'A = \frac{A'E}{\cos 45} \quad \text{but } \phi = \frac{A'A}{AD} \quad \therefore A'A = \phi \cdot AD$$

$$\therefore \phi = \frac{A'E}{\cos 45\,AD}$$

On the Δ^{le} ADC, $\cos 45^\circ = \frac{AD}{AC}$

$$AD = AC \cos 45^\circ$$

then $\phi = \frac{A'E}{\cos 45^\circ \cdot AC \cos 45^\circ}$ $\qquad \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}}$

$$= \frac{2A'E}{AC} \qquad \frac{1}{2}$$

Since deformation is very small, AC can be taken as equal to EC

$AC = EC$.

$$\phi = \frac{2A'E}{EC} = 2e$$

ie Shear strain = 2 . Longitudinal strain.

But $C = \frac{q \text{ or } \tau}{\phi}$ $\quad \frac{\tau}{C}$ or $\frac{q}{C} = 2e$. $\quad \frac{\tau}{C} = 2e \quad \therefore e = \frac{\tau}{2C}$

$\phi = \frac{q}{C}$ It is known that pure shear gives rise to diagonal tension 'q' along AC and diagonal compression 'q' along BD. Hence

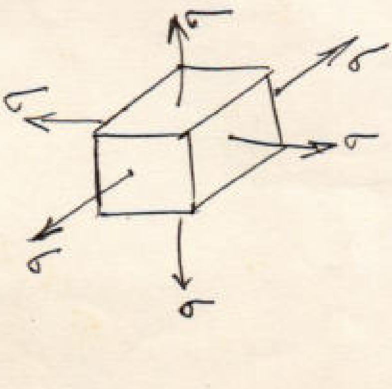
Net strain in diagonal $AC = e = \frac{q}{E} + \frac{q}{mE}$

$$e = \frac{q}{E}\left[1 + \frac{1}{m}\right]$$

$$\frac{q}{2C} = \frac{q}{E}\left[1 + \frac{1}{m}\right]$$

$$\boxed{E = 2C\left[1 + \frac{1}{m}\right]} \qquad \text{or } \boxed{E = 2C\left[1 + \mu\right]}$$

Relation between E and K:-



Consider a cube subjected to stress 'σ' in three mutually perpendicular directions x, y and z.

Now,

the net strain along x direction $e_x = \frac{\sigma}{E} - \frac{\sigma}{mE} - \frac{\sigma}{mE}$

$$e_x = \frac{\sigma}{E}\left[1 - \frac{2}{m}\right]$$

III^{ly}

$$e_y = \frac{\sigma}{E}\left[1 - \frac{2}{m}\right]$$

$$e_z = \frac{\sigma}{E}\left[1 - \frac{2}{m}\right]$$

$\therefore$ Volumetric strain $= e_v = e_x + e_y + e_z$

$$= \frac{3\sigma}{E}\left[1 - 2/m\right]$$

Then Bulk modulus $= K = \frac{\sigma}{e_v}$

$$= \frac{\sigma}{\frac{3\sigma}{E}\left[1-2/m\right]}$$

$$K = \frac{E}{3\left[1-2/m\right]}$$

$$\boxed{E = 3K\left[1-2/m\right]}$$

Relation between E, C and K:

We know that $E = 2C\left[1 + 1/m\right]$ —— ①

$E = 3K\left[1 - 2/m\right]$ —— ②

From eqn ①, $\frac{1}{m} = \frac{E}{2C} - 1$

Substituting the value of $1/m$ in eqn ②, we get,

$$E = 3K\left[1 - 2\left(\frac{E}{2C} - 1\right)\right]$$

$$= 3K\left[1 - \frac{E}{C} + 2\right]$$

$$= 3K\left[3 - \frac{E}{C}\right]$$

$$= 3K\left[\frac{3C - E}{C}\right]$$

$$E = \frac{9KC - 3KE}{C}$$

$$3KE + EC = 9KC$$

$$E\left[3K + C\right] = 9KC$$

$$\boxed{E = \frac{9KC}{3K+C}}$$

Calculate the modulus of rigidity and bulk modulus of a cylindrical bar of diameter 30mm and length 1.5mts if the longitudinal strain in a bar during a tension test is four times the lateral strain. Find the change in volume, when the bar is subjected to hydrostatic pressure of 100 N/mm². Take E = 100 kN/mm².

March 2000

Soln:

$$\frac{1}{m} = \frac{1}{4} = 0.25$$

$$E = 100\times10^3 \text{ N/mm}^2$$

$$E = 2C(1 + 1/m)$$

$$C = \frac{E}{2(1+1/m)}$$

$$C = \frac{100\times10^3}{2(1+0.25)} = \boxed{40000 \text{ N/mm}^2}$$

$$E = 3K(1 - 2/m)$$

$$K = \frac{E}{3[1-2\times1/m]} = \frac{100\times10^3}{3[1-0.5]} = 66666.6 \text{ N/mm}^2.$$

$$K = \frac{\sigma}{e_v}$$

$$e_v = \frac{100}{66666.66} = 1.5\times10^{-3}.$$

$$e_v = \frac{\delta V}{V} = 1.5\times10^{-3}.$$

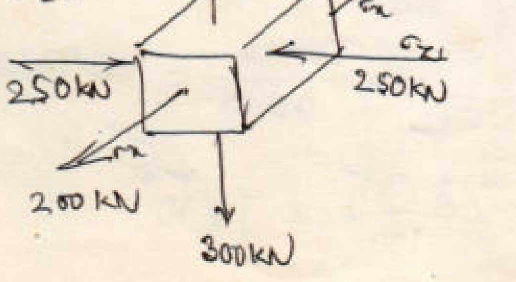
$$\delta V = \frac{\pi\times30^2}{4}\times1.5\times10^3\times1.5\times10^{-3}$$

$$\boxed{\delta V = 1590.43 \text{ mm}^3.}$$

A cube of a metal 100mm sides is subjected to forces of 200kN tensile, 250kN compressive and 300kN tensile on its perpendicular faces. Calculate the change in volume of the cube. $\mu = 0.26$, $E = 2.2\times10^5$ N/mm²

Aug 1999

Soln:



$$\sigma_x = \frac{200\times10^3}{100\times100} = 20 \text{ N/mm}^2 \text{ (Tension)}$$

$$\sigma_y = \frac{300\times10^3}{100\times100} = 30 \text{ N/mm}^2 \text{ (Tension)}$$

$$\sigma_z = \frac{250\times10^3}{100\times100} = 25 \text{ N/mm}^2 \text{ (Compressive)}$$

$$e_x = \frac{\sigma_x}{E} - \frac{\sigma_y}{E}\mu + \frac{\sigma_z}{E}\mu$$

$$e_x = \frac{20}{2.2\times10^5} - \frac{30\times0.26}{2.2\times10^5} + \frac{25\times0.26}{2.2\times10^5}$$

(Tension)

$$= 8.499\times10^{-5}$$

$$e_y = +\frac{\sigma_y}{E} - \mu\frac{\sigma_x}{E} + \mu\frac{\sigma_z}{E}$$

(Tension)

$$= \frac{1}{E}[30 - 0.26\times20 + 0.26\times25]$$

$$= 1.42\times10^{-4}$$

$$e_z = +\frac{\sigma_z}{E} + \mu\frac{\sigma_x}{E} + \mu\frac{\sigma_y}{E}$$

(Compression)

$$= \frac{1}{E}[25 + 0.26\times20 + 0.26\times30]$$

$$= 1.72\times10^{-4}$$

$$e_z = \frac{\sigma_z}{E} - \frac{1}{m}\frac{\sigma_x}{E} - \frac{1}{m}\frac{\sigma_y}{E}$$

$$= -\frac{\sigma_z}{E} - \frac{1}{m}\frac{\sigma_x}{E} - \frac{1}{m}\frac{\sigma_y}{E}$$

$$e_V = e_x + e_y - e_z$$

$$= 8.499\times10^{-5} + 1.42\times10^{-4} - 1.72\times10^{-4}$$

$$= 5.5\times10^{-5}$$

$$e_V = \frac{\delta V}{V} = 0.55\times10^{-4}$$

$$\delta V = 0.55\times10^{-6}\times100\times100\times100$$

$$\boxed{\delta V = 55\ mm^3}$$

A rod of diameter 100 mm and 1 m long is subjected to a pull of 200 kN in the direction of its length. The extension of rod was found to be 0.15 mm, while the decrease in diameter was 0.007 mm. Find the young's modulus, poisson's ratio, modulus of rigidity and bulk modulus for the material of the rod.

Soln:-

$$\sigma = \text{Stress} = \frac{P}{A} = \frac{200\times10^3}{7853.98} = 25.46\ N/mm^2$$

$$A = \frac{\pi\times100^2}{4} = 7853.98\ mm^2$$

$$\text{Linear strain} = e_{linear} = \frac{0.15}{1000} = 0.00015$$

$$\text{young's modulus} = E = \frac{\sigma}{e} = \frac{25.46}{0.00015} = 169733.33\ MPa$$

$$\text{Lateral strain} = \frac{\delta d}{d} = \frac{0.007}{100} = 7\times10^{-5}$$

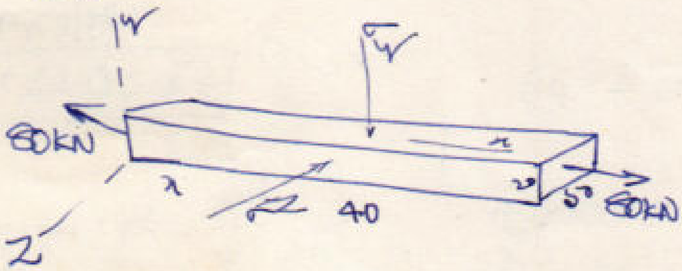
$$\text{Poisson's ratio} = \frac{1}{m} = \frac{7\times10^{-5}}{0.00015} = 0.466$$

Rigidity modulus $E = 2C(1+1/m)$

$$C = \frac{E}{2(1+1/m)} = \frac{169733.3}{2\times(1.466)}$$
$$= 57889.93\ N/mm^2.$$

$$E = 3K(1-2/m)$$
$$k = \frac{E}{3(1-2/m)} = \frac{169733.3}{3(1-2\times0.466)}$$
$$= 832025.95\ N/mm^2$$

A bar of rectangular cross section 20 mm × 50 mm is 400 mm long and is subjected to an axial tensile load of 80 kN. If E and C are $1\times10^5\ N/mm^2$ and $0.4\times10^5\ N/mm^2$, find the bulk ~~modulus~~ modulus and changes in dimensions and also volume.



$E = 1\times10^5\ N/mm^2$

$C = 0.4\times10^5\ N/mm^2$

$$E = \frac{9KC}{3K+C}$$

$$E(3K+C) = 9KC$$
$$3EK + EC = 9KC$$
$$\cancel{K = 9KC - EC}$$
$$3KE + EC - 9KC = 0$$
$$3K[E-3C] + EC = 0$$
$$3K = \frac{-EC}{E-3C}$$
$$K = \frac{1}{3}\frac{EC}{(3C-E)} = \frac{1}{3}\frac{1\times10^5\times0.4\times10^5}{(3\times0.4\times10^5 - 1\times10^5)}$$
$$= 66666.66\ N/mm^2$$

$$\boxed{K = \frac{\sigma}{e_v} \quad \therefore e_v = \frac{\sigma}{K}} =$$

$$\sigma_x = \frac{P}{A} = \frac{80\times10^3}{20\times50} = 80\ N/mm^2$$

* If σ_{normal} is identical in three mutually $\perp^r$ directions, then this is applicable

$$\cancel{e_v = \frac{\sigma}{K} = \frac{80}{66666.66} = 1.2\times10^{-3}}$$

$$\cancel{e_v = \frac{\delta v}{V} \quad \delta v = e_v \times V = 1.2\times10^{-3}\times 20\times50\times400}$$

$$\cancel{e_x = \frac{\sigma_x}{E}}$$

$$E = 2C(1+1/m)$$

$$\frac{1}{m} = \frac{E}{2C} - 1$$

$$= \frac{1\times10^5}{2\times0.4\times10^5} - 1$$

$$\boxed{\frac{1}{m} = 0.25}$$

Net strain $e_x = \frac{\sigma_x}{E} - \frac{1}{mE}\sigma_y - \frac{1}{mE}\sigma_z$

$$= \frac{80}{1\times10^5} - 0 - 0 = 8\times10^{-4}$$

$$e_x = \frac{\delta x}{x} \quad \therefore \delta x = e_x \cdot x$$

$$= 8\times10^{-4}\times400$$

$$\boxed{\delta x = 0.32\ mm}$$

$$e_y = +\frac{\sigma_y}{E} - \frac{1}{mE}(\sigma_x + \sigma_z)$$

$$= +\frac{0.25\times80}{1\times10^5} = (-)+2\times10^{-4}$$

$$e_y = \frac{\delta y}{y} \therefore \delta y = e_y \cdot y$$

$$= -2\times10^{-4}\times20$$

$$= +4\times10^{-3}$$

$$\boxed{\delta y = -0.004\ mm}$$

decrease

$$e_z = \frac{\sigma_z}{E} - \frac{1}{mE}(\sigma_y + \sigma_x)$$

$$= +\frac{0.25\times80}{1\times10^5} = (-)+2\times10^{-4}$$

$$e_z = \frac{\delta z}{z} \therefore \delta z = e_z \cdot z$$

$$= +2\times10^{-4}\times50$$

$$\boxed{\delta z = 0.010\ mm}$$

decrease

$$e_v = e_x + e_y + e_z$$

$$= 8\times10^{-4} - 2\times10^{-4} - 2\times10^{-4}$$

$$= 4\times10^{-4}$$

$$\delta v = e_v \times V$$

$$= 4\times10^{-4}\times20\times50\times400 = 160\ mm^3$$

A Circular rod of 100 mm dia and 500 mm long is subjected to a tensile force of 1000 kN. Determine C, K and δV if $\mu = 0.3$. $E = 2\times10^5\ N/mm^2$

Soln

$$E = 2C(1+\mu) = 3K(1-2\mu)$$

$$C = \frac{E}{2(1+\mu)} = 0.7692\times10^5\ N/mm^2$$

$$K = \frac{E}{3(1-2\mu)} = 1.6667\times10^5\ N/mm^2.$$

$$\text{Longitudinal stress} = \frac{P}{A} = \frac{1000\times10^3}{\frac{\pi}{4}\times100^2} = 127.324\ N/mm^2.$$

$$\text{Linear strain} = \frac{\sigma}{E} = \frac{127.324}{2\times10^5} = 63.662\times10^{-5}$$

$$\text{Lateral strain}\quad e_y = \frac{\sigma_y}{E} - \frac{\sigma_x}{mE} - \frac{\sigma_z}{mE}$$

$$= -63.662\times10^{-5}\times0.3 = 1.909\times10^{-4}$$

$$e_z = -1.909\times10^{-4}$$

$$e_v = e_x + e_y + e_z$$

$$= (63.662\times10^{-5}) - (2\times1.909\times10^{-4})$$

$$= 2.546\times10^{-4}$$

$$e_v = \frac{\delta V}{V}$$

$$\text{Change in volume} = \delta V = e_v\cdot V = 2.546\times10^{-4}\times\frac{\pi}{4}\times100^2\times500$$

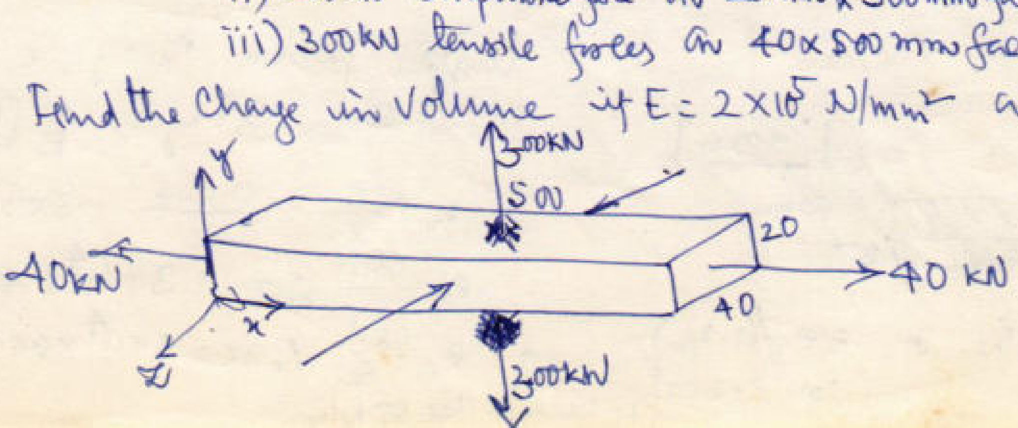
$$\boxed{V = 1000\ mm^3}$$

A 500 mm long bar has rectangular cross section 20 mm × 40 mm. This bar is subjected to i) 40 kN tensile force on 20 mm × 40 mm faces

ii) 200 kN Compressive force on 20 mm × 500 mm faces and

iii) 300 kN tensile forces on 40 × 500 mm faces

Find the change in volume if $E = 2\times10^5\ N/mm^2$ and $\mu = 0.3$



$$\sigma_x = \frac{40\times10^3}{40\times20} = 50\,N/mm^2$$

$$\sigma_y = \frac{300\times10^3}{500\times40} = 15\,N/mm^2$$

$$\sigma_z = \frac{200\times10^3}{500\times20} = 20\,N/mm^2$$

Tensile strain +ve
Compressive −ve

$$e_x = \frac{\sigma_x}{E} - \frac{\sigma_y}{mE} + \frac{\sigma_z}{mE}$$

$$= \frac{1}{E}\left[50 - 15\times0.3 + 20\times0.3\right] = 2.57\times10^{-4}$$

$$e_y = \frac{\sigma_y}{E} + \frac{\sigma_z}{mE} - \frac{\sigma_x}{mE}$$

$$= \frac{1}{E}\left[15 + 20\times0.3 - 50\times0.3\right] = 3\times10^{-5}$$

$$e_z = \frac{\sigma_z}{E} + \frac{\sigma_x}{mE} + \frac{\sigma_y}{mE} =$$

$$= \frac{1}{E}\left[20 + 50\times0.3 + 15\times0.3\right] = 1.975\times10^{-4}$$

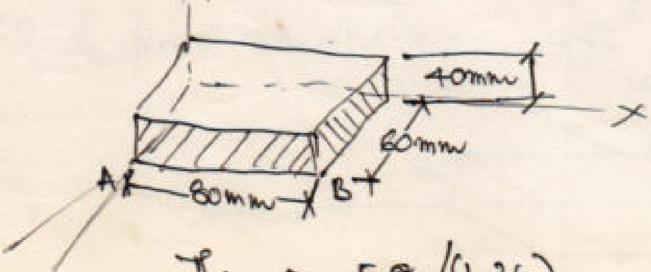
$$\cancel{\delta x = 4.845\times10^{-4}\times 20\times40\times500}$$

$$e_v = 2.57\times10^{-4} + 3\times10^{-5} - 1.975\times10^{-4}$$

$$\delta V = 8.95\times10^{-5}\times 40\times20\times500$$

$$\boxed{\delta V = 35.8\,mm^3}$$

The Steel block shown in fig is subjected to a uniform pressure on all its faces. Knowing that the change in length of edge AB is 0.024 mm, determine a) Change in length of other two edges. (b) the pressure 'P' applied to the faces of the block. Assume E = 200 GPa and $\frac{1}{m} = 0.29$.



Uniform pressure $\sigma_x = \sigma_y = \sigma_z = \sigma$

The net strain $= e_x = e_y = e_z = \frac{\sigma}{E}\left[1 - 2/m\right]$

$$e_x = \frac{\delta x}{x} = \frac{0.024}{80} = -3\times10^{-4} \text{ (Decrease)}$$

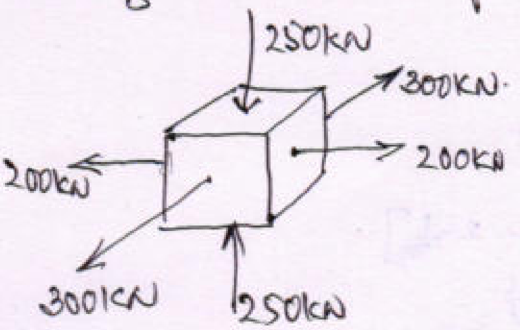
$$e_y = \frac{\delta y}{y} \therefore \delta y = 3\times10^{-4}\times 40 = 0.012\,mm$$

$$e_z = \frac{\delta z}{z} \therefore \delta z = 3\times10^{-4}\times60 = 0.018\,mm$$

Then $\sigma = E e_x/(1-2/m)$

$$= \left\{\frac{200\times10^3\times3\times10^{-4}}{(1-2\times0.29)}\right\} = 142.85\,N/mm^2$$

* A cube of a metal 100 mm sides is subjected to forces of 200 kN tensile, 250 kN compressive and 300 kN tensile on its perpendicular faces. Calculate the change in volume of the cube. $\mu = 0.26$, $E = 2.2 \times 10^5$ N/mm²



Soln:

$$\sigma_x = +\frac{200 \times 10^3}{100^2} = +20 \text{ N/mm}^2 \ (T)$$

$$\sigma_y = -\frac{250 \times 10^3}{100^2} = -25 \text{ N/mm}^2 \ (C)$$

$$\sigma_z = +\frac{300 \times 10^3}{100^2} = +30 \text{ N/mm}^2 \ (T)$$

$$e_x = \frac{\sigma_x}{E} - \mu\left(\frac{\sigma_y}{E} + \frac{\sigma_z}{E}\right)$$

$$= \frac{1}{E}\left[20 - 0.26(-25+30)\right]$$

$$e_x = \frac{1}{2.2 \times 10^5}\left[18.7\right] = 8.5 \times 10^{-5}$$

$$e_y = \frac{\sigma_y}{E} - \mu\left(\frac{\sigma_z}{E} + \frac{\sigma_x}{E}\right)$$

$$= \frac{1}{E}\left[-25 - 0.26(30+20)\right]$$

$$= \frac{1}{2.2 \times 10^5}\left[38\right] = -1.72 \times 10^{-4}$$

$$e_z = \frac{\sigma_z}{E} - \mu\left[\frac{\sigma_x}{E} + \frac{\sigma_y}{E}\right]$$

$$= \frac{1}{E}\left[30 - 0.26(20-25)\right]$$

$$= \frac{1}{2.2 \times 10^5}\left[31.3\right] = 1.422 \times 10^{-4}$$

$$e_v = e_x + e_y + e_z$$

$$= 8.5 \times 10^{-5} - 1.72 \times 10^{-4} + 1.422 \times 10^{-4}$$

$$e_v = 5.52 \times 10^{-5}$$

$$e_v = \frac{dv}{V}$$

$$dv = e_v \times V$$

$$= 5.52 \times 10^{-5} \times 100^3$$

$$\boxed{dv = 55.2 \text{ mm}^3}$$

* A circular rod of 100 mm dia and 500 mm long is subjected to a tensile force of 1000 kN. Determine C, K and δV if $\mu = 0.3$ and $E = 2 \times 10^5$ N/mm²

Soln:

$$C = \frac{E}{2(1+\mu)} = 0.7692 \times 10^5 \text{ N/mm}^2$$

$$K = \frac{E}{3(1-2\mu)} = 1.67 \times 10^5 \text{ N/mm}^2$$

$$\sigma_x = \frac{1000 \times 10^3}{\frac{\pi}{4} \times 100^2} = 127.324 \text{ N/mm}^2$$

$$\sigma_y = 0$$

$$\sigma_z = 0$$

$$e_x = \frac{1}{E}\left[\sigma_x - \mu(\sigma_y + \sigma_z)\right] = \frac{1}{2 \times 10^5}\left[127.32\right] = 6.366 \times 10^{-4}$$

$$e_y = \frac{1}{E}\left[\sigma_y - \mu(\sigma_z + \sigma_x)\right] = \frac{1}{2 \times 10^5}\left[0 - 0.3(127.32)\right] = -1.909 \times 10^{-4}$$

$$e_z = \frac{1}{E}\left[\sigma_z - \mu(\sigma_x + \sigma_y)\right] = \frac{1}{2 \times 10^5}\left[0 - 0.3(127.32)\right] = -1.909 \times 10^{-4}$$

$$e_v = e_x + e_y + e_z = 2.548 \times 10^{-4}$$

$$e_v = \frac{dv}{V} \quad \therefore dv = e_v \times V$$

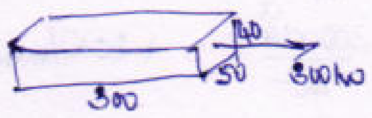
$$= 2.548 \times 10^{-4} \times \left(\frac{\pi}{4} \times 100^2 \times 500\right)$$

$$\boxed{dV = 1000.59 \text{ mm}^3}$$

A steel bar 300mm long, 50mm wide and 40mm thick is subjected to a pull of 300kN in the direction of its length. Determine the change in volume. Take $E = 2\times10^5$ N/mm² and Poisson's ratio = 0.25

Soln:

$$\text{Stress} = \sigma_x = \frac{300\times10^3}{50\times40} = 150\,N/mm^2.$$



$$E = 3K\left[1-2\cdot\frac{1}{m}\right]$$

$$K = \frac{2\times10^5}{3[1-2\times0.25]}$$

$$K = 133.33\times10^3\ N/mm^2.$$

$$K = \frac{\sigma}{e_v}$$

Statically indeterminate members

When the number of unknowns exceed the number of equilibrium equations then those members or setup are called Statically indeterminate members.

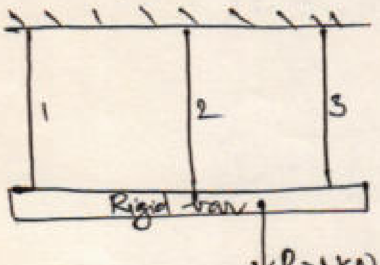
points to be remembered for solving these type of problems:

1) The FBD of the entire structure or part of it should be drawn with all the internal and external forces acting on it.

2) If there are unknowns more than three, additional equations should be developed by making use of geometric relations between the elastic deformations produced by the loads.

A load of $P = 1KN$ is applied to a rigid bar that is suspended by three wire as shown in fig. All the wires are initially of equal length L and of the same size. Compute the force carried by each wire. Take $L = 4m$, $E = 200\,GPa$ and $A = 80mm^2$.

Aug 2001



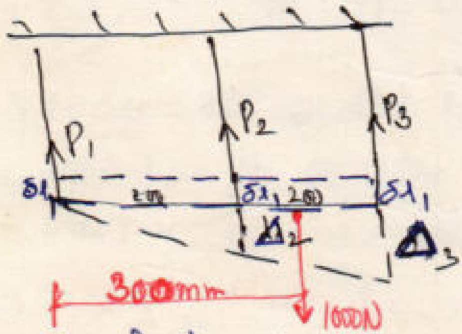
$A_1 = A_2 = A_3 = A$

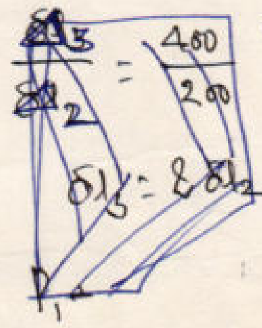
$L_1 = L_2 = L_3 = L$

$\Sigma V = 0$

$$\boxed{P_1 + P_2 + P_3 = 1KN} \quad —①$$

Soln:-





Taking moment about ①,

$$P_2 \times 200 + P_3 \times 400 = 1000 \times 300 \quad —②$$

Let δl_1 be the increase in elongation of wire 1.

Let δl_2 be the increase in length of wire 2 $\quad \delta l_2 = \delta l_1 + \Delta_2$

Let δl_3 be the increase in length of wire 3. $\quad \delta l_3 = \delta l_1 + \Delta_3$

From $\parallel^{ler}$ triangles, $\dfrac{\Delta_2}{200} = \dfrac{\Delta_3}{400}$ $\quad \boxed{\Delta_3 = 2\Delta_2} \quad —②'$

If P_1 is the force in wire (1), then

Let P' is the force to produce deflection Δ_2

$\|^{ly}$ $2P'$ is the force to produce deflection Δ_3 $\because \Delta_3 = 2\Delta_2$

$$\left.\begin{array}{llll} \delta l_1 & \rightarrow \delta l_1 & \frac{P_1 l}{AE} \longrightarrow & P_1 \\ \delta l_2 & \rightarrow \delta l_1 + \Delta_2 & \frac{P_1 l}{AE} + \frac{P' l}{AE} \longrightarrow & P_1 + P' \\ \delta l_3 & \rightarrow \delta l_1 + \Delta_3 & \rightarrow \delta l_1 + 2\Delta_2 & P_1 + 2P' \end{array}\right\} \quad \text{— (3)}$$

$$\frac{P_1 l}{AE} + \frac{2P' l}{AE}$$

Then

$$P_1 + (P_1 + P') + (P_1 + 2P') = 1000N$$

$$3P_1 + 3P' = 1000N \quad \text{— (4)}$$

$$(P_1 + P')200 + (P_1 + 2P')400 = 1000 \times 300$$

$$2P_1 + 2P' + 4P_1 + 8P' = 3000 \quad \text{— (5)}$$

$$6P_1 + 10P' = 3000$$

$$3P_1 + 5P' = 1500 \quad \text{— (6)}$$

Subtract (4) from (6)

$$2P' = 500$$

$$\boxed{P' = 250N} \quad \text{— (7)}$$

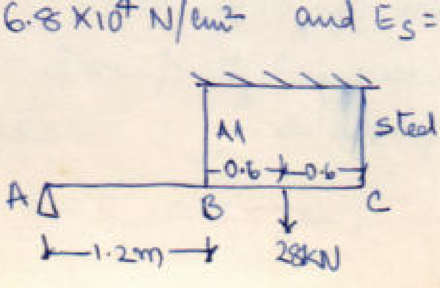
From (4)

$$P_1 = \frac{1000 - 750}{3} = 83.33\,N$$

$$P_2 = P_1 + P' = 250 + 83.33 = 333.33N$$

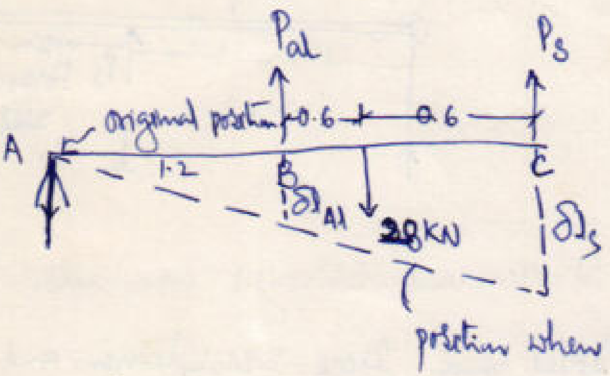
$$P_3 = P_1 + 2P' = 83.33 + 500 = 583.33N$$

A rigid bar ABC is hinged at A and distances AB = BC = 1.2m is hung by two rods of each 1m length which are made of Al and steel at B and C respectively. The bar carries a load of 28 kN midway between B and C. Determine the load carried by the two rods if $A_{Al} = 3cm^2$, $A_S = 2cm^2$, $E_{Al} = 6.8 \times 10^4\ N/cm^2$ and $E_S = 2 \times 10^5\ N/cm^2$.



Soln:-

The bar ABC is rigid and does not bend. The aluminium and steel rods will elongate under the action of load. There are three unknowns. $\Sigma M=0$, $\Sigma V=0$ are the two equations.



From III^{lar} triangles,

$$\frac{\delta l_{AL}}{1.2} = \frac{\delta l_s}{2.4}$$

$$\delta l_{AL} = 0.5\,\delta l_s \quad —①$$

Let the force in steel and aluminium rods be P_s and P_{Al} respectively.

$E_{al} = 6.8\times10^4\ N/cm^2$

$= 6.8\times10^2\ N/mm^2$

$$\therefore\ \delta l_{Al} = \frac{P_{al} L_{al}}{E_{al} A_{al}} = \frac{P_{al}\times1000}{\frac{6.8\times10^4}{10\times10}\times3\times10\times10} = 4.90\times10^{-3} P_{al}$$

$E_s = 2\times10^5\ N/cm^2$

$= 2\times10^3\ N/mm^2$

$$\therefore\ \delta l_s = \frac{P_s L_s}{A_s E_s} = \frac{P_s\times1000}{\frac{2\times10^5}{10\times10}\times2\times100} = 2.5\times10^{-3} P_s$$

$$\boxed{\delta l_{al} = \delta l_s\times0.5} \ —①$$

$$4.90\times10^{-3} P_{al} = 0.5\,(2.5\times10^{-3})\,P_s$$

$$\boxed{P_{al} = 0.255\,P_s} \qquad \boxed{P_s = 3.92\,P_{al}}$$

Taking moment about A,

$\Sigma M_A = 0$

$$P_s\times2.4 + P_{al}\times1.2 = 28\times1.8$$

$$P_s(2.4 + 0.255\times1.2) = 28\times1.8$$

$$\boxed{P_s = \frac{28\times1.8}{2.706} = 18.62\,kN}$$

$$\boxed{P_{al} = 0.255\times18.62 = 4.75\ kN}$$

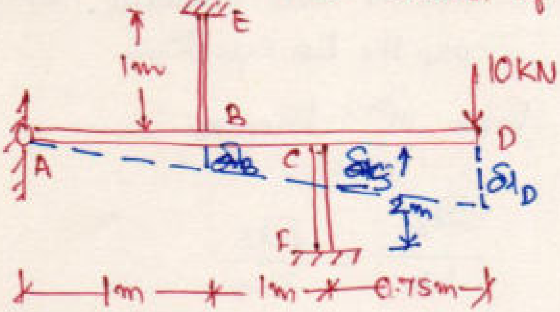
To find reaction at hinge.

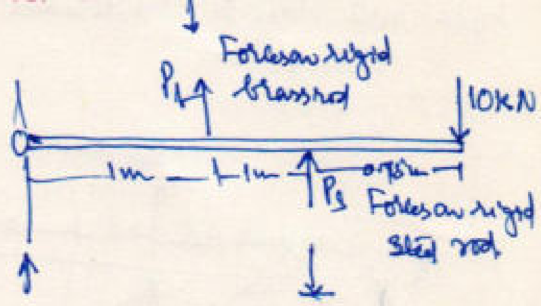
$$P_{al} + P_s + R_A = 28$$

$$R_A = 28 - (18.62 + 4.75)$$

$$\boxed{R_A = 4.63\ kN}$$

~~Two~~ The rigid bar ABCD shown in fig is hinged at A and is connected to brass rod BE and steel rod CF. Find the stress developed in brass and steel bars if $A_b = 1000\,mm^2$, $E_b = 1\times10^5\,N/mm^2$, $A_s = 600\,mm^2$, $E_s = 2\times10^5\,N/mm^2$ when a load of 10KN acts at end D.





Soln: Since the bar ABCD is rigid, it remains straight even after loading. The deformed position is as shown in dotted lines. Here EB is in tension and FC is in compression and FBD is as shown in fig.

Taking moment about A,

$$P_b \times 1 + P_s \times 2 = 10 \times 2.75 \times 10^3$$

$$\boxed{P_b + 2P_s = 27.5\times10^3}$$

From $\parallel^{lar}$ triangles,

$$\frac{\delta l_b}{1} = \frac{\delta l_s}{2}$$

$$\delta l_b = 0.5\,\delta l_s$$

$$\frac{P_b l_b}{A_b E_b} = 0.5\,\frac{P_s l_s}{A_s E_s}$$

$$P_b = 0.5\,\frac{P_s l_s}{A_s E_s}\cdot\frac{A_b E_b}{l_b}$$

$$= 0.5 \times \frac{P_s \times 2000}{600 \times 2\times10^5} \times \frac{1000 \times 1\times10^5}{1000}$$

$$P_b = 0.833\,P_s$$

$$P_s(0.833 + 2) = 27.5\times10^3$$

$P_s = 9707.82\,N$	$\sigma_s = 16.178\,N/mm^2$
$P_b = 8085.95\,N$	$\sigma_b = 8.8085\,N/mm^2$

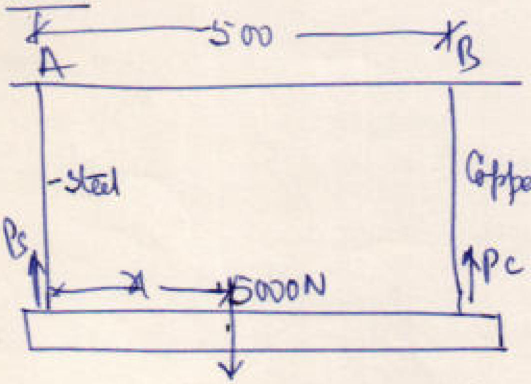
* Reactn at A $\quad -10 + P_s + P_b + R_A = 0 \qquad R_A = +10 - 17.79 = -7.79\,KN \downarrow$

Two Vertical rods one of steel and other of Copper are each rigidly fixed at their top 500mm apart. Dia and length of each rod are 20mm and 3.5m respectively. A cross bar is fixed at lower ends of the rods.

i) Determine the location of 5000N load to be placed on the Cross bar so that the cross bar remains horizontal. Calculate the stresses in both rods.

ii) If the load is placed at the centre of cross bar what will be the values of stresses in both rods. $E_C = 1.05 \times 10^5$ N/mm². $E_S = 2.1 \times 10^5$ N/mm²

Soln: Case 1: The cross bar is made to remain horizontal after the application of the load.



$l_S = l_C = 3500$ mm

$A_S = A_C = 314.15$ mm²

Let the load of 5000N be placed at a distance 'x' from 'A'.

Let P_S and P_C be the loads (axial forces) acting along steel and Copper bar respectively. $\therefore P_S + P_C = 5000N$

We also know that $\delta l_S = \delta l_C$

$l_S = l_C = l$

$$\frac{P_S l_S}{A_S E_S} = \frac{P_C l_C}{A_C E_C}$$

$$P_S = P_C \cdot \frac{E_S}{E_C}$$

$$P_S = \frac{2.1 \times 10^5}{1.05 \times 10^5} \cdot P_C$$

$$\boxed{P_S = 2P_C}$$

Then $3P_C = 5000N$

$P_C = 1666.66N$ $\quad \sigma_C = 5.30$ N/mm²

$P_S = 3333.34N$ $\quad \sigma_S = 10.61$ N/mm²

Taking moment about A.

$$5000 \cdot x = 1666.66 \times 500$$

$$x = \frac{1666.66 \times 500}{5000} = 166.67 \text{ mm}$$

Case 2: When the load is placed at the Centre of the cross bar, then taking moment about A,

$$5000 \times 250 = 500 \times P_c$$

$P_c = 2500\ N$ $\quad$ $\sigma_c = 7.95\ N/mm^2$

$P_s = 2500\ N$ $\quad$ $\sigma_s = 7.95\ N/mm^2$

A steel bar of 20mm dia is subjected to a tensile test. Determine stress, strain, E and percentage elongation from the following data:

a) gauge length = 200mm

b) Extension at a load of 100KN = 0.14mm

c) Total extension = 50 mm.

Also determine the percentage decrease in area of cross section if the diameter of the rod at failure is 16mm. Further determine the breaking load if ultimate stress of bar material is 600 N/mm².

Soln:

$$\text{Area} = \frac{\pi \times 20^2}{4} = 314.15 \qquad \text{Stress} = \sigma = \frac{100\times10^3}{314.159} = 318.309 \text{ N/mm}^2.$$

$$\text{Strain} = e = \frac{\delta l}{l} = \frac{0.14}{200} = 7\times10^{-4}$$

$$E = \frac{318.309}{7\times10^{-4}} = 454727\times14 \text{ N/mm}^2$$

$$\text{percentage elongation} = \frac{\text{Length of specimen at failure} - \text{original length}}{\text{original length}} \times 100$$

$$= \frac{\text{Total extension}}{\text{Original length}} \times 100$$

$$= \frac{50}{200} \times 100 = 25\%$$

$$\text{percentage reduction in area} = \left(\frac{\text{Original Area of C/s} - \text{Final area of C/s}}{\text{Original area of C/s}}\right)\times100$$

$$= \left(\frac{\frac{\pi\times20^2}{4} - \frac{\pi\times16^2}{4}}{\frac{\pi\times20^2}{4}}\right)100$$

$$= 35.94\ \%$$

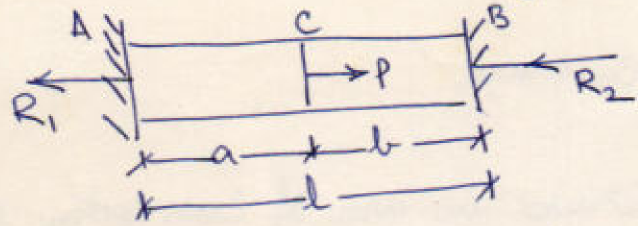
Breaking load = Ultimate stress × original area of C/s

$$= 600\times314.15$$

$$= 188490 \text{ N}$$

$$\text{Ultimate stress} = \frac{\text{Ultimate load}}{\text{Area of C/s}}$$

A homogeneous uniform bar is fixed at both the ends and carries an axial force 'P' as shown in fig. Determine the reactions R_1 and R_2.

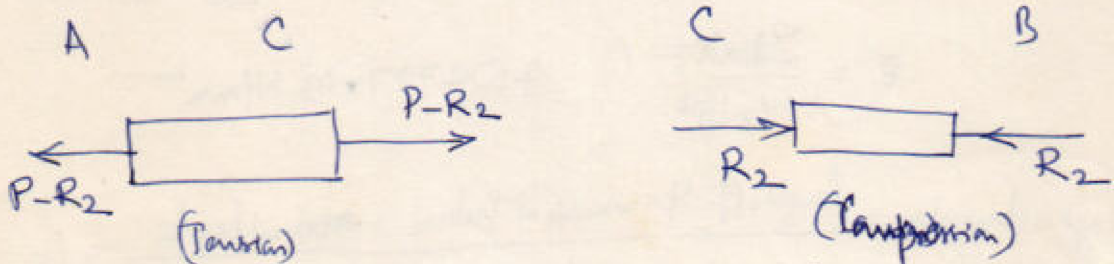


Soln: For equilibrium, the algebraic sum of all the horizontal forces must be equal to zero.

$$-R_1 - R_2 + P = 0$$

$$R_1 = P - R_2 \quad \text{—(1)}$$

FBD's of segments are as follows.



The net change in the length of the bar is equal to zero

$$\frac{P_{ac} L_{ac}}{A_{ac} E_{ac}} - \frac{P_{CB} L_{CB}}{A_{CB} E_{CB}} = 0 \qquad A_{ac} = A_{CB}, \quad E_{ac} = E_{CB}$$

$$\frac{(P-R_2)(a)}{A\,E} - \frac{R_2(b)}{AE} = 0$$

$$(P-R_2)(a) - R_2(b) = 0$$

$$Pa - R_2 a - R_2 b = 0$$

$$Pa - R_2(b+a) = 0$$

$$R_2 = \frac{Pa}{(b+a)} = \frac{Pa}{l}.$$

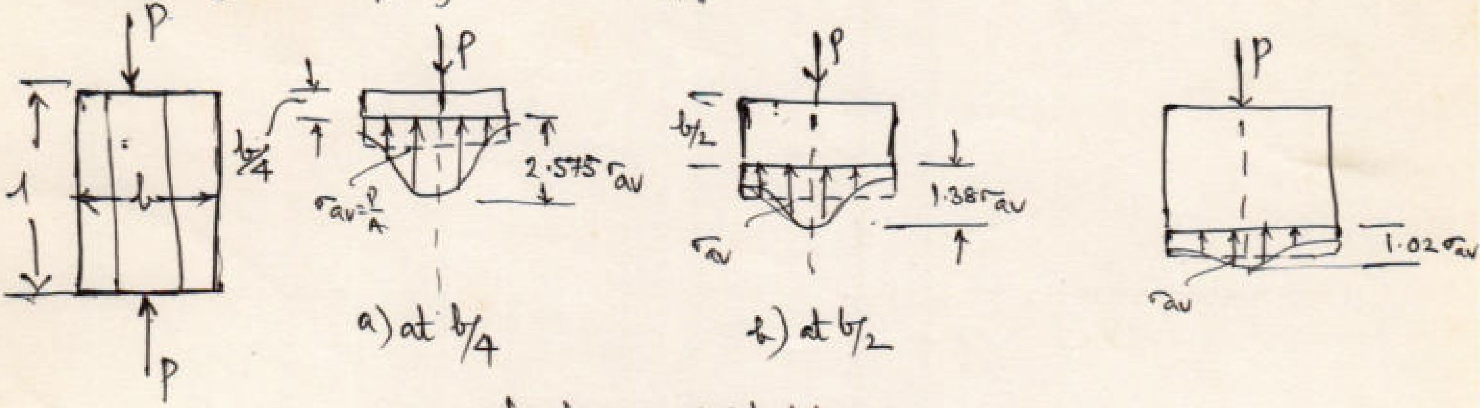
$$\boxed{R_2 = \frac{Pa}{l}}$$

$$R_1 = P - \frac{Pa}{l} = \frac{lP - Pa}{l} = \frac{P(l-a)}{l} = \frac{Pb}{l}$$

$$\boxed{R_1 = \frac{Pb}{l}}$$

St. Venant's Principle:

This Principle states that "the stresses always tend to Concentrate in the immediate vicinity of the load applied and becomes uniform at Sections farther off from the load".

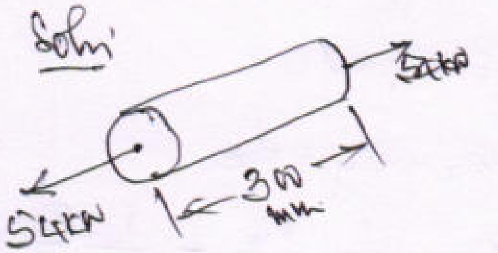


The stresses that are Considered for designs and Calculations, are average stresses $\sigma_{av} = \frac{P}{A}$.

A bar 30mm in diameter was subjected to tensile load of 54 kN and measured extension on a 300mm gauge length was 0.112 mm and change in diameter was 0.00366 mm. Calculate Poisson's ratio and the values of three modulii.

12 marks
June-July 2019.

Soln:



Longitudinal strain $= \frac{\delta l}{l} = \frac{0.112}{300}$

$e_{long} = 3.73 \times 10^{-4}$

$\delta l = 0.112$ mm

$l = 300$ mm

$\delta d = 0.00366$ mm

$d = 30$ mm

C/s area $= \frac{\pi \times 30^2}{4} = 706.85 \text{ mm}^2$

$P = 54$ kN

Lateral strain $(e_{lateral}) = \frac{\delta d}{d} = \frac{0.00366}{30} = 1.22 \times 10^{-4}$

Poisson's ratio $= \frac{1}{m} = \mu = \frac{e_{lateral}}{e_{longitudinal}}$

$= \frac{1.22 \times 10^{-4}}{3.73 \times 10^{-4}}$

$\boxed{\frac{1}{m} = 0.327}$

Now

$E = \frac{\sigma}{e}$

$\sigma = \frac{P}{A} = \frac{54 \times 10^3}{706.85} = 76.39 \text{ N/mm}^2$

$\boxed{E = \frac{76.39}{3.73 \times 10^{-4}} = 2.04 \times 10^5 \text{ N/mm}^2}$

Modulus of rigidity $= C = \frac{E}{2(1+\frac{1}{m})} = \frac{2.04 \times 10^5}{2(1+0.327)}$

$\boxed{C = 76.86 \times 10^3 \text{ N/mm}^2}$

Bulk modulus $= K = \frac{E}{3[1-2(\frac{1}{m})]}$

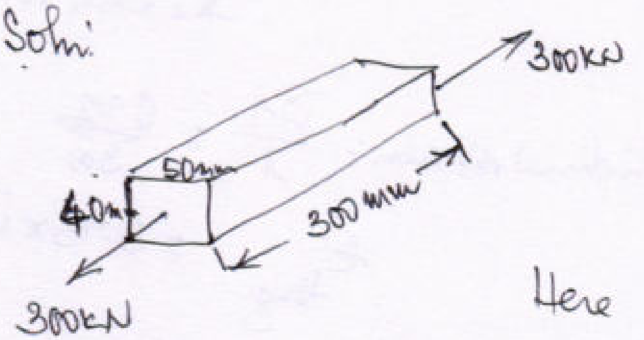
$= \frac{2.04 \times 10^5}{3[1-2(0.327)]}$

$\boxed{K = 1.96 \times 10^5 \text{ N/mm}^2}$

A steel bar 300 mm long, 50 mm wide and 40 mm thick is subjected to a pull of 300 kN in the direction of its length. Determine the change in volume. Take $E = 2\times10^5\ N/mm^2$ and $\frac{1}{m} = 0.25$

June-July 2019 (06 m)

Soln:



$$\delta V = e_v \times V$$

$$e_v = e_x + e_y + e_z$$

Here $e_x = \frac{\sigma_x}{E} - \frac{1}{m}\frac{\sigma_y}{E} - \frac{1}{m}\frac{\sigma_z}{E}$

$$\sigma_x = \frac{300\times10^3}{40\times50} = 150\ N/mm^2 \qquad \sigma_y = 0 \quad \sigma_z = 0$$

(Because no loads in y and z directions)

$$\boxed{e_x = \frac{150}{2\times10^5} - 0 - 0 = 75\times10^{-5}}$$

$$e_y = \frac{\sigma_y}{E} - \frac{1}{m}\frac{\sigma_z}{E} - \frac{1}{m}\frac{\sigma_x}{E}$$

$$\boxed{e_y = 0 - 0 - (0.25)\left(\frac{150}{2\times10^5}\right) = -1.875\times10^{-4}}$$

$$e_z = \frac{\sigma_z}{E} - \frac{1}{m}\frac{\sigma_x}{E} - \frac{1}{m}\frac{\sigma_y}{E}$$

$$e_z = 0 - (0.25)\left(\frac{150}{2\times10^5}\right) - 0 = -1.875\times10^{-4}$$

$$e_v = 75\times10^{-5} - 1.875\times10^{-4} - 1.875\times10^{-4}$$

$$\boxed{e_v = 3.75\times10^{-4}}$$

$$\delta V = 3.75\times10^{-4}\times(300\times40\times50)$$

$$\boxed{\delta V = 225\ mm^2}$$

* For a given material E = 100 kN/mm² and shear modulus is 40kN/mm². Find the bulk modulus and lateral contraction of round bar of 50mm dia and 2.5m long when stretched by 2.5mm. Take μ=0.25

Soln:

Jan 2016 06 marks

$E = 100 kN/mm^2 = 100\times10^3 N/mm^2$

$C = 40 kN/mm^2 = 40\times10^3 N/mm^2$

$$k = \frac{E}{3\left[1-2\left(\frac{1}{m}\right)\right]} = \frac{100\times10^3}{3\left[1-2(0.25)\right]} = 6.6\times10^4 N/mm^2$$

Now

$$\text{Longitudinal strain} = e_{longitudinal} = \frac{2.5}{2500} = 1\times10^{-3}$$

$\delta l = 2.5mm$

$l = 2.5m = 2500 mm$

$$\text{Poisson's ratio} = 0.25 = \frac{e_{longitudinal}}{e_{lateral}}$$

$$e_{lateral} = \frac{1\times10^{-3}}{0.25} = 4\times10^{-3}$$

$$e_{lateral} = \frac{\delta d}{d}$$

$$\delta d = 4\times10^{-3}\times50$$

Lateral Contraction or Change in dia $= \boxed{\delta d = 0.2 m}$ ✓

Module-2

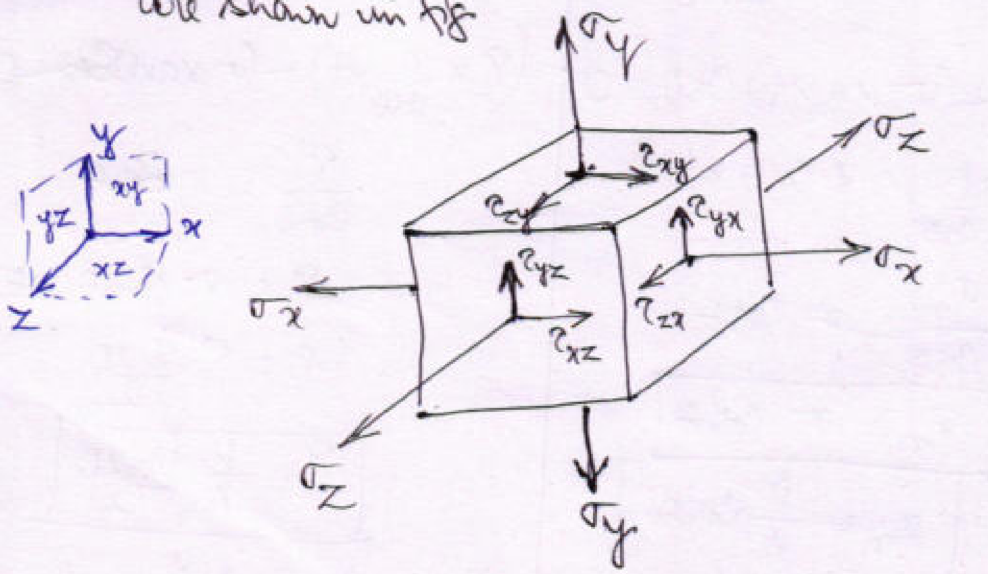
Compound Stresses: Introduction, State of stress at a point, General two dimensional stress system, Principal stresses and planes, Mohr's Circle of stresses. Theory of Failures: Maximum Shear stress theory and Maximum Principal stress theory

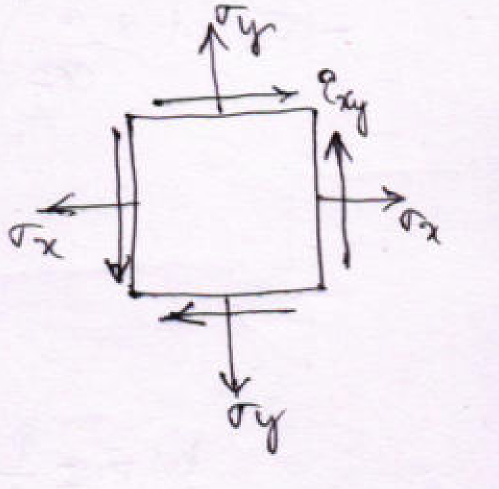
C. Nagaraj

In actual practice, a structural member is subjected to direct stresses in different directions. At any point in a strained material, a Combination of tensile, Compressive and shear stresses act on various planes of the material. Hence it is of practical importance to determine the resultant stresses which may be greater than applied stresses and also the planes on which they act when subjected to Complex stress system.

In general stress system, in the case of three dimensional stresses three normal stresses and three shear stresses act on a body. The systems in which direct stresses and shear stresses act on a body simultaneously are called Compound Stresses or Combined stresses.

In a 3D stress system, the various stresses acting on a body are shown in fig





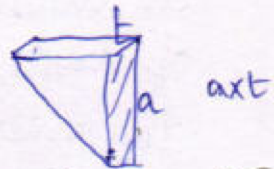
In many problems, 2D stress system idealisation is possible and study is limited to 2D stress system only in this chapter as shown above.

Stresses on an inclined plane

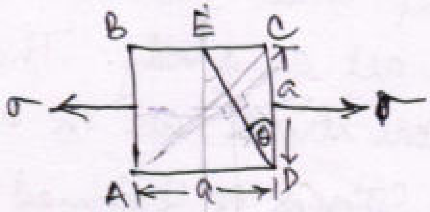
It is required to find the stresses on an inclined plane inclined at an angle θ to a known plane in a strained element. The normal (σ) and tangental stresses (τ) (shear stress) on this plane is to be determined. The following three types of stress conditions are considered:

i) Uniaxial direct stress
ii) Biaxial direct stresses
iii) General two dimensional stress system.

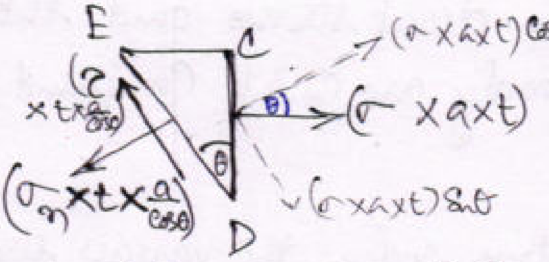
Element subjected to uniaxial direct stress:



Consider an element subjected to uniaxial direct stress as shown in fig. Let DE be the plane at an angle θ (measured anticlockwise) to the plane of stress σ. Consider an element ABCD of thickness 't' and sides 'a'



Consider the equilibrium of element CDE.



Let σ_n and τ be the normal and shear stresses on the plane DE

Sum of forces normal to DE = 0

$$-\left(\sigma_n \times t \times \frac{a}{\cos\theta}\right) + (\sigma \times a \times t)\cos\theta = 0$$

$$\sigma_n \frac{at}{\cos\theta} = \sigma \cos\theta\, at$$

$$\frac{\sigma_n}{\cos\theta} = \sigma \cos\theta$$

$$\boxed{\sigma_n = \sigma \cos^2\theta}$$

But $\sigma = \frac{P}{A}$ $\quad \boxed{\sigma_n = \frac{P}{A}\cos^2\theta}$

Where P = Load

Normal stress is maximum when $\theta = 0^\circ$

$$(\sigma_n)_{max} = \frac{P}{A} = \sigma$$

Sum of forces parallel to DE = 0

$$\left(\tau \times \frac{a}{\cos\theta} \times t\right) - (\sigma \times a \times t)\sin\theta = 0$$

$$\frac{\tau}{\cos\theta} = \sigma \sin\theta$$

$$\tau = \sigma \sin\theta \cos\theta$$

$$\tau = \sigma \frac{\sin 2\theta}{2}$$

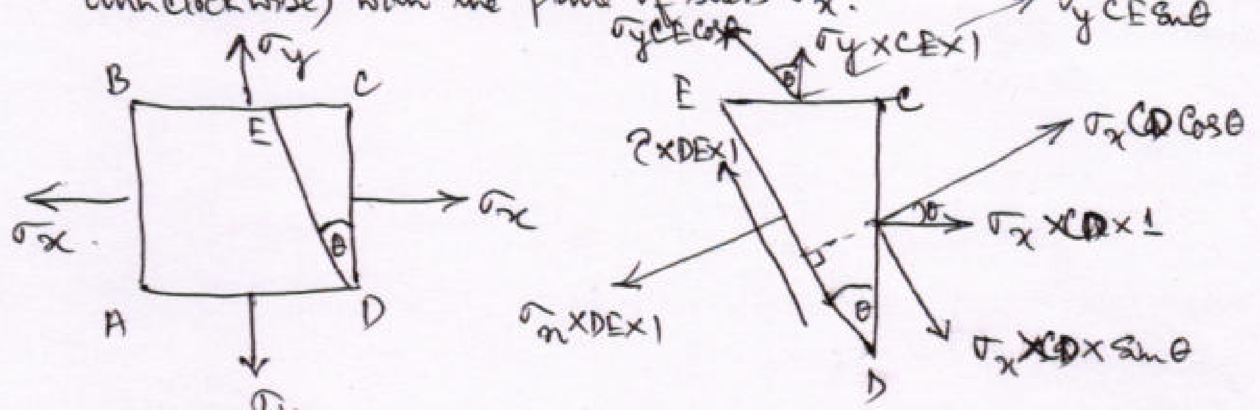
$$\boxed{\tau = \frac{P}{A}\frac{\sin 2\theta}{2}}$$

Shear stress will be maximum when $2\theta = 90^\circ$ i.e. $\theta = 45^\circ$

$$\boxed{\tau_{max} = \frac{P}{2A} = \frac{\sigma}{2}}$$

Elements subjected to Biaxial direct stresses

Consider the element of unit thickness subjected to direct ~~tensile~~ stresses σ_x and σ_y as shown in fig. Consider a plane DE at an angle θ (measured anticlockwise) with the plane of stress σ_x.



Consider the element DCE shown in fig. Consider the equilibrium of the element DCE as shown in fig.

Sum of forces normal to plane DE = 0.

$$-\sigma_n \times DE \times 1 + \sigma_x CD\cos\theta + \sigma_y CE\sin\theta = 0$$

$$\sigma_n DE = \sigma_x CD\cos\theta + \sigma_y CE\sin\theta$$

$$\sigma_n = \sigma_x \cdot \frac{CD}{DE}\cos\theta + \sigma_y \frac{CE}{DE}\sin\theta$$

But $\frac{CD}{DE} = \cos\theta$, $\frac{CE}{DE} = \sin\theta$

$$\sigma_n = \sigma_x\cos^2\theta + \sigma_y\sin^2\theta$$

$$\sigma_n = \sigma_x\left[\frac{1+\cos 2\theta}{2}\right] + \sigma_y\left[\frac{1-\cos 2\theta}{2}\right]$$

$$\boxed{\sigma_n = \left(\frac{\sigma_x+\sigma_y}{2}\right) + \cos 2\theta\left(\frac{\sigma_x-\sigma_y}{2}\right)}$$

Sum of forces parallel to DE = 0

$$\tau \times DE \times 1 + \sigma_y CE\cos\theta - \sigma_x CD\sin\theta = 0$$

$$\tau \times DE = \sigma_x CD\sin\theta - \sigma_y CE\cos\theta$$

$$\tau = \sigma_x \frac{CD}{DE}\sin\theta - \sigma_y \frac{CE}{DE}\cos\theta$$

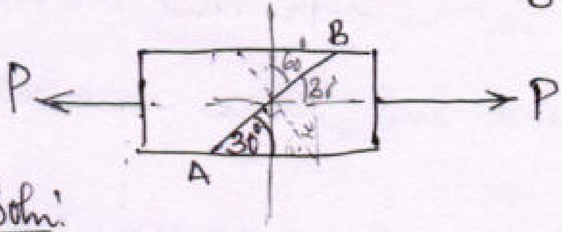
$$\tau = \sigma_x\cos\theta\sin\theta - \sigma_y\cos\theta\sin\theta$$

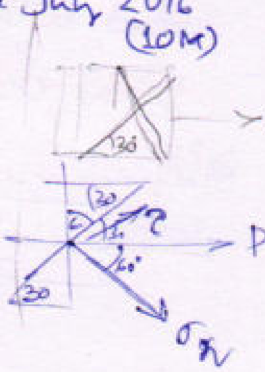
$$\tau = \cos\theta\sin\theta(\sigma_x - \sigma_y)$$

$$\tau = \left(\frac{\sigma_x-\sigma_y}{2}\right)\sin 2\theta$$

* Two wooden pieces 100 mm × 100 mm in cross section are glued together along line AB as shown in fig. What maximum axial force 'P' can be applied if the allowable shearing stress along AB is 1.2 N/mm²?

June-July 2013, 2016 (10M)





$$\tau = \frac{P}{A}\frac{\sin 2\theta}{2}$$

Soln:

Area = 100 × 100 = 10,000 mm²

Here $\theta = 60^\circ$ (w.r.t plane ⊥ to direction of load)

$$\sigma_n = \frac{P}{A}\cos^2\theta$$

$$\tau = \frac{P}{A}\frac{\sin 2\theta}{2}$$

Now

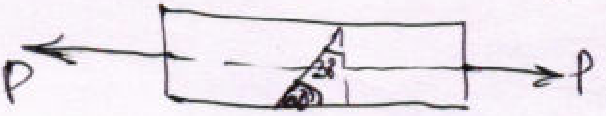
$$1.2 = \frac{P}{(100\times100)}\frac{\sin(60\times2)}{2}$$

$$P = \frac{1.2\times(100\times100)\times2}{\sin 120^\circ}$$

$$\boxed{P = 27712.81\ N}$$

* A rectangular bar of cross sectional area of 11,000 mm² is subjected to a tensile load 'P' as shown in fig. The permissible normal and shear stresses on the oblique plane BC are given as 7 N/mm² and 3.0 N/mm² respectively. Determine the safe value of 'P'.

June-July 2019 (6M)



$$\sigma_n = \frac{P}{A}\cos^2\theta = 7$$

$$P = \left(\frac{7\times11000}{\cos^2 30^\circ}\right)$$

$$\boxed{P = 1.02\times10^5\ N}$$

$$\tau = \frac{P}{A}\frac{\sin 2\theta}{2}$$

$$3.0 = \frac{P}{A}\frac{\sin 2\theta}{2}$$

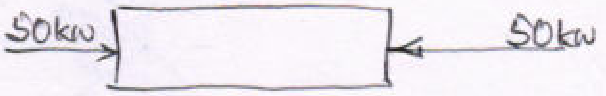
$$P = \frac{3\times11000\times2}{\sin(2\times30)}$$

$$\boxed{P = 76.21\times10^3\ N}$$

Safe value of 'P' is the least of two values i.e., $\boxed{P = 76.21\times10^3\ N}$

A Short bar of rectangular section 25mm x 50mm is subjected to a Compressive force of 50 kN. Find the stresses on planes whose normal makes angles of (i) + 40° (ii) - 30° with longitudinal axis. What is the maximum shear stress in the bar and on which plane does it act? Show the planes and stresses in a neat sketch.

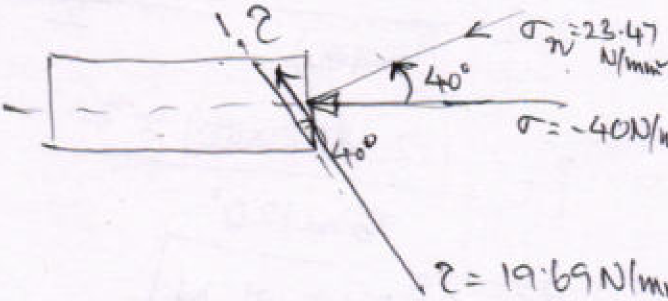
Soln:



Area of bar = 25 × 50 = 1250 mm²

$$\sigma = \frac{-50\times10^3}{1250} = -40 N/mm^2 \quad \text{(Compressive)}$$

I Case: when normal makes 40° with longitudinal axis



$\sigma_n = \sigma \cos^2\theta$

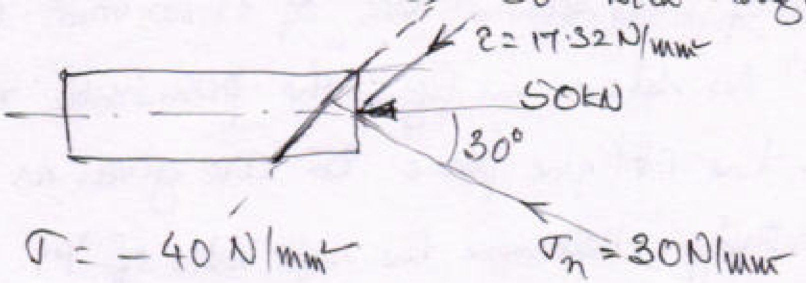
$\sigma_n = -40 \cos^2 40°$

$$\boxed{\sigma_n = -23.47 N/mm^2}$$

$$\tau = \sigma \frac{\sin 2\theta}{2} = \frac{-40(\sin 80°)}{2}$$

$$\boxed{\tau = -19.69 N/mm^2}$$

II When normal makes -30° with longitudinal axis



$\sigma_n = \sigma \cos^2\theta$

$$\boxed{\sigma_n = -40 \cos^2(-30) = -30 N/mm^2}$$

$$\tau = \sigma \frac{\sin 2\theta}{2} = -40 \frac{(\sin(-60))}{2}$$

$$\boxed{\tau = 17.32 N/mm^2}$$

III Maximum Shear stress will be on a plane making 45° (or 135°) with the plane ⊥ʳ to 'σ'

$$\tau_{max} = \frac{\sigma}{2} = -\frac{40}{2} = -20 N/mm^2$$

Accompanying $\sigma_n = \sigma \cos^2 45° = -40 \cos^2 45 = -20 N/mm^2$.

Resultant due to σ_n and τ

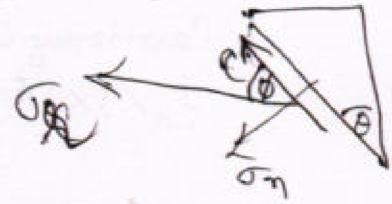
$$\sigma_r = \sqrt{\sigma_n^2 + \tau^2}$$

$$= \sqrt{(\sigma_x \cos^2\theta + \sigma_y \sin^2\theta)^2 + [(\sigma_x - \sigma_y)\cos\theta\sin\theta]^2}$$

$$\sigma_r = \sqrt{(\sigma_x^2 \cos^4\theta + \sigma_y^2 \sin^4\theta + \cancel{2\sigma_x\sigma_y \cos^2\theta\sin^2\theta} + \sigma_x^2 \cos^2\theta\sin^2\theta + \sigma_y^2 \cos^2\theta\sin^2\theta - \cancel{2\sigma_x\sigma_y \cos^2\theta\sin^2\theta})}$$

$$\sigma_r = \sqrt{(\sigma_x^2 \cos^2\theta(\cos^2\theta + \sin^2\theta) + \sigma_y^2 \sin^2\theta(\cos^2\theta + \sin^2\theta)}$$

$$\boxed{\sigma_r = \sqrt{\sigma_x^2 \cos^2\theta + \sigma_y^2 \sin^2\theta}}$$



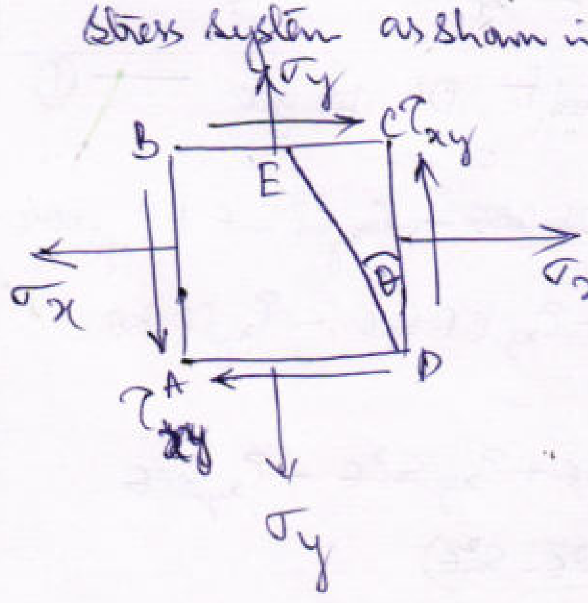
If the angle between σ_r and the given plane is ϕ,

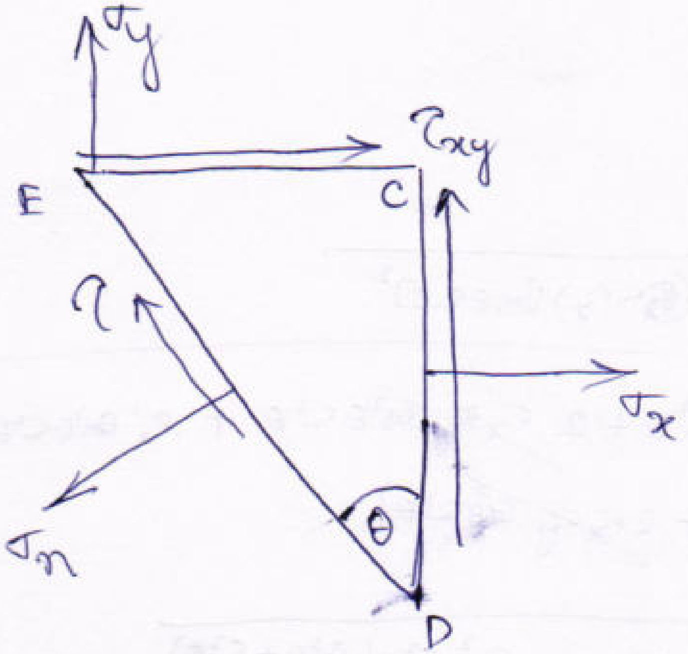
then $\tan\phi = \dfrac{\sigma_n}{\tau}$

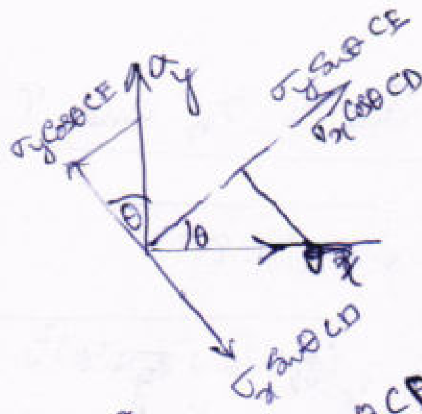
Hence the resultant makes angle $\phi + \theta$ with the plane of σ_x

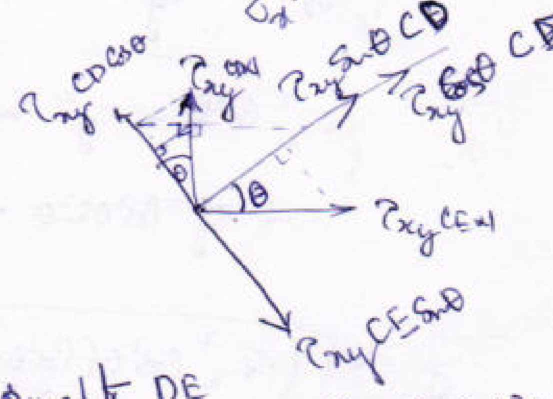
Elements subjected to General two dimensional Stress System

Consider an element of unit thickness subjected to general two dimensional stress system as shown in the fig. The sides are subjected to direct stresses σ_x and σ_y and shear stress τ_{xy} as shown in the fig. An arbitrary plane DE is choosen at angle θ and it is required to find normal and shear stresses on the DE









Considering equilibrium of forces normal to DE

$$-\sigma_n \times 1 \times DE + \sigma_x \times 1 \times CD\cos\theta + \sigma_y \times 1 \times CE\sin\theta + \tau_{xy}\cos\theta\, CE + \tau \cos\theta = 0$$

$$\sigma_n DE = \sigma_x CD\cos\theta + \sigma_y CE\sin\theta + \tau_{xy} CE\cos\theta + \tau_{xy} CD\sin\theta$$

$\div$ by DE

$$\sigma_n \frac{DE}{DE} = \sigma_x \frac{CD}{DE}\cos\theta + \sigma_y \frac{CE}{DE}\sin\theta + \tau_{xy}\frac{CE}{DE}\cos\theta + \tau_{xy}\frac{CD}{DE}\sin\theta$$

$$\sigma_n = \sigma_x\cos^2\theta + \sigma_y\sin^2\theta + \tau_{xy}\cos\theta\sin\theta + \tau_{xy}\cos\theta\sin\theta$$

$$= \sigma_x\left[\frac{1+\cos 2\theta}{2}\right] + \sigma_y\left[\frac{1-\cos 2\theta}{2}\right] + 2\tau_{xy}\cos\theta\sin\theta$$

$$\boxed{\sigma_n = \left[\frac{\sigma_x+\sigma_y}{2}\right] + \left[\frac{\sigma_x-\sigma_y}{2}\right]\cos 2\theta + \tau_{xy}\sin 2\theta} \quad \text{—— (1)}$$

Considering equilibrium of forces parallel to DE, we get

$$+\tau \times 1 \times DE = \sigma_x CD\sin\theta + \sigma_y CE\cos\theta - \tau_{xy} CE\sin\theta + \tau_{xy} CD\cos\theta = 0$$

$$\tau DE = \sigma_x CD\sin\theta - \sigma_y CE\cos\theta + \tau_{xy} CE\sin\theta - \tau_{xy} CD\cos\theta$$

$\div$ by DE

$$\tau = \sigma_x\cos\theta\sin\theta - \sigma_y\cos\theta\sin\theta + \tau_{xy}\sin^2\theta - \tau_{xy}\cos^2\theta$$

$$\tau = (\sigma_x-\sigma_y)\cos\theta\sin\theta - \tau_{xy}(\cos^2\theta - \sin^2\theta)$$

$$\boxed{\tau = (\sigma_x-\sigma_y)\frac{\sin 2\theta}{2} - \tau_{xy}\cos 2\theta} \quad \text{—— (2)}$$

To determine the planes having maximum and minimum values of direct stress, the eqn (1) is to be differentiated w.r.t θ and equating to zero,

$$\frac{d\sigma_n}{d\theta} = 0 + (-)\left[\frac{\sigma_x - \sigma_y}{2}\right] 2\sin 2\theta + \tau_{xy}\, 2\cos 2\theta = 0$$

$$(\sigma_x - \sigma_y)\sin 2\theta = 2\tau_{xy}\cos 2\theta$$

$$\frac{\sin 2\theta}{\cos 2\theta} = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} \quad \text{ie} \quad \tan 2\theta = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

This equation provides for two values of 2θ differing by 180° or θ by 90°, the planes along which direct stresses have maximum and minimum values

Note that the same values of θ are also obtained by equating ? to zero, which indicates that shear stress is zero on these planes. Thus it is concluded that shear stresses are zero on the planes of minimum and maximum values of direct stress. They are known as principal planes. The normal stresses on those planes are principal stresses.

Principal Stresses:

When a body is subjected to direct and shear stress system, there exist ~~three~~ two mutually perpendicular planes on which the resultant is normal (ie. shear stress is zero). These planes are known as principal planes and the normal stresses are known as principal stresses. Larger of these two stresses is called major principal stress and smaller one is called minor principal stress. The corresponding planes are known as major and minor principal planes.

Here $\tau = 0$ on Principal plane.

Then

$$\frac{(\sigma_x - \sigma_y)}{2}\sin 2\theta - \tau_{xy}\cos 2\theta = 0$$

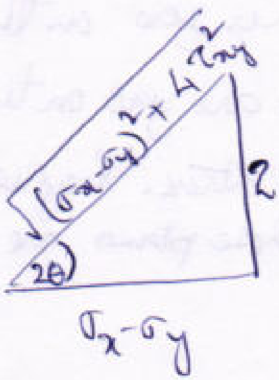
$$\tau_{xy}\cos 2\theta = \left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta$$

$$\cancel{2\tau_{xy}\frac{\cos 2\theta}{\sin 2\theta} = (\sigma_x - \sigma_y)}$$

$$2\tau_{xy} = \frac{\sin 2\theta}{\cos 2\theta}(\sigma_x - \sigma_y)$$

$$\boxed{\tan 2\theta = \frac{2\tau_{xy}}{(\sigma_x - \sigma_y)}}$$

ie



There are two values of 2θ differing by $180°$. Let $2\theta_1$ and $2\theta_2$ be the Solutions. Then

$$\sin 2\theta_1 = \frac{2\tau_{xy}}{\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}}$$

$$\cos 2\theta_1 = \frac{\sigma_x - \sigma_y}{\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}}$$

$$\sin 2\theta_2 = \frac{-2\tau_{xy}}{\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}}$$

$$\cos 2\theta_2 = \frac{-(\sigma_x - \sigma_y)}{\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}}$$

If $2\theta_1$ and $2\theta_2$ differ by 180° then, θ_1 and θ_2 differ by 90° giving the direction of two principal planes.

To get the magnitudes of principal stresses, σ_1 and σ_2, the values of $2\theta_1$ and $2\theta_2$ are to be substituted in the expression for σ_n ie Eqn ①

Then

$$\sigma_1 = \left(\frac{\sigma_x+\sigma_y}{2}\right) + \left(\frac{\sigma_x-\sigma_y}{2}\right)\frac{(\sigma_x-\sigma_y)}{\sqrt{(\sigma_x-\sigma_y)^2+4\tau_{xy}^2}} + \tau_{xy}\frac{2\tau_{xy}}{\sqrt{(\sigma_x-\sigma_y)^2+4\tau_{xy}^2}}$$

$$\sigma_1 = \frac{\sigma_x+\sigma_y}{2} + \frac{1}{2}\frac{(\sigma_x-\sigma_y)^2+4\tau_{xy}^2}{\sqrt{(\sigma_x-\sigma_y)^2+4\tau_{xy}^2}}$$

$$\sigma_1 = \frac{\sigma_x+\sigma_y}{2} + \frac{1}{2}\sqrt{(\sigma_x-\sigma_y)^2+4\tau_{xy}^2}$$

$$\sigma_1 = \frac{\sigma_x+\sigma_y}{2} + \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2}$$

$|||^{ly}$ substituting for $\sin 2\theta_2$ and $\cos 2\theta_2$,

$$\sigma_2 = \left(\frac{\sigma_x+\sigma_y}{2}\right) - \left(\frac{\sigma_x-\sigma_y}{2}\right)\frac{(\sigma_x-\sigma_y)}{\sqrt{(\sigma_x-\sigma_y)^2+4\tau_{xy}^2}} - \tau_{xy}\cdot\frac{2\tau_{xy}}{\sqrt{(\sigma_x-\sigma_y)^2+4\tau_{xy}^2}}$$

Simplifying

$$\sigma_2 = \left(\frac{\sigma_x+\sigma_y}{2}\right) - \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2}$$

Planes of maximum shear stress:

To get the planes of maximum shear, differentiate the shear stress Eqn ② w.r.t to θ and equate it to zero, i.e.,

$\theta = \theta'$ $$\frac{d\tau}{d\theta} = 0$$

$$\frac{d}{d\theta}\left[(\sigma_x-\sigma_y)\frac{\sin 2\theta'}{2} - \tau_{xy}(\cos 2\theta')\right] = 0$$

$$(\sigma_x-\sigma_y)\,2\cos 2\theta' - (-2\sin 2\theta')\,\tau_{xy} = 0$$

ie ~~tan2θ~~ $2\cos 2\theta \frac{(\sigma_x-\sigma_y)}{2} = -2\sin 2\theta\, \tau_{xy}$

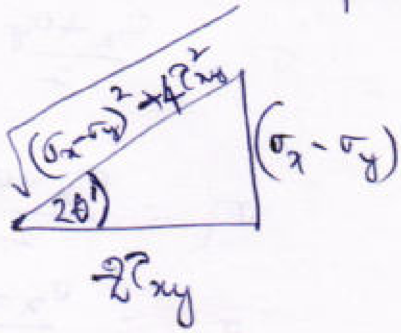
$$\tan 2\theta' = \frac{-(\sigma_x-\sigma_y)}{2\tau_{xy}}$$

Here $\tan 2\theta \times \tan 2\theta' = -1$

ie the values of 2θ and $2\theta'$ differ by $90°$. In other words, the planes of maximum shear stress are at 45^0 to principal planes.

Here

$$\sin 2\theta' = \frac{\sigma_x-\sigma_y}{\sqrt{(\sigma_x-\sigma_y)^2+4\tau_{xy}^2}}$$



$$\cos 2\theta' = \frac{-\tau_{xy}}{\sqrt{(\sigma_x-\sigma_y)^2+4\tau_{xy}^2}}$$

Substituting these in eqn (2),

$$\tau_{max} = \frac{(\sigma_x-\sigma_y)}{2}\frac{(\sigma_x-\sigma_y)}{\sqrt{(\sigma_x-\sigma_y)^2+4\tau_{xy}^2}} - \tau_{xy}\frac{(-\tau_{xy})}{\sqrt{(\sigma_x-\sigma_y)^2+4\tau_{xy}^2}}$$

$$\tau_{max} = \frac{1}{2}\frac{(\sigma_x-\sigma_y)^2+4\tau_{xy}^2}{\sqrt{(\sigma_x-\sigma_y)^2+4\tau_{xy}^2}}$$

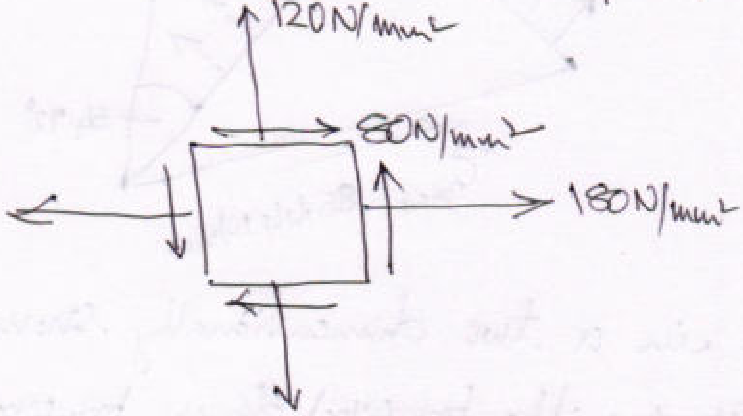
$$\tau_{max} = \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

ie $\tau_{max} = \frac{1}{2}$ the difference of σ_1 and σ_2 $= \frac{\sigma_1-\sigma_2}{2}$

1. The state of stress at a point in a strained material is as shown in the fig. Determine i) the direction of principal planes

ii) the magnitude of principal stresses

iii) the magnitude of maximum shear stress and its direction. Indicate all the above planes by a sketch.



Soln: $\sigma_x = 180 N/mm^2$ $\sigma_y = 120 N/mm^2$ $\tau_{xy} = 80 N/mm^2$

Then
$$\tan 2\theta = \frac{2\tau_{xy}}{(\sigma_x - \sigma_y)}$$

$$\tan 2\theta = \frac{2\times 80}{180-120} = \frac{160}{60} = 2.67$$

$$2\theta = 69.44$$

$$\theta = 34.72^\circ \text{ and } 34.72 + 90 = 124.72^\circ$$

Principal planes exist at 34.72° and 124.72°

Principal stresses are
$$\sigma_1 \text{ or } \sigma_2 = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{180+120}{2} \pm \sqrt{\left(\frac{180-120}{2}\right)^2 + 80^2}$$

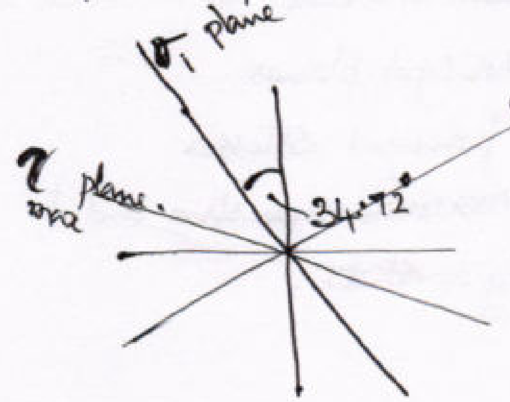
$$\sigma_1 \text{ and } \sigma_2 = 150 \pm \sqrt{7300}$$

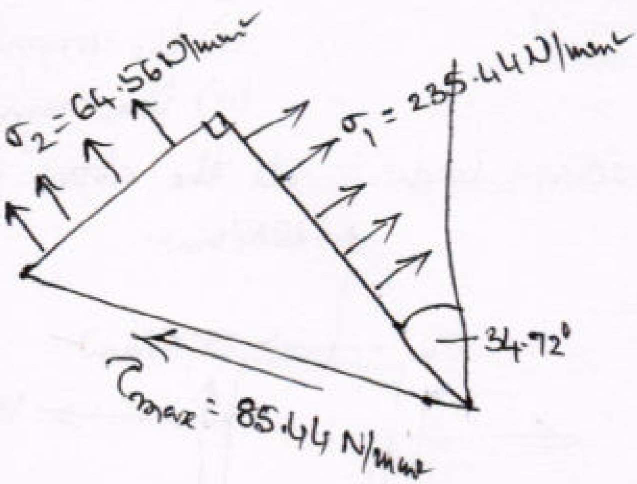
$$= 150 \pm 85.44$$

σ_1 and σ_2 = 235.44 N/mm² (Tensile) and 64.56 N/mm² (Tensile)

$$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{235.44 - 64.56}{2} = 85.44 N/mm^2$$

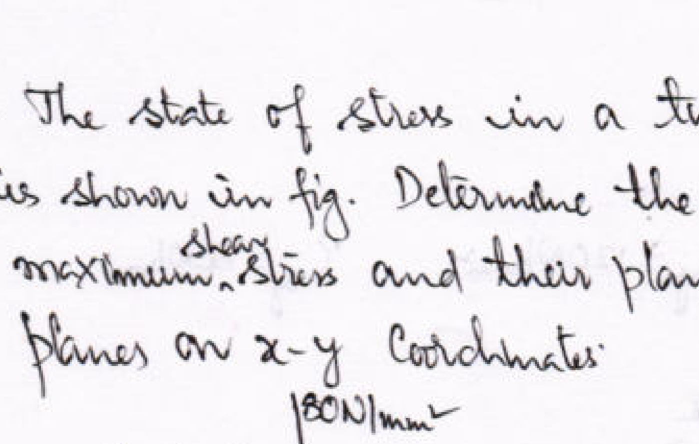
planes of maximum shearing stresses are at 45° to the principal planes.





2. The state of stress in a two dimensionally stressed body is shown in fig. Determine the principal planes, principal stress, maximum shear stress and their planes. Schematically represent these planes on x-y coordinates.

July 2008
June 2010



Soln.

$\sigma_x = -120\,N/mm^2 \quad \sigma_y = -80\,N/mm^2 \quad \tau_{xy} = -60\,N/mm^2$

$$\tan 2\theta = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{2(-60)}{-120-(-80)} = \frac{-120}{-40} = 3$$

$2\theta = 71.56^\circ$

$\theta = 35.78$ and 125.78°

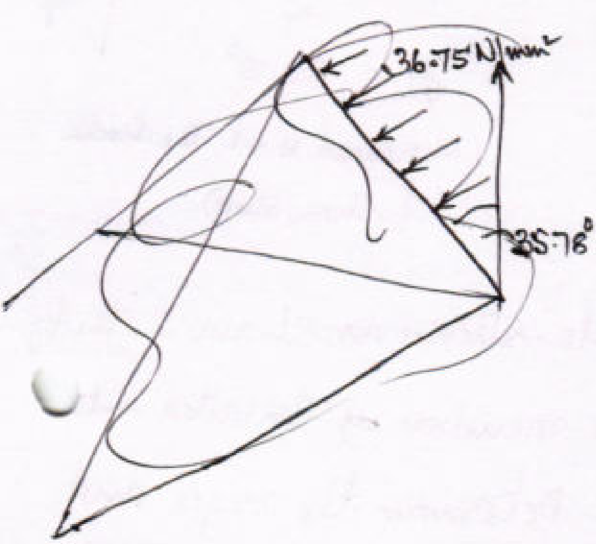
$$\sigma_1 \text{ and } \sigma_2 = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

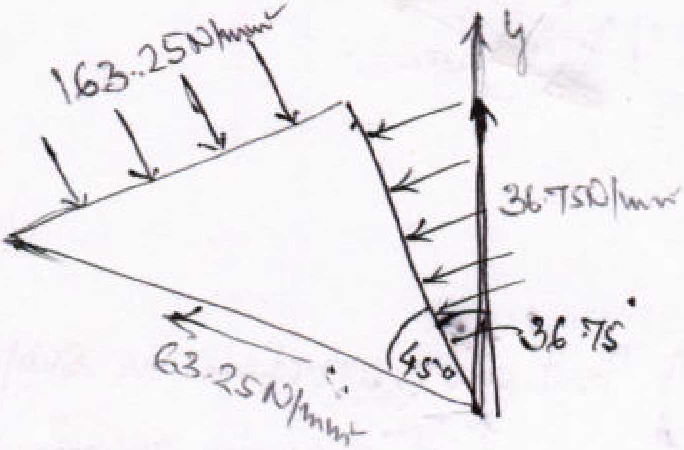
$$= \frac{-120-80}{2} \pm \sqrt{\left(\frac{-120+80}{2}\right)^2 + (-60)^2}$$

σ_1 and σ_2 $= -100 \pm 63.24 = -163.24\,N/mm^2$ (σ_2) and $-36.75\,N/mm^2$ (σ_1)

$$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{-36.75 - (-163.75)}{2}$$

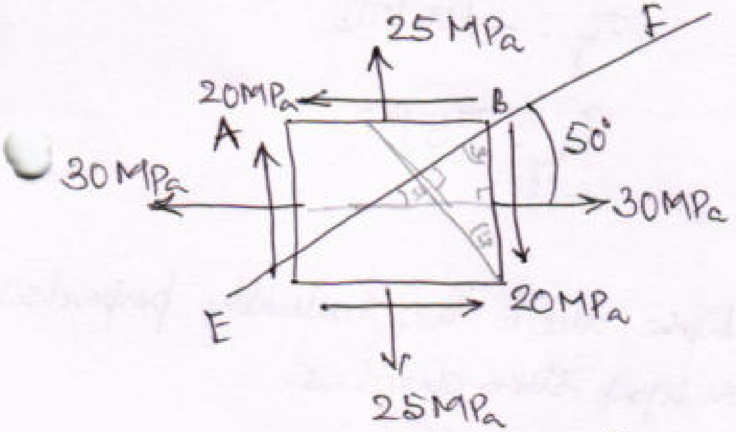
$$\tau_{max} = +63.125\ N/mm^2$$





3. The state of stress at a point is shown in fig. If the plane EF cuts the element, determine the normal and shear stresses on the plane and show them clearly on the portion of the element ABFE

December 2010

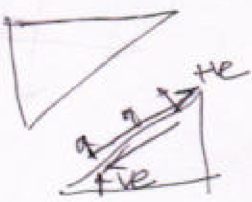


$\sigma_x = 30\ MPa$

$\sigma_y = 25\ MPa$

$\tau_{xy} = -20\ MPa$

$\theta = 50°$



Normal stress $= \sigma_n = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta + \tau_{xy}\sin 2\theta$

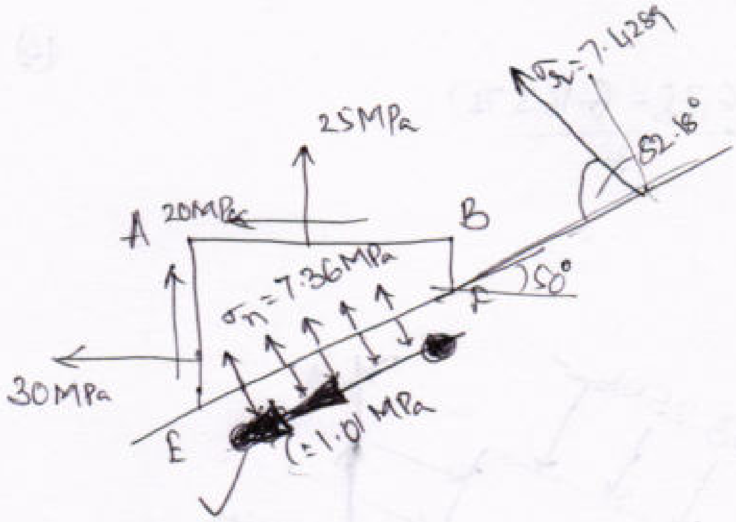
$$\sigma_n = \left(\frac{30+25}{2}\right) + \left(\frac{30-25}{2}\right)\cos 100° + (-20)\sin 100$$

$$\boxed{\sigma_n = 7.3697\ MPa}$$ Tensile

Shear stress $\tau = (\sigma_x - \sigma_y)\frac{\sin 2\theta}{2} - \tau_{xy}\cos 2\theta$

$$= \frac{(30-25)}{2}\sin 100 - (-20)\cos 100$$

$$\boxed{\tau = -1.01\ MPa}$$



σ_R = Resultant stress = $\sqrt{\sigma_n^2+\tau_{xy}^2}$

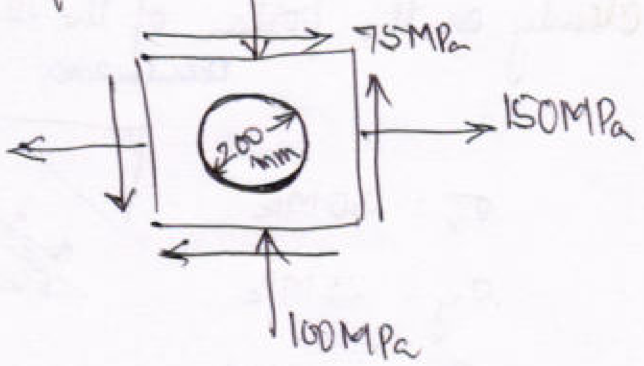
$$\boxed{\sigma_R = 7.4289 \text{ MPa}}$$

$$\tan\phi = \frac{\sigma_n}{\tau_{xy}}$$

$$\phi = 82.18^\circ$$

(Measured w.r.t the direction of shear stress)

4. A Point in a machine is subjected to stresses as shown in the fig. A circle of of diameter 200 mm on the member is converted into ellipse after the application of stresses. Determine the major and minor axes of the ellipse and their orientations. $E = 2\times10^5$ MPa and poisson's ratio $\mu = 0.3$



$\sigma_x = 150$ MPa

$\sigma_y = -100$ MPa

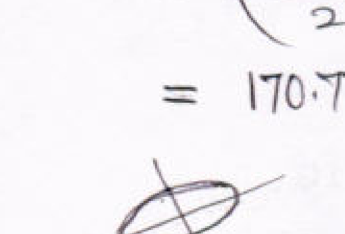
$\tau_{xy} = 75$ MPa

The circle is converted to ellipse due to two mutually perpendicular principal strains acting along principal stress directions.

$$\sigma_1 \text{ and } \sigma_2 = \left(\frac{\sigma_x+\sigma_y}{2}\right) \pm \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2}$$

$$= \left(\frac{150+(-100)}{2}\right) \pm \sqrt{\left(\frac{150+100}{2}\right)^2+75^2}$$

$$= 170.77 \text{ N/mm}^2 \quad \text{and} -120.77 \text{ N/mm}^2$$



Principal strains along direction 1 and 2

$$\epsilon_1 = \frac{\sigma_1}{E} - \frac{\nu}{E}\sigma_2 = \frac{170.77}{2\times10^5} - \frac{0.3(-120.77)}{2\times10^5} = 1.03\times10^{-3}$$

$$\epsilon_2 = \frac{\sigma_2}{E} - \frac{\nu\sigma_1}{E} = \frac{-120.77}{2\times10^5} - \frac{0.3(170.77)}{2\times10^5} = -8.6\times10^{-4}$$

Major axis = dia + change in dia

$= 200 + 1.03\times10^{-3}\times200 = 200.206\,mm$

Minor axis = dia + change in dia

$= 200 - 8.6\times10^{-4}\times200 = 199.828\,mm$

Orientation

$$\tan 2\theta = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{2\times75}{150-(-100)} = 0.6$$

$$2\theta = 30.96^\circ$$

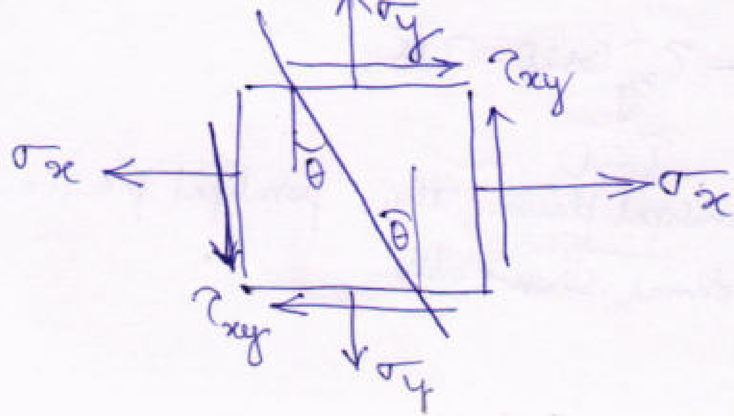
$$\theta_1 = 15.48^\circ \quad \text{and } \theta_2 = 105.48^\circ$$

Derive expressions for normal stress and shear stress on a plane inclined at θ to the vertical axis in a biaxial stress system with shear stress as shown in fig. Also S.T

i) Sum of Normal stresses on any two mutually perpendicular planes are always constant.

ii) Principal planes are planes of maximum normal stresses also.

Soln Derivation of σ_n and τ for the system shown is in page 5 and 6



$$\sigma_n = \left[\frac{\sigma_x+\sigma_y}{2}\right] + \left[\frac{\sigma_x-\sigma_y}{2}\right]\cos 2\theta + \tau_{xy}\sin 2\theta \qquad (1)$$

$$\tau = (\sigma_x - \sigma_y)\frac{\sin 2\theta}{2} - \tau_{xy}\cos 2\theta \qquad (2)$$

Let σ_n' be the stress on a plane at $\theta+90°$. Then replacing θ by $(\theta+90°)$ in Equation ①, we get,

$$\sigma_n' = \frac{\sigma_x+\sigma_y}{2} + \frac{\sigma_x-\sigma_y}{2}\cos(2\theta+180) + \tau_{xy}\sin(2\theta+180)$$

$$= \left(\frac{\sigma_x+\sigma_y}{2}\right) - \left(\frac{\sigma_x-\sigma_y}{2}\right)\cos 2\theta - \tau_{xy}\sin 2\theta$$

Then,
Sum of normal stresses is $\sigma_n + \sigma_n'$, ie.

$$\left(\frac{\sigma_x+\sigma_y}{2}\right) + \cancel{\left(\frac{\sigma_x-\sigma_y}{2}\right)\cos 2\theta} + \cancel{\tau_{xy}\sin 2\theta}$$

$$+$$

$$\left(\frac{\sigma_x+\sigma_y}{2}\right) - \cancel{\left(\frac{\sigma_x-\sigma_y}{2}\right)\cos 2\theta} - \cancel{\tau_{xy}\sin 2\theta}$$

$$\boxed{\sigma_n + \sigma_n' = \sigma_x + \sigma_y}$$

Thus sum of normal stresses on any two mutually perpendicular planes is constant and is equal to $(\sigma_x+\sigma_y)$

To find maximum value of σ_n

$$\frac{d\sigma_n}{d\theta} = 0$$

ie.
$$\left(\frac{\sigma_x-\sigma_y}{2}\right) 2(\sin 2\theta) + \tau_{xy}(2\cos 2\theta) = 0$$

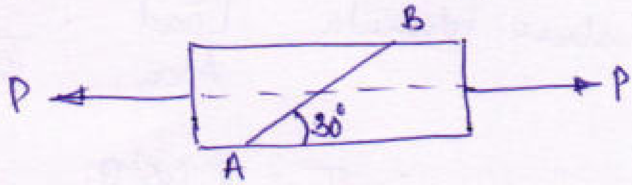
$$\frac{\sigma_x-\sigma_y}{2}\sin 2\theta - \tau_{xy}\cos 2\theta = 0 \text{ ie.}$$

$$\tau = 0$$

Which is the condition for principal planes. Thus principal planes are the planes of maximum normal stresses also.

Two wooden pieces 100 mm x 100 mm in cross section are glued together along line AB as shown in fig. What maximum axial force 'P' can be applied if the allowable shearing stress along AB is 1.2 N/mm²? 10 m

July 2013



$A = 1 \times 10^4 \ mm^2$

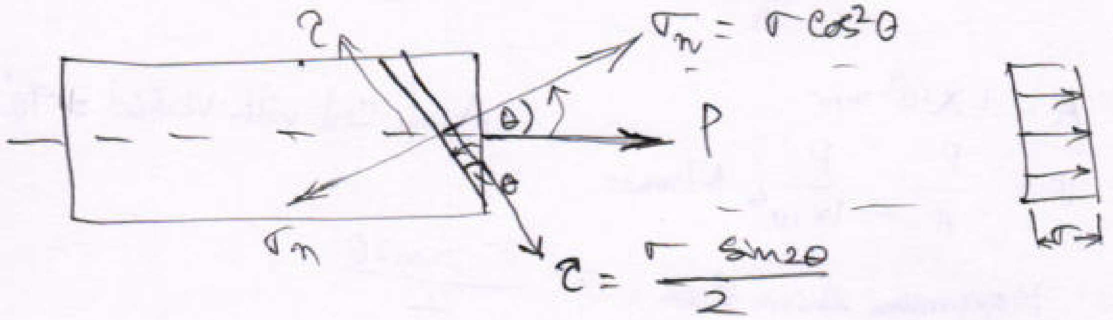
Angle cut with vertical $= 90° - 30° = 60°$

$\sigma = \frac{P}{A} = \frac{P}{1 \times 10^4} \ N/mm^2$

Maximum shear stress : $\tau = \frac{\sigma \sin 2\theta}{2}$

Summary

1. When a member is subjected to uniaxial tensile or compressive force, the stress on a plane normal to the axis of the member is obtained by average stress formula, $\frac{\text{Load}}{\text{Area}} = \frac{P}{(b \times d)}$ or $\frac{P}{(a \times a)} = \sigma$



If $\theta = 45^\circ$, then $\sigma_n = \frac{\sigma}{2}$

$\tau = \tau_{max} = \frac{\sigma}{2}$

2. Any oblique plane is identified by the angle that the normal to the plane makes with the longitudinal axis. Positive angles are measured in anticlockwise direction. (as shown in fig above)

3. The stress on the oblique plane is a normal stress (σ_n) and tangential stress is shear stress (τ). $\sigma_n = \sigma \cos^2\theta$

$\tau = \frac{\sigma \sin 2\theta}{2}$

As the value of the angle changes, different planes and different values of stress are obtained.

when $\theta = 0$, $\sigma_{max} = \sigma$ and $\tau = 0$

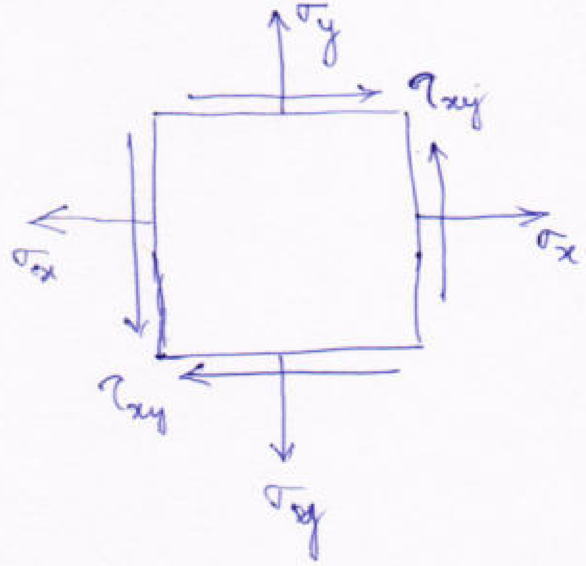
$\theta = 45^\circ$ $\sigma_n = \frac{\sigma}{2}$ $\tau_{max} = \frac{\sigma}{2}$

i.e, Max shear stress plane (of angle 45°) have a normal stress $\sigma_n = \frac{\sigma}{2}$.

4. Tensile stress is taken as positive.

Mohr's circle of stress:

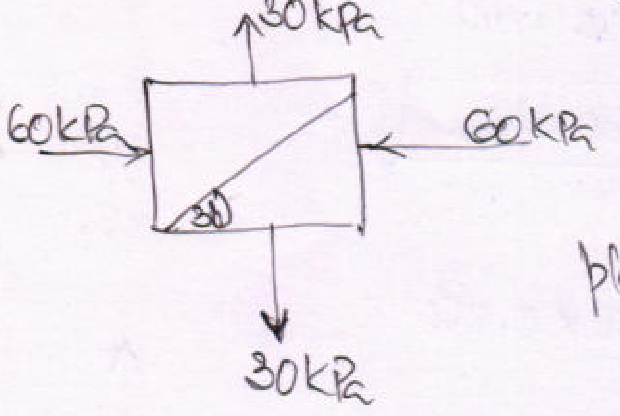
Consider a rectangular ABCD subjected to tensile stress and shear stress as shown in fig.



Sign Conventions:

1. Tensile stresses are considered positive

* The stresses acting on a strained material is as shown in fig. Find the normal and tangential stress acting on a plane AB.



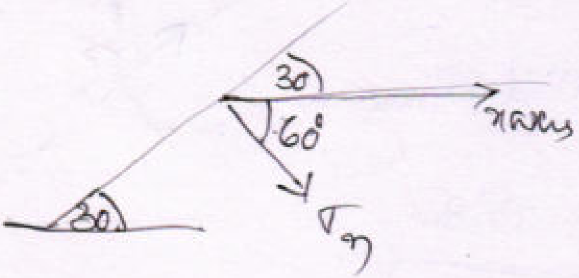
Soln:

The angle that the normal to the plane makes with x axis is

$\theta = -60^\circ$

$\sigma_x = -60\ kPa$

$\sigma_y = 30\ kPa$



$$\sigma_n = \left(\frac{\sigma_x+\sigma_y}{2}\right) + \left(\frac{\sigma_x-\sigma_y}{2}\right)\cos 2\theta$$

$$\sigma_n = \left(\frac{-60+30}{2}\right) + \left(\frac{-60-30}{2}\right)\cos(2\times-60)$$

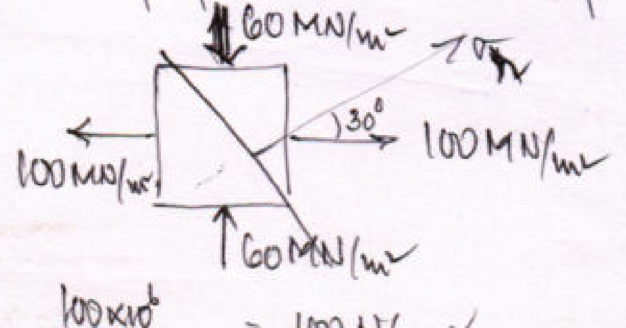
$$\sigma_n = -15 + (-45)(\cos(-120))$$

$$\boxed{\sigma_n = -15 + 22.5 = 7.5\ kPa}$$

$$\tau = \left(\frac{\sigma_x-\sigma_y}{2}\right)\sin 2\theta = \left(\frac{-60-30}{2}\right)\sin(2\times-60)$$

$$\boxed{\tau = (-45)(-0.866) = 38.97\ kPa}$$

* The direct stresses at a point in a piece of steel are 100 MN/m² tensile, 60 MN/m² compressive and zero. Find the intensity and direction of the stress across a plane, a normal of which is inclined at 30° to the axis of the 100 MN/m² direct stress, this plane being perpendicular to the plane of zero stress.



Soln

$\sigma_x = +100\ N/mm^2$

$\sigma_y = -60\ N/mm^2$

$\theta = 30^\circ$

$$\frac{100\times10^6}{1000\times1000} = 100\ N/mm^2$$

$$\sigma_n = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \left(\frac{\sigma_x - \sigma_y}{2}\right) \cos 2\theta$$

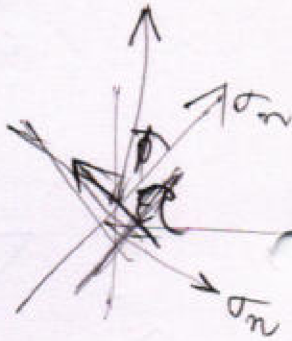
$$= \left(\frac{100 - 60}{2}\right) + \left(\frac{100 - (-60)}{2}\right) \cos(2 \times 30^\circ)$$

$$\boxed{\sigma_n = 20 + 80 \times 0.5 = 60 \, N/mm^2}$$

$$\tau = \left(\frac{\sigma_x - \sigma_y}{2}\right) \sin 2\theta$$

$$= \left(\frac{100 - (-60)}{2}\right) \sin 60 = 80 \times 0.866$$

$$\boxed{\tau = 69.28 \, N/mm^2}$$

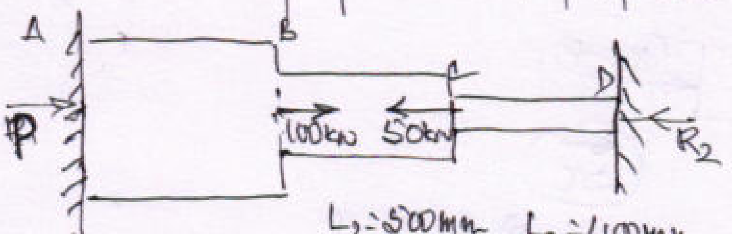


$$\sigma_R = \sqrt{\sigma_n^2 + \tau^2} = 91.66 \, N/mm^2$$

$$\tan\phi = \frac{\sigma_n}{\tau} = \frac{60}{69.28} = 0.866$$

$$\boxed{\theta = 40^\circ 53^\circ}$$

Two forces 50 kN and 100 kN are applied to a bar fixed between two unyielding supports. Compute the stresses induced in different materials. The material properties and properties of bar are indicated in fig.



$L_1 = 600$ mm, $A_1 = 1000\,mm^2$, $E_1 = 70$ GPa

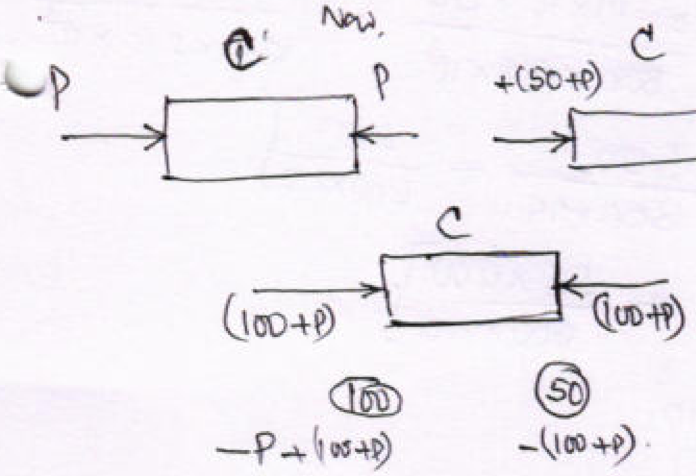
$L_2 = 500$ mm, $A_2 = 800\,mm^2$, $E_2 = 95$ GPa

$L_3 = 400$ mm, $A_3 = 600\,mm^2$, $E_3 = 210$ GPa

Soln: Let the reaction at A be P

$$+P + 100 - 50 - R_2 = 0$$
$$R_2 = (50+P)$$

Now,



$$0 = -\delta l_1 - \delta l_2 - \delta l_3$$

$$0 = -\left[\frac{P \times 600 \times 10^3}{1000 \times 70 \times 10^3} + \frac{(100+P) \times 500 \times 10^3}{800 \times 95 \times 10^3} + \frac{(50+P) \times 400 \times 10^3}{600 \times 210 \times 10^3}\right]$$

$-100 - P$
$+50 + P$
$+(50+P) = 0$

Since the supports are unyielding $\delta l = 0$.

$$0 = -\left[P\left[\frac{600 \times 10^3}{1000 \times 70 \times 10^3} + \frac{500 \times 10^3}{800 \times 95 \times 10^3} + \frac{400 \times 10^3}{600 \times 210 \times 10^3}\right] + \left[\frac{100 \times 500 \times 10^3}{800 \times 95 \times 10^3} + \frac{50 \times 400 \times 10^3}{600 \times 210 \times 10^3}\right]\right]$$

$$0 = -\left[P\left(1.832 \times 10^{-2}\right) + \left(8.166 \times 10^{-1}\right)\right]$$

$$P = \frac{-8.166 \times 10^{-1}}{1.832 \times 10^{-2}} = -44.57 \text{ kN}$$

(Direction to be reversed)

Portion AB is in tension.

$$\sigma_{AB} = \frac{44.57 \times 10^3}{1000} = 44.57 \text{ N/mm}^2 \quad T$$

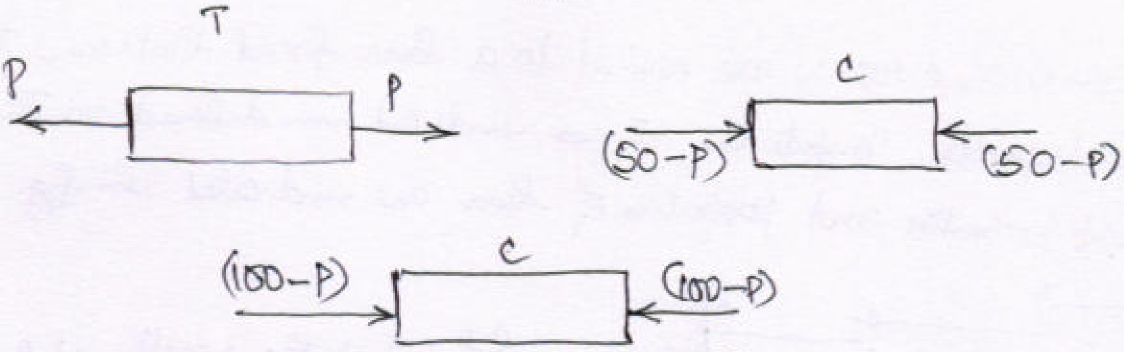
$(100+P) = (100-44.57) = 55.43$ kN

$$\sigma_{BC} = \frac{55.43 \times 10^3}{1000} = 55.43 \text{ N/mm}^2 \quad C$$

$(50+P) = (50-44.57) = 5.43$ kN

$$\sigma_{CD} = \frac{5.43 \times 10^3}{1000} = 5.43 \text{ N/mm}^2 \quad C$$

or



$-P+100-50+R_2=0$

$R_2=-50+P$

$R_2=-(50-P)$

$\overrightarrow{100}$ $\qquad$ $\overleftarrow{50}$

$+P-P+100=\overrightarrow{100}$ $\qquad$ $-(100-P)+50-P$

$$\delta l=0=\delta l_1-\delta l_2-\delta l_3$$

$$0=\frac{P\times 600\times 10^3}{1000\times 70\times 10^3}-\frac{(100-P)\times 10^3\times 500}{800\times 95\times 10^3}-\frac{(50-P)\times 10^3\times 400}{600\times 210\times 10^3}$$

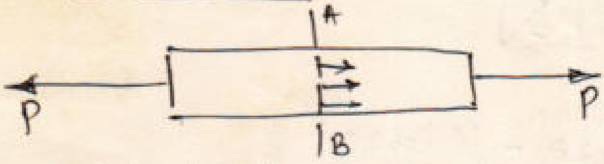
$$0=P\left[\frac{600}{1000\times 70}+\frac{500}{800\times 95}+\frac{400}{600\times 210}\right]-\left[\frac{100\times 500}{800\times 95}+\frac{50\times 400}{600\times 210}\right]$$

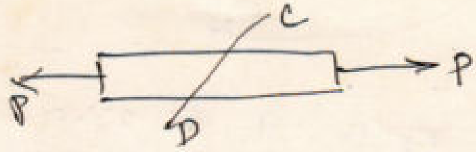
$$18.32\times 10^{-3}\,P=816.62\times 10^{-3}$$

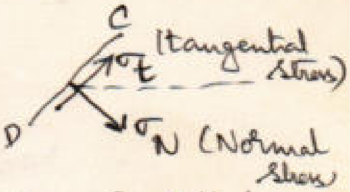
$$\boxed{P=44.54\,kN}$$

Compound Stresses (Complex Stresses)

Introduction:





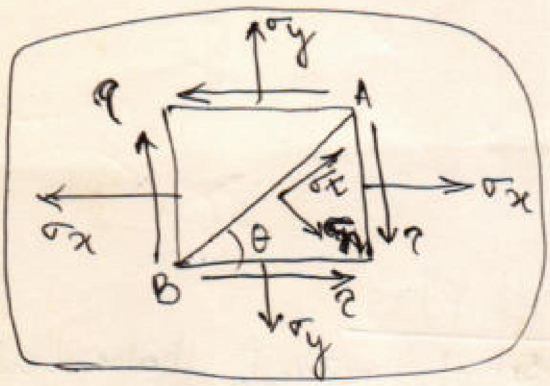


Normal Stress: The stress which acts normal to a given plane is called normal stress or direct stress. The normal stress may be either ~~+ve or~~ tensile(+) or Compressive (-ve)

From the fig, on the plane AB, normal stress exists

Tangential stress or shear stress: The stress which acts tangential to a given plane is called tangential stress or shear stress.

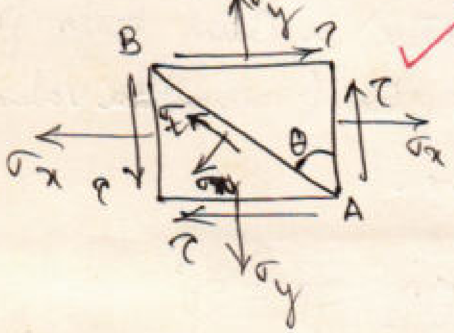
Consider a 2 dimensional stress system as shown in fig



Note: Tensile stresses are positive

The plane AB under consideration makes an anticlockwise angle θ with plane of stress

or



Normal stress on the plane AB = σ_N

$$\sigma_N = \frac{\sigma_x+\sigma_y}{2} + \frac{\sigma_x-\sigma_y}{2}\cos 2\theta + \tau \sin 2\theta.$$

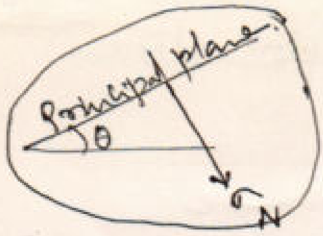
Tangential stress on plane AB = σ_t

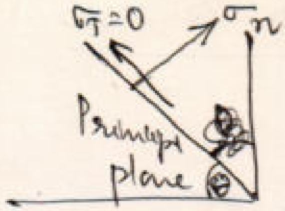
$$\sigma_t = \left(\frac{\sigma_x-\sigma_y}{2}\right)\sin 2\theta - \tau \cos 2\theta$$

Principal stresses and Principal planes:

The planes on which only normal stress exist are called principal planes. The tangential stresses on these planes are zero.

The normal stresses acting on principal planes are called as principal stresses.



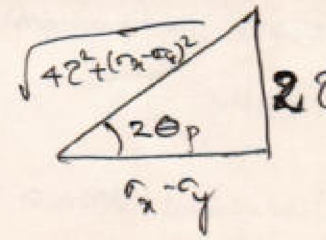


Now $\sigma_T = 0$ ie $\frac{\sigma_x - \sigma_y}{2} \sin 2\theta - \tau \cos 2\theta = 0$

Then $\theta \Rightarrow \theta_p$

$$\frac{\sigma_x - \sigma_y}{2} \sin 2\theta_p = \tau \cos 2\theta_p$$

$$\boxed{\tan 2\theta_p = \frac{2\tau}{\sigma_x - \sigma_y}}$$



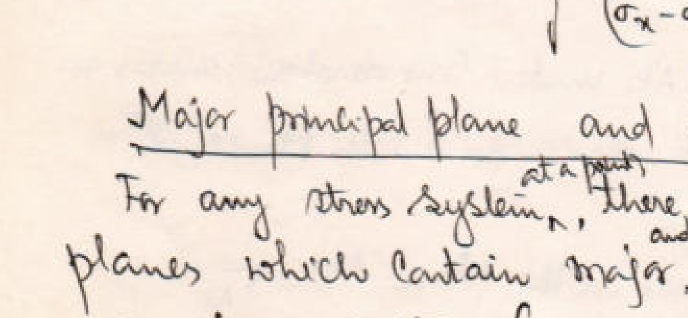
Then

$$\sin 2\theta_p = \frac{2\tau}{\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2}}$$

$$\cos 2\theta_p = \frac{\sigma_x - \sigma_y}{\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2}}$$

<u>Major principal plane and Minor principal plane.</u>

For any stress system at a point, there exists a set of mutually perpendicular planes which contain major and minor principal stresses. ~~S~~ The plane with major principal stress (maximum value of normal stress) is called major principal plane and the plane with minor principal stress (minimum value of normal stress) is called Minor principal plane.



$$\theta p_2 = \theta p_1 + 90^\circ$$

$$\sigma_{n_1} > \sigma_{n_2}$$

Major principal stress
or
Minor principal stress

$$\sigma_{n_1}, \sigma_{n_2} = \frac{(\sigma_x + \sigma_y) \pm \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2}}{2}$$

For Major principal stress use +ve sign and for minor principal stress, use -ve sign.

Inclination of planes of Maximum and ~~minimum~~ Maximum shear stress (tangential stress)

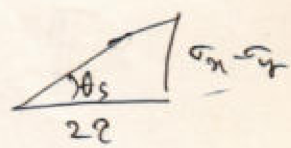
For any stress system at a point, there exists a set of two mutually perpendicular planes that hold maximum shear stress ~~with~~ a constant normal stress. These planes are called planes of maximum shear stress.

$$\sigma_t = \frac{\sigma_x - \sigma_y}{2} \sin 2\theta - \tau \cos 2\theta$$

For σ_t to be maximum or minimum, $\frac{d\sigma_t}{d\theta} = 0$,

$$\Rightarrow \frac{\sigma_x - \sigma_y}{2} 2\cos 2\theta - \tau(-2\sin 2\theta) = 0$$

$$\tan 2\theta_s = \frac{-(\sigma_x - \sigma_y)}{2\tau}$$



$$\theta_{s_2} = \theta_{s_1} + 90^\circ$$

Here θ_p and θ_s differ by 45^0

$$\text{Maximum Shear Stress} = \frac{\sigma_{n_1} - \sigma_{n_2}}{2}$$

Formulae:

1. Normal Stress $= \sigma_n = \dfrac{\sigma_x+\sigma_y}{2} + \dfrac{\sigma_x-\sigma_y}{2}\cos 2\theta + \tau \sin 2\theta$

2. Tangential Stress $= \sigma_t = \dfrac{\sigma_x-\sigma_y}{2}\sin 2\theta - \tau \cos 2\theta$

3. Principal stress and principal planes

$$\sigma_{n_1} \text{ and } \sigma_{n_2} = \frac{(\sigma_x+\sigma_y) \pm \sqrt{(\sigma_x-\sigma_y)^2 + 4\tau^2}}{2}$$

For major principal stress, use +ve sign.

For minor principal stress, use -ve sign.

* Angle of principle planes.

$$\tan 2\theta_p = \frac{+2\tau}{\sigma_x-\sigma_y} \qquad \left[\begin{array}{l} \theta_{p_1} = \text{Inclination of major P.P} \\ \theta_{p_2} = \theta p_1 + 90 \text{ - ,, — minor PP} \end{array}\right]$$

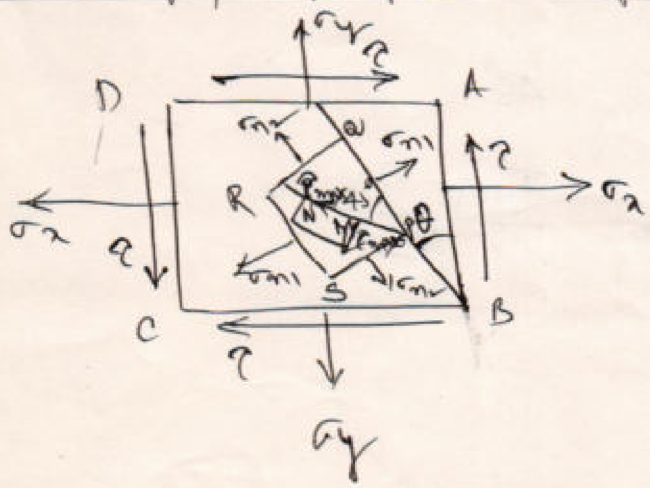
* Angle of shear planes.

$$\tan 2\theta_s = \frac{-(\sigma_x-\sigma_y)}{2\tau} \qquad \left[\begin{array}{l} \theta_{s_1} = \text{Inclination of max shear stress} \\ \theta_{s_2} = \theta_{s_1}+90 \text{ - ,, —} \end{array}\right]$$

4. Maximum Shear Stress $= \dfrac{\sigma_{n_1} - \sigma_{n_2}}{2}$

The planes carrying maximum shear stress are inclined at $(\theta + 45^0)$ and $(\theta + 135^0)$ to the plane carrying the maximum normal stress

Orientation of Principal planes and planes carrying maximum shear stress



PQ = Major Principal plane inclined at θ to AB

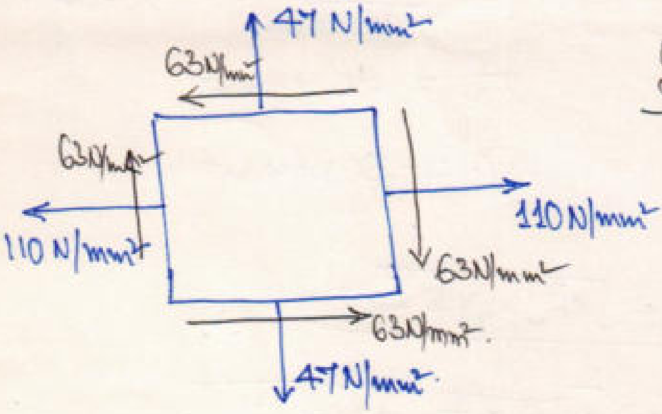
PS = Minor Principal plane inclined at $\theta + 90^0$ to AB

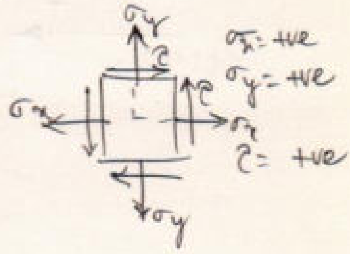
MN = Plane carrying τ_{max} and inclined at $(\theta+45)$ to AB.

MF = plane carrying τ_{max} and inclined at $(\theta+135)$ to AB

A rectangular block of material is subjected to a tensile stress of 110 N/mm² on one plane and tensile stress of 47 N/mm² on the plane at right angles to the former as shown in fig, each of the above stress is accompanied by a shear stress of 63 N/mm². Find the direction and magnitude of each principal stress and magnitude of maximum shear stress. Sketch the planes and mark the stresses on the planes

March 2000





Soln:

Here $\sigma_x = 110\ \mathrm{N/mm^2}$

$\sigma_y = 47\ \mathrm{N/mm^2}$

$\tau = -63\ \mathrm{N/mm^2}$

Principal stress σ_{n_1} or σ_{n_2}

$$= \frac{(\sigma_x+\sigma_y) \pm \sqrt{(\sigma_x-\sigma_y)^2 + 4\tau^2}}{2}$$

$$= \frac{(110+47) \pm \sqrt{(63)^2 + 4(-63)^2}}{2}$$

$$= \frac{157 \pm 140.87}{2} = 148.93\,\mathrm{MPa} \rightarrow \sigma_{n_1}$$

$$8.063\,\mathrm{MPa} \rightarrow \sigma_{n_2}$$

Angle of principal plane.

$$\tan 2\theta_P = \frac{2\tau}{\sigma_x - \sigma_y} = \frac{2(-63)}{63} = -2$$

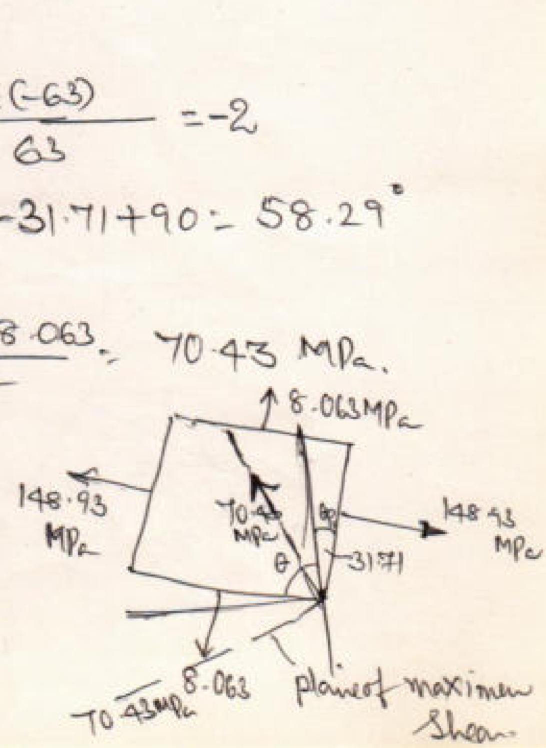
$\theta_{P_1} = -31.71$ $\quad \theta_{P_2} = -31.71 + 90 = 58.29°$

Max Shear stress:

$$\sigma_{t\,max} = \frac{\sigma_{n_1} - \sigma_{n_2}}{2} = \frac{148.93 - 8.063}{2} = 70.43\ \mathrm{MPa}.$$

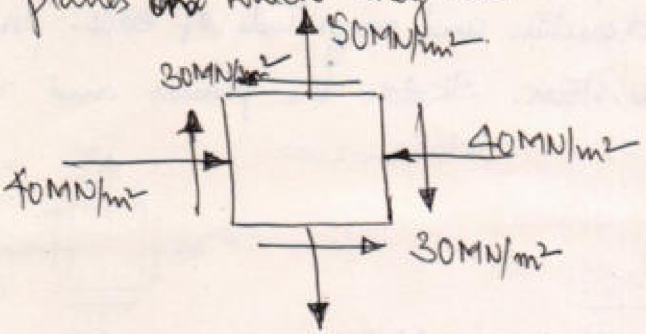
$\theta_{S_1} = -31.71 + 45 = 13.28°$

$\theta_{S_2} = 13.28 + 90° = 103.28°$



An element is subjected to biaxial stress and accompanied with the shear stress as shown in fig. Determine a) the principal stresses and their directions b) the direction of the plane on which they occur. Show the stresses and the planes ~~on~~ which they act.

March 2001



Soln.

Here

$\sigma_x = -40\ MN/m^2$

$= -\dfrac{40 \times 10^6}{1000 \times 1000} = -40\ N/mm^2$

$\sigma_y = 50\ N/mm^2$

$\tau = -30\ N/mm^2$

Principal stresses:

$$\sigma_{n_1} \text{ or } \sigma_{n_2} = \frac{(\sigma_x + \sigma_y) \pm \sqrt{(\sigma_x - \sigma_y) + 4\tau^2}}{2}$$

$$= \frac{(-40+50) \pm \sqrt{(-90)^2 + 4(-30)^2}}{2}$$

$$= \frac{10 \pm 108.16}{2} = 59.08 \longrightarrow \sigma_{n_1} \text{ (Tensile)}$$

$$-49.08 \longrightarrow \sigma_{n_2} \text{ (Compressive)}$$

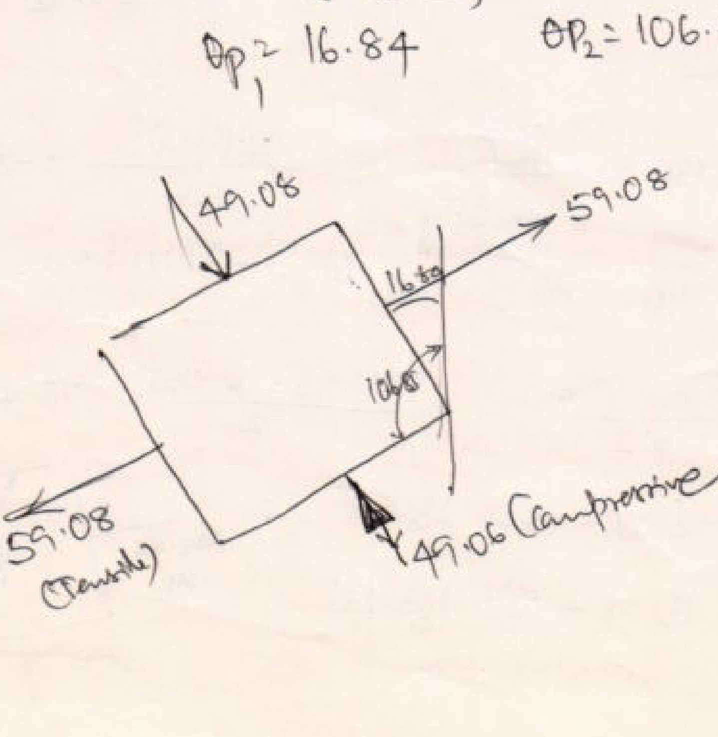
Direction:

Angle of principal plane:

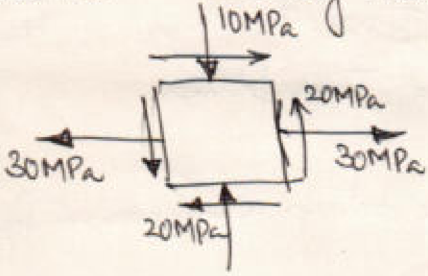
$$\tan 2\theta_p = \frac{2\tau}{(\sigma_x - \sigma_y)}$$

$$= \frac{2 \times (-30)}{(-40-50)} = 0.666$$

$\theta_{p_1} = 16.84 \qquad \theta_{p_2} = 106.85.$



The state of stress at a point in a material under plane stress is as defined. The normal stress along x and y axes are 30 MPa (Tensile) and 10 MPa (Compressive) respectively. The accompanying shear is 20 MPa. Determine
i) the principal planes and the stresses on them.
ii) the critical shear planes and stresses on them. Aug 2001
Show the results by means of properly oriented differential elements.



Here $\sigma_x = 30\text{MPa}$

$\sigma_y = -10\text{MPa}$

$\tau = +20\text{MPa}$

principal stress:

$$\sigma_{n_1} \text{ or } \sigma_{n_2} = \frac{(\sigma_x + \sigma_y) \pm \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2}}{2}$$

$$= \frac{20 \pm \sqrt{40^2 + 4(20)^2}}{2}$$

$$\sigma_{n_1} \text{ or } \sigma_{n_2} = \frac{20 \pm 56.57}{2} = 38.28 \text{ MPa} \rightarrow \sigma_{n_1}$$
$$-18.28 \text{ MPa} \rightarrow \sigma_{n_2}$$

$$\tan 2\theta_p = \frac{2\tau}{\sigma_x - \sigma_y} = \frac{2 \times 20}{40} = 2$$

$\theta_{p_1} = 22.5$ $\quad \theta_{p_2} = 112.5^\circ$

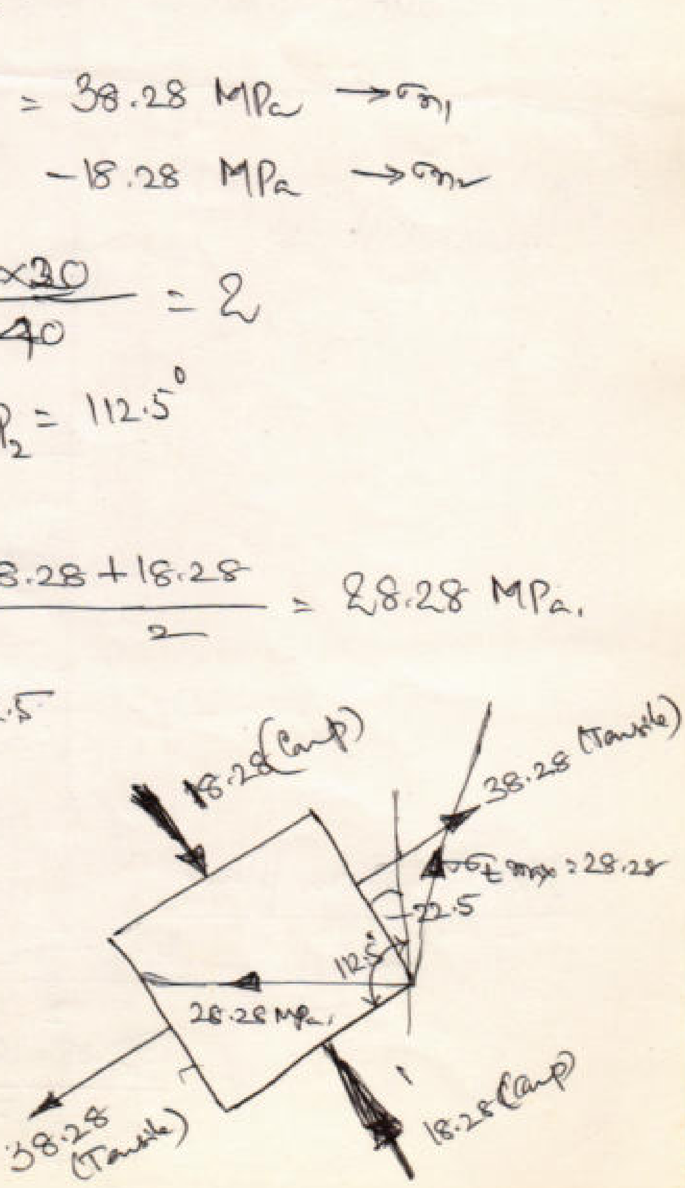
Max shear:

$$\tau_{max} = \frac{\sigma_{n_1} - \sigma_{n_2}}{2} = \frac{38.28 + 18.28}{2} = 28.28 \text{ MPa.}$$

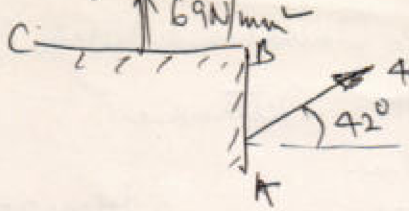
$\theta_{S_1} = 22.5 \cancel{+} 45 = \cancel{22.5}$

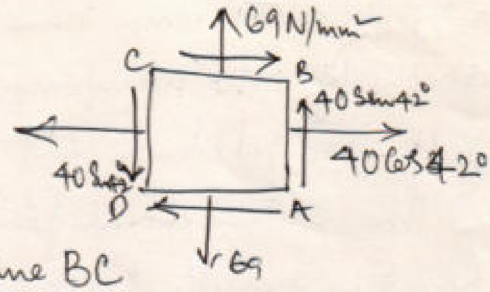
$$\tan 2\theta_{S_1} = \frac{-40}{2 \times 20} = -\cancel{2.5}\,1$$

$\theta_{S_1} = -22.5$ $\quad \theta_{S_2} = 67.5$



The intensity of stresses on planes AB and BC is as shown.





Determine i) the resultant stress on Plane BC

ii) the principal stresses and their directions. Feb-2002

iii) The max^m shear stresses and their planes.

<u>Soln</u>

To keep element in equilibrium, stresses are setup as shown in fig..

$\sigma_x = 40\cos 42^\circ = 29.72\ N/mm^2$

$\sigma_y = 69\ N/mm^2 = 69\ N/mm^2$

$\tau = 40\sin 42 = 26.76\ N/mm^2$.

Resultant Stress on Plane BC $= \sqrt{\sigma_{y_{BC}}^2 + \tau^2}$

$= \sqrt{69^2 + 26.76^2} = 74\ N/mm^2$.

Principal Stresses:

$$\sigma_{n_1} \text{ and } \sigma_{n_2} = \frac{(\sigma_x + \sigma_y) \pm \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2}}{2}$$

$$= \frac{(29.72 + 69) \pm \sqrt{(39.28)^2 + 4(26.76)^2}}{2}$$

$$= \frac{98.72 \pm 66.38}{2} = 82.55\ N/mm^2 \rightarrow \sigma_{n_1}$$

$16.17\ N/mm^2 \rightarrow \sigma_{n_2}$

$$\tan 2\theta_p = \frac{2\tau}{\sigma_x - \sigma_y} = \frac{2(26.76)}{-39.28} = -1.3625$$

$2\theta_p = -53.72$

$\theta_{p_1} = -26.86 \quad \theta_{p_2} = 63.13$

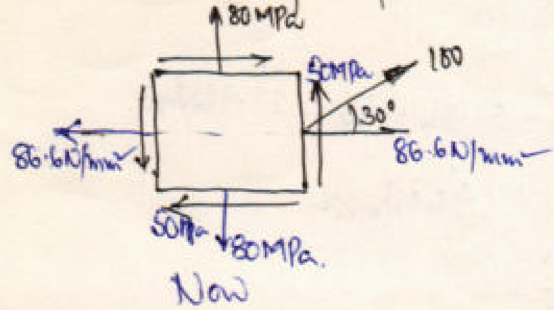
<u>Max^m Shear plane:</u>

$$\tau_{t\,max} = \frac{\sigma_{n_1} - \sigma_{n_2}}{2}$$

$$= \frac{82.55 - 16.17}{2} = 33.19$$

$$\tan 2\theta_s = \frac{-(\sigma_x - \sigma_y)}{2\tau} = \frac{39.28}{2(26.76)} = 18.13 \qquad \theta_{s_2} = 108.13^\circ$$

At a point in a strained material, the state of stress is as shown in the fig. Determine the principal stresses and orientations of principal axis. Aug 99



Soln.

Tensile stress along x direction = $\sigma_x = 100\cos 30^\circ = 86.6$ N

$\tau = 100 \sin 30^\circ = 50$ MPa

$\sigma_y = 80$ MPa

Now

Principal stress:

$$\sigma_{n_1} \text{ or } \sigma_{n_2} = \frac{(\sigma_x+\sigma_y) \pm \sqrt{(\sigma_x-\sigma_y)^2 + 4\tau^2}}{2}$$

$$= \frac{(166.6) \pm \sqrt{(6.66)^2 + 4(50)^2}}{2}$$

$$= \frac{166.6 \pm 100.22}{2} = 133.41 \text{ MPa} — \sigma_{n_1} \text{ (Tensile)}$$

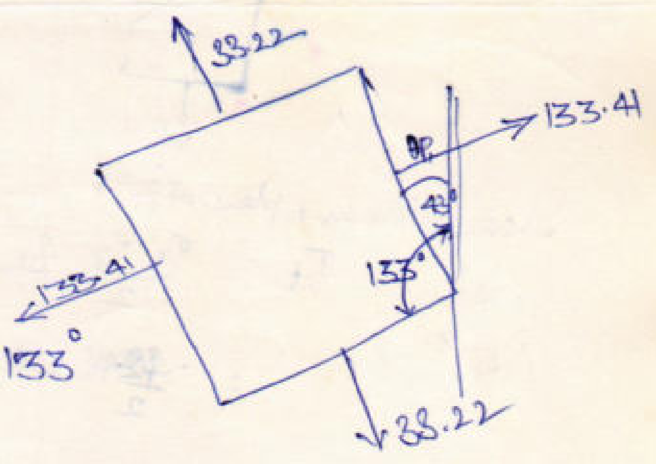
$$33.22 \text{ MPa} — \sigma_{n_2} \text{ (Tensile)}$$

Orientation of Principal planes:

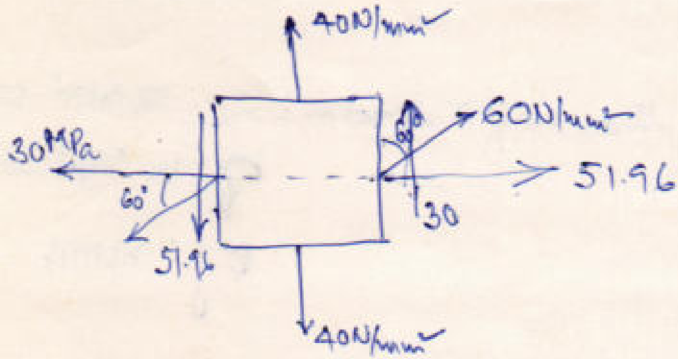
$$\tan 2\theta_p = \frac{2\tau}{(\sigma_x-\sigma_y)}$$

$$= \frac{2\times 50}{6.66}$$

$\theta_{p_1} = 43^\circ$ $\quad \theta_{p_2} = 133^\circ$



A point in a Strained material is Subjected to the stresses as Shown in fig. Locate the principal planes and evaluate the principal Stresses

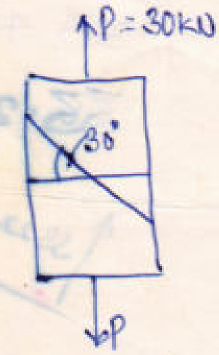


$\sigma_x = 51.96 - 30 = 21.96 N/mm^2$

$\sigma_y = 40 N/mm^2$

$\tau =$

A Circular bar of diameter 30mm is Subjected to an axial force of 30KN as shown in the fig. Find the shear stresses on a plane making 30° to the plane of axial stresses and also on the plane which has maximum shear stress.



$$\sigma = \frac{30 \times 10^3}{\frac{\pi \times 30^2}{4}} = 42.44 N/mm^2$$

$\sigma_y = 30KN$

$\sigma_x = 0$

$\tau = 0$

Shear stress on a plane at 30° to

$\theta = 30°$

$\sigma_y = 30$

$$\sigma_t = \frac{\sigma_x - \sigma_y}{2} \sin 2\theta - \tau \cos 2\theta$$

$$= \frac{-42.44}{2} \sin 60° = \cancel{-18}\ -18.37 KN.$$

The maximum shear stress occurs on a plane where $\theta = 45°$

$$\therefore \text{ Maxm Shear Stress} = \frac{\sigma_x - \sigma_y}{2} \sin 2\theta - \tau \cos 2\theta$$

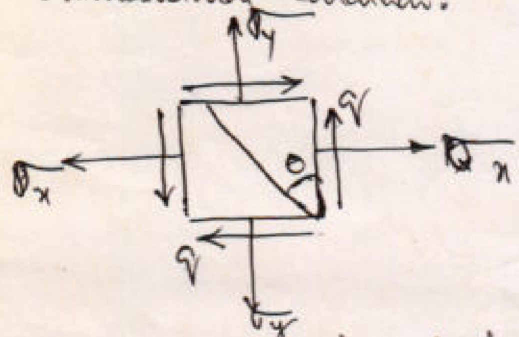
$$= -\frac{42.44}{2} \sin 90°$$

$$= -21.22 N/mm^2$$

Mohr's Circle of stresses:

Explain the Construction of Mohr's Circle for Compound Stress System in two dimensional Condition.

March 2001
March 2000



Sign Convention:

i) Tensile stress (normal) is +ve

ii) Shear stress producing clockwise moment is +ve

To Construct Mohr's Circle:

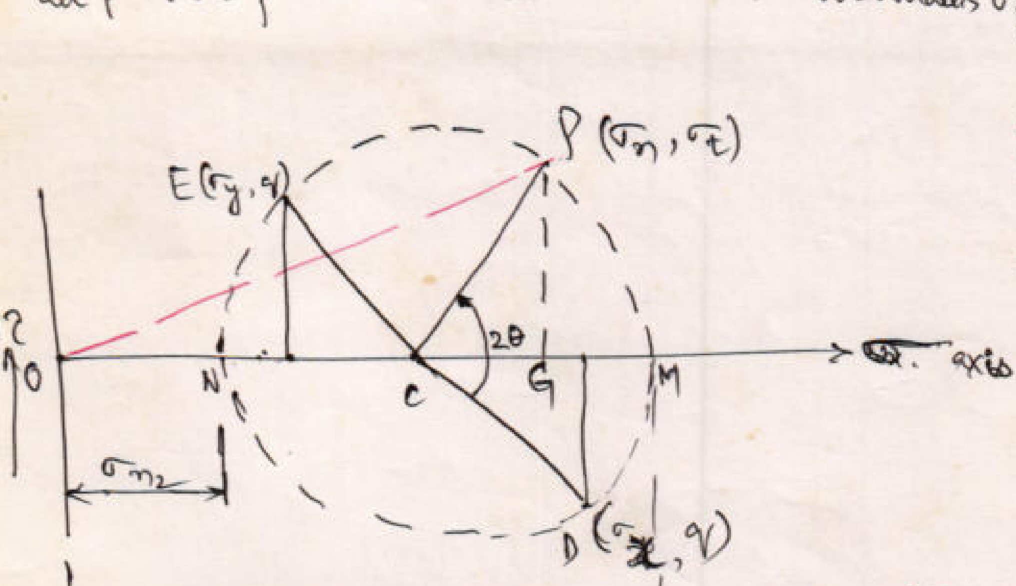
a) Draw X and Y axis and denote X axis as direct stress axis (σ) and Y axis as τ axis.

b) Mark point D (σ_x, q) representing the state of stress on the plane of σ_x

c) Mark point E (σ_y, q) representing the state of stress on the plane of σ_y.

d) Join DE, let the point of intersection with normal stress axis (σ axis) be 'C'

Let P be a point such that $\angle DCP = 2\theta$. Coordinates of P is σ_n and σ_t.



$OG = \sigma_n$

$PG = \sigma_t$

Here OM = Major principal stress

ON = Minor principal stress.

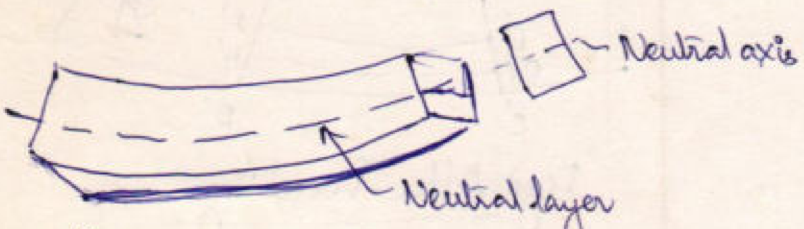
$OP = \sqrt{\sigma_n^2 + \sigma_t^2}$

(Resultant stress)

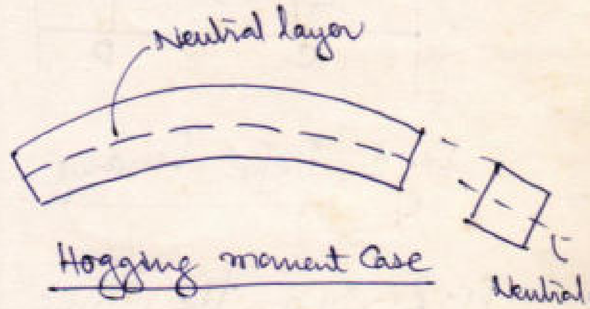
Stresses in Beams

Simple bending theory:

Bending is (always or) normally associated with shear. However for simplicity, the effect of shear is neglected and moment alone is considered to find stresses due to bending. Such a theory which deals with determination of stresses at a section due to pure moment is called simple bending theory



Sagging moment case

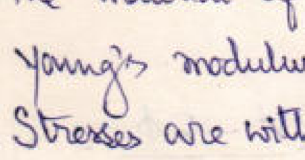


Hogging moment case

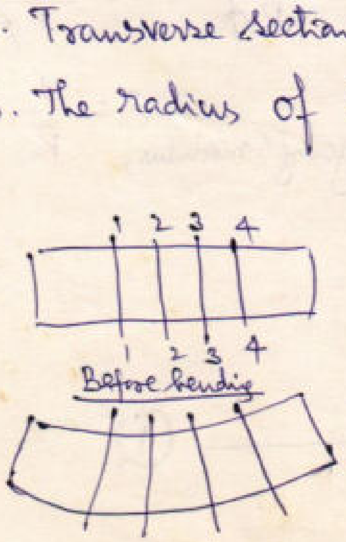
When a beam is subjected to sagging moment, the bottom fibres get elongated (Tension) and top fibres get compressed. (Compression). In between these two fibres, there is a layer where stresses are zero. Such a layer is called neutral layer. Its intersection with cross section is called Neutral axis.

Assumptions in simple theory of bending:

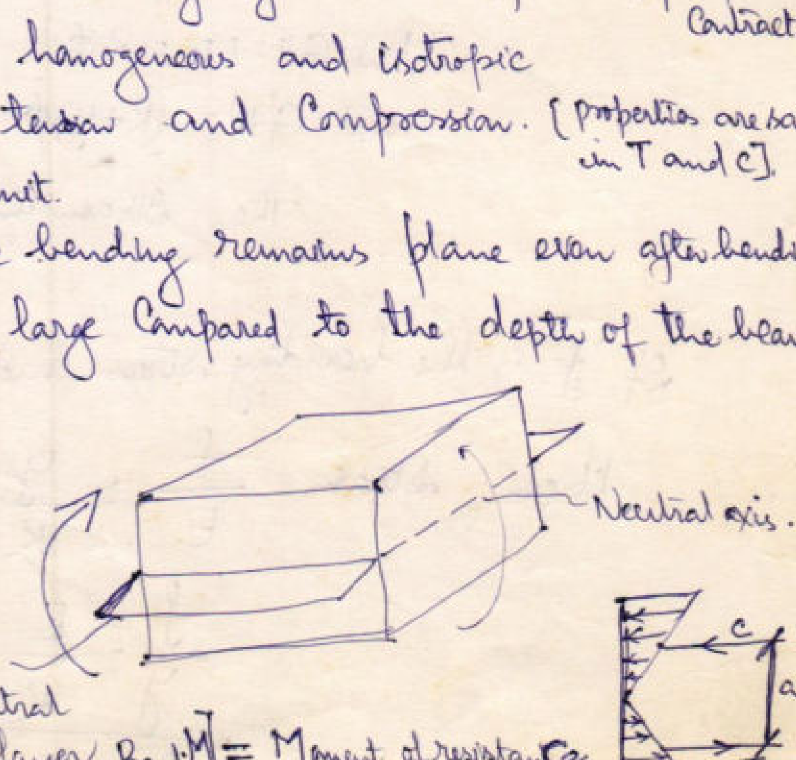
1. The beam is initially straight and every layer in it is free to expand or contract.
2. The material of the beam is homogeneous and isotropic
3. Young's modulus is same in tension and compression. [Properties are same in T and C].
4. Stresses are within elastic limit.
5. Transverse section plane before bending remains plane even after bending.
6. The radius of curvature is large compared to the depth of the beam.



Before bending



After bending.



Bend. M (due to loads) = Moment of resistance

C or T × a

: Derivation of bending Equation: Bernoulli's Bending Equation

Consider a portion of a beam between Sections AC and BD as shown in fig 1. Let EF be the neutral layer (neutral axis) and GH be a fibre at a distance 'y' from neutral axis.

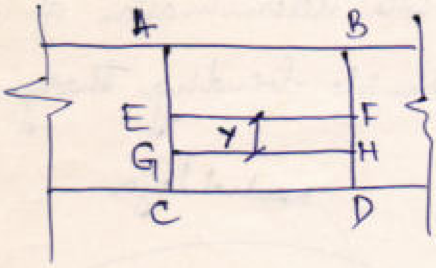


Fig 1. Before bending

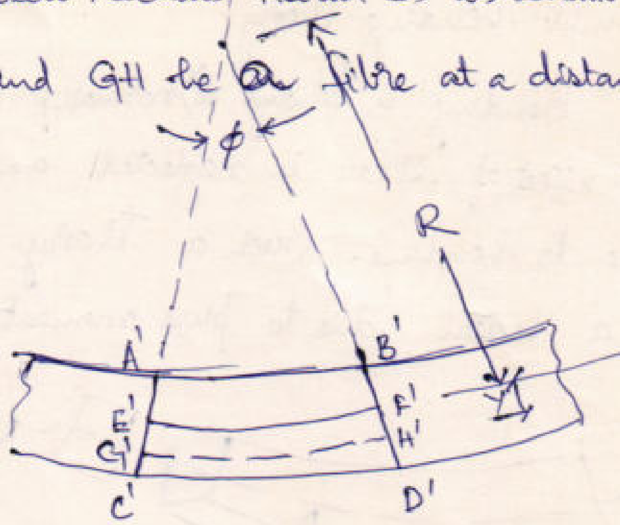


Fig 2. After bending

Fig 2. Shows the same portion after bending. Let 'R' be the radius of Curvature and ϕ be the angle subtended by C'A' and D'B' at Centre of radius of Curvature.

EF is neutral layer and hence no change in length. $E'F' = EF$

$$EF = E'F'$$
$$= R\phi$$

Now, Strain in layer $GH = \frac{G'H' - GH}{GH}$

But $GH = EF = R\phi$

and $G'H' = (R+y)\phi$

Then Strain in layer $GH = \frac{(R+y)\phi - R\phi}{R\phi} = \frac{y\phi}{R\phi}$

$$e = \frac{y}{R}$$

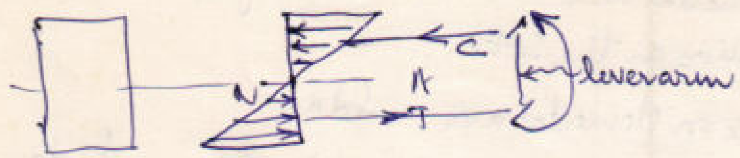
If 'f' is the bending stress and 'E' is the young's modulus,

then strain $= \frac{f}{E} = \frac{y}{R}$

$$\frac{f}{y} = \frac{E}{R} \qquad \text{—— (1)}$$

or $\boxed{f = \frac{E}{R} \cdot y}$

Thus bending stress varies linearly across the depth.



Further, let the fibre GH posses an elementary area δa and 'f' be the bending stress on this fibre.

Force on this fibre = $f\,da$.

Moment of the resisting force about N.A = $f.da.y$

but : $f = \frac{E}{R}.y$ $\qquad = \frac{E}{R}\,\delta a\, y^2$

Moment generated by entire area of cross-section $M' = \frac{E}{R}\int y^2 da$.

and $\int y^2 da$ is moment of inertia about centroid i.e. $I = \int y^2 da$.

$$M' = \frac{E}{R}.I$$

For equilibrium, the moment of resistance M' should be equal to applied moment M. i.e. $M' = M$.

Hence $\quad M = \frac{E}{R}.I$

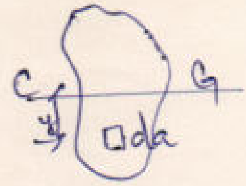
$$\frac{M}{I} = \frac{E}{R} \quad — ②$$

From ① and ②,

$$\boxed{\frac{M}{I} = \frac{f}{y} = \frac{E}{R}}$$

Where M = Bending moment
I = moment of inertia about centroidal axis.
y = Distance of the fibre from N.A
E = young's modulus
R = Radius of curvature

Location of Neutral axis: (To prove N Axis is the Centroidal axis)



Consider an elemental area da at a distance y from N-Axis.
Let f be the Stress on the Section.

$\therefore$ Force on elemental area = f da.

$\therefore$ Total force on cross section of the beam = $\int f\,da$.

But $f = \frac{E}{R} \cdot y$

Then Total force on section $= \int \frac{E}{R} \cdot y\,da$.

$= \frac{E}{R} \int y\,da$

Since there is no axial force on the beam and the above force is in axial directn for Equilibrium

$$\frac{E}{R} \int y\,da = 0.$$

Here $\frac{E}{R} \neq 0$ and $\int y\,da = 0$ only.

$\int y\,da$ is the first moment of area about neutral axis will become zero Only when Centroidal axis coincides with N.Axis.

Moment Carrying Capacity of a Section:

Section modulus: (Sept 1999)

The ratio of MI of transverse section of a beam about its N Axis to the distance of extreme fibre from the N. Axis is known as Section modulus.

$$\text{Section modulus} = Z = \frac{\text{Moment of inertia}}{\text{Distance of extreme fibre from N.A}} = \frac{I}{y_{max}}$$

$$Z = \frac{I}{y_{max}}$$

We know that

$$M = \frac{I}{y} \cdot f \quad \text{or} \quad M = \frac{I}{y_{max}} f.$$

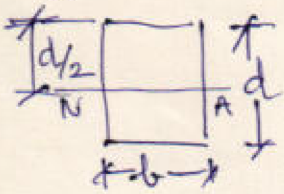
M_{max} =

$$\boxed{M_{max} = \frac{I}{y_{max}} \cdot f_{per}}$$

Moment Carrying Capacity of a section depends on Section modulus.

Expressions for section modulus:

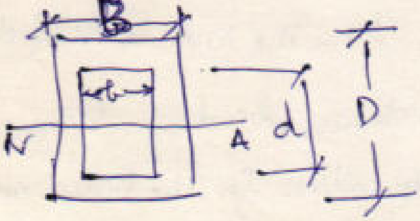
a. Rectangular section of width 'b' and depth 'd'



$$Z = \frac{I}{y_{max}} = \frac{bd^3/12}{d/2} = \frac{bd^2}{6}$$

b. Hollow rectangular section:

M(2010)



$$I = \frac{BD^3 - bd^3}{12} \quad \text{and } y = \frac{D}{2}$$

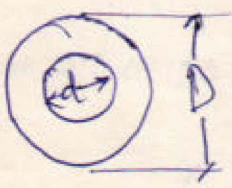
$$Z = \frac{I}{y_{max}} = \frac{BD^3 - bd^3}{12} \Big/ \frac{D}{2}$$

$$= \frac{(BD^3 - bd^3)}{6D}$$

c. Circular section:

$$Z = \frac{I}{y_{max}} = \frac{\pi d^4/64}{d/2} = \frac{\pi d^3}{32}$$

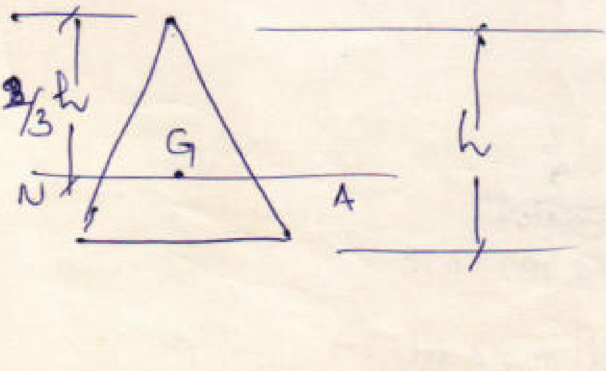
d. Hollow circle:



$$Z = \frac{\frac{\pi D^4}{64} - \frac{\pi d^4}{64}}{D/2} = \frac{\pi}{64D}(D^4 - d^4) \cdot 2$$

$$\boxed{Z = \frac{\pi}{32D}[D^4 - d^4]}$$

e. Triangular section



$$Z = \frac{I}{y_{max}} = \frac{\frac{bh^3}{36}}{\frac{2}{3}h} = \frac{bh^2}{24}$$

Moment of resistance :

from bending eqn

$$\frac{M}{I} = \frac{f}{y}$$

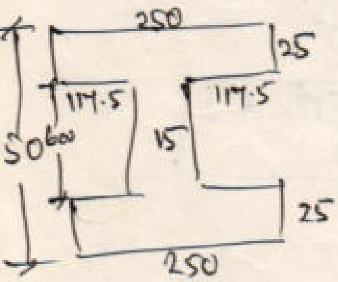
$$M = \frac{I}{y} \cdot f$$

The fibre at the ~~extreme~~ farthest distance from the N.A is subjected max bending stress. To avoid failure of beams, the maximum bending stress induced in extreme fibre should be less than the allowable stress for the beam material.

$$M_{max} = \frac{I}{y} \cdot f_{allowable} \qquad \text{or } f_{permissible}.$$

Thus Maximum bending moment that a beam can withstand without failure is called moment of resistance.

A rolled steel joist of I section has the following dimensions: Flange 250mm wide and 25mm thick. Web of 15mm thick and overall depth of 650mm. If this beam carries a UDL of 50KN/m on a span of 6m, Calculate the maxm bending stress produced.



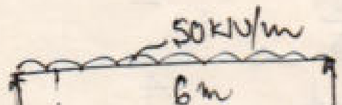
$$I = \left[\frac{250 \times 650^3}{12} - \frac{2 \times 117.5 \times 600^3}{12}\right] = 1491354167\ mm^4$$ Feb-2002

$$y_{max} = \frac{650}{2} = 325\ mm \qquad Z = \frac{I}{y_{max}}$$

$$=$$

$$M = fZ$$

$$f = \frac{M}{Z}$$



$$Max BM = \frac{wl^2}{8}$$

$$= \frac{50KN \times 6 \times 6}{8}$$

$$= 225\ KN\text{-}m$$

$$= 225 \times 10^3\ KN\text{-}mm$$

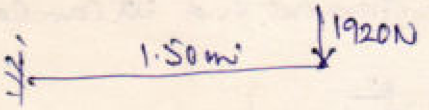
$$= 225 \times 10^6\ N\text{-}mm.$$

$$f = \frac{225 \times 10^6}{\cancel{1491354167}}$$

$$4588782.052$$

$$= 49.0326\ N/mm^2$$

A Cast iron Cantilever of length 1.50mt fails when a load of 1920N is applied at the free end. Determine the stress at failure if the section of the Cantilever is 40mm × 60mm



$M = 1500 \times 1920$
$= 2880 \times 10^3$ N-mm

Bending Eqn

$M = fZ$

$= f\frac{bd^2}{6}$

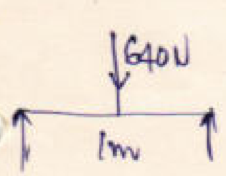
$f = \frac{6M}{bd^2}$

$= \frac{6 \times 2880 \times 10^3}{40 \times 60^2}$

$= 120 N/mm^2$.

A Cast iron test beam 20mm × 20mm in section and 1mt long and supported at the ends fails when a central load of 640N is applied. What UDL will break a Cantilever of the same material 50mm wide, 100mm deep and 2mts long?

Soln: Considering the test beam,



$M = \frac{WL}{4} = \frac{640 \times 1000}{4} = 160 \times 10^3$ N-mm

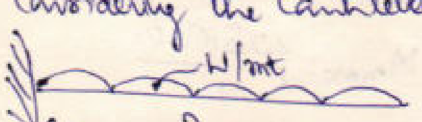
$M = fZ$

$M = f\frac{bd^2}{6}$

$f = \frac{6M}{bd^2} = \frac{6 \times 160 \times 10^3}{20 \times 20^2} = 120 N/mm^2$

Max bending stress or permissible bending stress $= 120 N/mm^2$

Considering the Cantilever, let UDL be W N/mt.



$M_{max} = \frac{Wl^2}{2} = \frac{W \times 2^2}{2} \times 1000 = 2W$ N-m
$= 2 \times 10^3$ W N-mm

$\boxed{M = 2000W \text{ N-mm}}$

$M = fZ$

$2000W = 120 \times 50 \times \frac{100^2}{6}$

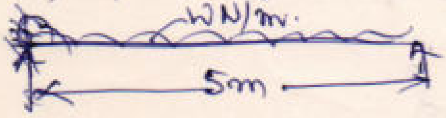
$W = 5000$ N/mt.

A simply supported beam of span 5m has a cross section 150mm × 250 mm. If the permissible stress is 10 N/mm², find

a) Max intensity of UDL it can carry.

b) Max Concentrated load 'P' applied at 2m from one end it can carry.

a) To find intensity of UDL



$$M_{max} = \frac{wl^2}{8} \quad N\text{-}m$$

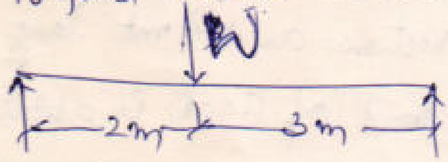
$$M = fz$$

$$\frac{w \times 5^2}{8} \times 10^3 = \frac{1}{6} f bd^2$$

$$= \frac{1}{6} \times 10 \times 150 \times 250^2$$

$$\boxed{w = 5000\ N/m}$$

b) To find maxm Concentrated load at 2m from one end



$$M_{max} = \frac{P \times 2 \times 3}{5} \times 1000 \ N\text{-}mm$$

$$M_{Max} = \frac{Wab}{l}$$

$$\frac{6P}{5} \times 1000 = \frac{1}{6} f bd^2$$

$$W = \frac{1 \times 5 \times 10 \times 150 \times 250^2}{6 \times 6 \times 1000}$$

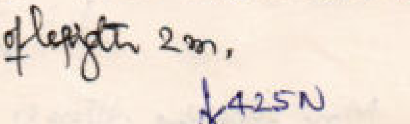
$$\boxed{W = 13020.79\ N}$$

Aug 1999

A simply supported beam 15mm × 20mm is 1.5m long and it fails at a Concentrated load of 425N is applied at Centre. Determine what UDL can break a Cantilever of same material of size 50mm × 110mm and of length 2m.

Soln

a) Taking Simply supported beam



$$M = fz$$

$$M_{max} = \frac{425 \times 1500}{4}$$

$$= 159375\ N\text{-}mm$$

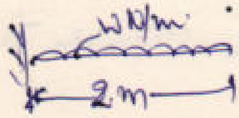
$$f = \frac{M}{z}$$

$$= \frac{M}{\frac{1}{6} bd^2} = \frac{6M}{bd^2}$$

$$f = \frac{6 \times 159375}{15 \times 20^2} = 159.37\ N/mm^2$$

Maxm Permissible bending stress = 159.37 N/mm²

Considering a Cantilever of the same material, let W N/m be the UDL



$M_{max} = \frac{Wl^2}{2} = \frac{4W}{2} \times 1000$

$= 2000 W$ N-mm (W is in N/mt)

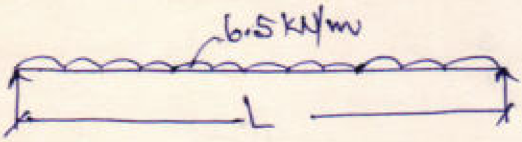
$$M = fZ$$

$$2000W = 159.37 \times \frac{50 \times 110^2}{6}$$

$$W = \frac{159.37 \times 50 \times 110^2}{2000 \times 6} = 8034 \text{ N/mt}$$

Determine the maximum allowable span 'L' for a beam simply supported beam of rectangular cross section, 140mm x 240mm subjected to a udl of 6.5 kN/m, if allowable bending stress is 8.2 MPa.

June-July 2009



$M_{max} = \frac{6.5 \times l^2}{8}$

$= 0.8125\, l^2$ kN-m. (l is in mts)

$= 0.8125\, l^2 \times 10^3 \times 10^3$ N-mm

We know that

$$M = fZ$$

$$M = \frac{1}{6} bd^2 \cdot f$$

$$0.8125\, l^2 \times 10^6 = \frac{1}{6} \times 140 \times 240^2 \times 8.2$$

$$0.8125 \times 10^6 \times l^2 = 11020800$$

$$l = 3.68 \text{ mt}$$

A thin strip 0.5mm thick and 3.14 m long is bent into a circular shape. Determine the maximum stress induced. Take E = 200 GPa

Soln:

The strip 3.14m of the strip becomes perimeter.

$$2\pi R = 3.14 \text{ m}$$

$$R = \frac{3.14}{2 \cdot \pi} = 0.5 \text{ m}$$

From bending eqn

$$\frac{f}{y} = \frac{E}{R}$$

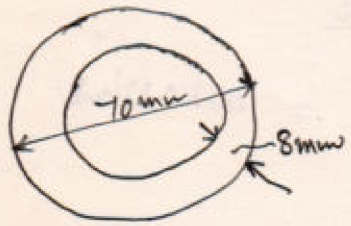
$y = \frac{0.5}{2}$

$= 0.25$ mm

$$f = \frac{200 \times 10^3}{500} \times 0.25 = 100 \text{ N/mm}^2$$

A circular pipe of external dia 70mm and thickness 8mm is used as a simply supported beam over an effective span 2.5m. Find the maximum concentrated load that can be applied at the centre of the span if permissible stress in tube is 150 N/mm²

Soln:



External dia = 70mm
Internal dia = 54mm.

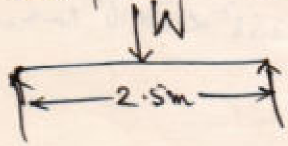
$f_{per} = 150 N/mm^2$

$$I = \frac{\pi}{64}(70^4 - 54^4)$$
$$= 761195.33 \text{ mm}^4$$
$$y_{max} = \frac{D}{2} = \frac{70}{2} = 35mm$$
$$Z = \frac{I}{y_{max}} = 21748.43 \text{ mm}^3$$

From bending eqn:

$$M_{max} = Z f_{per}$$
$$M_{max} = 150 \times 21748.43 \times 10^3$$
$$= 3.2 \times 10^9 \text{ N-mm} \quad \text{——— (1)}$$

When pipe is used as SSB



$$\text{Max moment} = \frac{Wl}{4} = \frac{W \times 2500}{4} \quad \text{——— (2)}$$

Equating (1) and (2)

$$\frac{W \times 2500}{4} = 3.2 \times 10^9$$
$$W = 5119623.2 \text{ N} = 5.22 \times 10^3 \text{ kN}$$

Let W = kN

$M = \frac{Wl}{4}$

$= \frac{2.5W}{4}$ kN-m

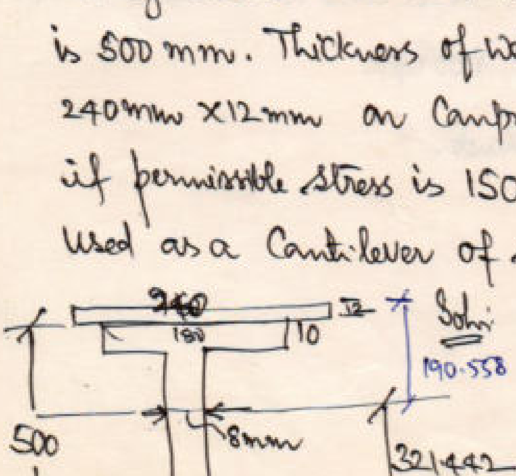
$= \frac{2.5W}{4} \times 10^6$

$= 3.2 \times 10^9$

$W = \frac{3.2 \times 10^9 \times 4}{2.5 \times 10^6}$

$\boxed{W = 5120 \text{ kN}}$

A symmetric I section has flanges of size 180mm × 10mm and its over all depth is 500 mm. Thickness of web is 8mm. It is strengthened with a plate of size 240mm × 12mm on compression side. Find the moment of resistance of the section if permissible stress is 150N/mm². How much UDL it can carry if it is used as a cantilever of span 3m?



Soln:

$$\bar{Y} = \frac{240 \times 12 \times 506 + 180 \times 10 \times 495 + 480 \times 8 \times 250 + 180 \times 10 \times 5}{240 \times 12 + 180 \times 10 + 480 \times 8 + 180 \times 10}$$

$$= \frac{3317280}{10320} = 321.442 \text{ mm}$$

⑥

$$I = \frac{240\times12^3}{12} + 240\times12(506-321.442)^2 + \frac{180\times10^3}{12} + 180\times10(495-321.442)^2$$

$$+ \frac{1}{12}\times480\times\cancel{8}^3 + 480\times\cancel{8}(321.442-250)^2 + \frac{180\times10^3}{12} + 180\times10(321.442-5)^2$$

$$= 3.5\times10^8 \text{ mm}^4 \checkmark$$

$$Z = \frac{I}{y_{max}} = \frac{3.5\times10^8}{321.442} = 1088843.39 \text{ mm}^3$$

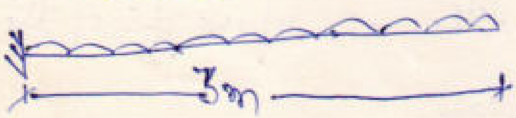
$$M_{max} = f_{per} Z$$

$$M_{max} = 150\times1088843.39$$

$$= 163326508.5 \text{ N-mm}$$

$$\boxed{\text{Maximum Moment of resistance} = 1.633\times10^8 \text{ N-mm}}$$

When the above I section is used as a Cantilever,



$$\frac{Wl^2}{2} = \text{Max BM}$$

ie. $\frac{9W}{2} = 1.633\times10^8$ N-mm.

$$W = \frac{1.633\times10^5}{4.5} \text{ N-m}$$

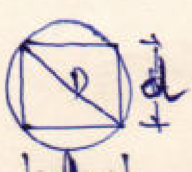
$W = 36288.88$ N or 36.288 kN

Find the width and depth of the strongest rectangular beam that can be cut out of a cylindrical log of wood whose dia is 500mm. (D).

Aug 2001 Jan 2007 (10 marks)

Soln: Let us take ~~D~~ dia as 'D'

Let 'b' be the width and 'd' be the depth of strongest rectangular section as shown in fig.



Then $D^2 = b^2 + d^2$

$$d^2 = D^2 - b^2$$

Section modulus of a rectangular section.

$$Z = \frac{1}{6}bd^2$$

$$= \frac{1}{6}b(D^2-b^2) = \frac{1}{6}(bD^2-b^3)$$

For the beam to be strongest, the section modulus 'Z' must be maximum

For Z to be maximum, $\frac{dZ}{db} = 0$

$$\frac{dZ}{db} = \frac{1}{6}(D^2 - 3b^2) = 0$$

ie $D^2 - 3b^2 = 0$ or $D^2 = 3b^2$

$$D = \sqrt{3}b$$

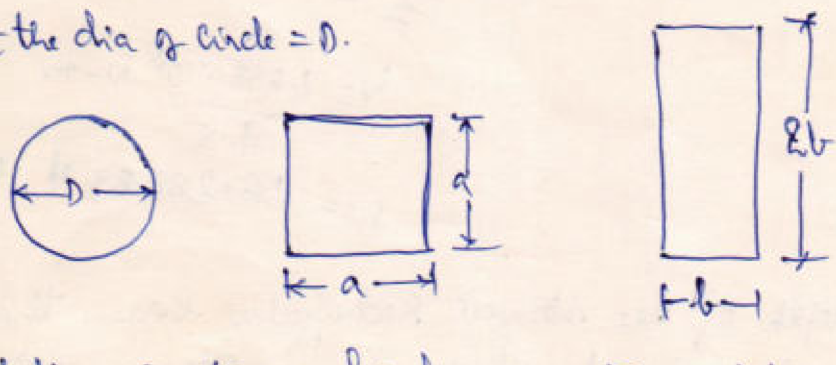
$$\boxed{b = \frac{D}{\sqrt{3}}}$$ Now $b = \frac{500}{\sqrt{3}} = 288.675$ mm

also
$$d^2 = D^2 - b^2$$
$$= D^2 - \frac{D^2}{3}$$
$$= \frac{3D^2 - D^2}{3} = \frac{2D^2}{3} \quad \therefore \boxed{d = \sqrt{\frac{2}{3}}D} \text{ and } \boxed{d = 408.24 \text{ mm}}$$

Three beams have the same length, the same allowable stress and the same bending moment. The cross sections of the beams are a circle, a square and a rectangle with depth twice the width. Find the ratio of weights of the circular and rectangular beams w.r.t the square beam.

Soln:

Let the dia of circle $= D$.



Let the sides of the square $= a$

Let the width of rectangular section $= b$

and depth $= 2b$.

If there is same bending moments on all beams and the allowable stress to be same, the "section modulus" must be same for all section

$M_1 = f_1 Z_1$

$Z_1 = \frac{M_1}{f_1}$

$Z_2 = \frac{M_2}{f_2}$

$Z_3 = \frac{M_3}{f_3}$

$$Z \text{ for circular section} = \frac{\frac{\pi D^4}{64}}{D/2} = \frac{\pi D^3}{32}$$

$$Z \text{ for square section} = \frac{\frac{a^4}{12}}{a/2} = \frac{a^3}{6}$$

$$Z \text{ for rectangular section} = \frac{bd^3/12}{d/2} = \frac{1}{6} b (2b)^2$$

Here $d = 2b$

$$= \frac{4b^3}{6} = \frac{2b^3}{3}$$

$$\therefore \frac{\pi D^3}{32} = \frac{a^3}{6} = \frac{2}{3} b^3$$

$$\frac{\pi D^3}{32} = \frac{a^3}{6} \qquad \therefore \quad D = \sqrt[3]{\frac{a^3 \, 32}{6\pi}} = 1.193a$$

$$\frac{2}{3} b^3 = \frac{a^3}{6} \qquad \therefore \quad b = \sqrt[3]{\frac{a^3}{4}} = 0.6299a.$$

Ratio of circular to square section:-

$$\therefore \frac{\text{Weight of circular beam}}{\text{Weight of square beam}} = \frac{\left(\frac{\pi d^2}{4} \times l\right) w}{(a^2 \times l)\, w}$$

$$= \frac{\pi (1.193)^2 \cancel{a^2}}{4 \cancel{a^2}} \cdot \frac{\cancel{l}}{\cancel{l}} \cdot \frac{\cancel{w}}{\cancel{w}}$$

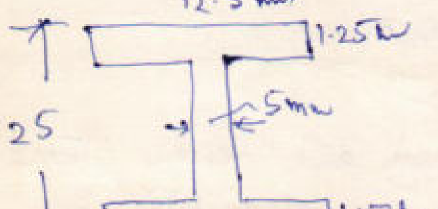
$$= 1.118$$

Ratio of rectangular section to square section.

$$\therefore \frac{(b.d \times l)\, w}{(a^2 \times l)\, w} = \frac{0.6299 \cdot \cancel{a} \times 2 \times 0.6299 \times \cancel{a} \times \cancel{l}}{\cancel{a^2} \times \cancel{l}}$$

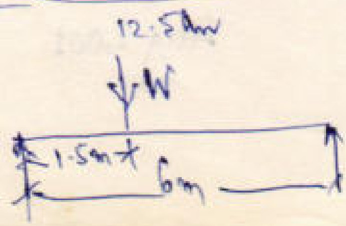
$$= 0.7935$$

A beam of I section shown in fig has an overall depth of 25 cm. The flanges are 12.5 cm wide and 1.25 cm thick and the web is 5 mm thick. The beam rests freely on supports 6 mt apart. Find the maximum load that may be applied at a point 1.5 mt from one support without producing a maximum flange stress greater than 80 N/mm²



$$I = \frac{1}{12}\left[12.5 \times 25^3 - 2 \times \frac{(22.5^3 \times 6)}{}\right]$$

$$= 4885.41 \text{ cm}^4.$$



$$\text{Max BM} = \frac{W \times 1.5 \times 4.5}{6}$$

$$= 1.125 W \text{ N-m}$$

$$= 1.125 W \times 10^3 \text{ N-mm}$$

from formula -

$$M = \frac{I}{y} \cdot f$$

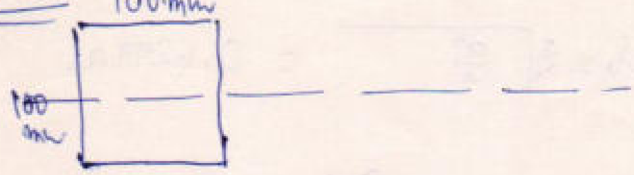
$$W \times 1.125 \times 10^3 = \frac{4885.4 \times 10^4}{12.5 \times 10} \times 80$$

$$= 27792.50 \text{ N}$$

A beam is of square section of side 100 mm. If the permissible stress is 70 N/mm², find moment of resistance of the section. Is there any improvement in moment of resistance, if the section is placed with one of the diagonals vertical.

Soln:

Square sec:



$f = 70\,N/mm^2$

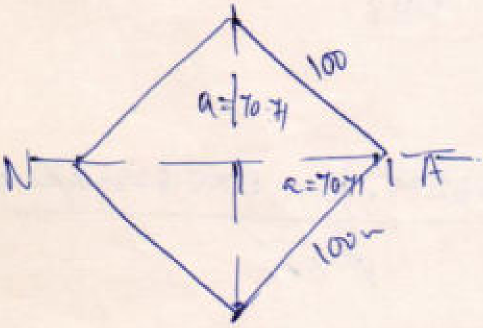
$Z = \frac{1}{6} bd^2 = \frac{1}{6} \times 100 \times 100^2$

$=$

M by square section $= \overset{f \times Z}{70 \times 166666.66}$

$= 11.66 \times 10^6$ N-mm.

Square section with vertical diagonal



$a^2 + a^2 = 100^2$

$2a^2 = 100^2$

$\sqrt{2}\,a = 100$

$a = \frac{100}{\sqrt{2}} = 70.71$

MI of the section about NA

$=$ MI of 2 triangles about their base

$= 2 \frac{bh^3}{12}$

$= \frac{2 \times (2 \times 70.71) \times 70.71^3}{12}$

$= 8333013.6\ mm^4$

$y_{max} = 70.71$

$Z = \frac{8333013}{70.71} = 117848.86\ mm^3$

$M = fZ$

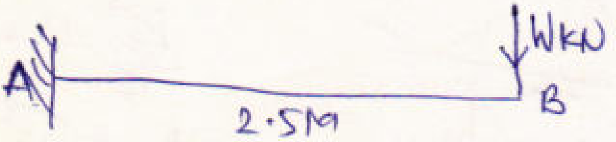
$= 70 \times 117848.86 = 8.24 \times 10^6$ N-mm

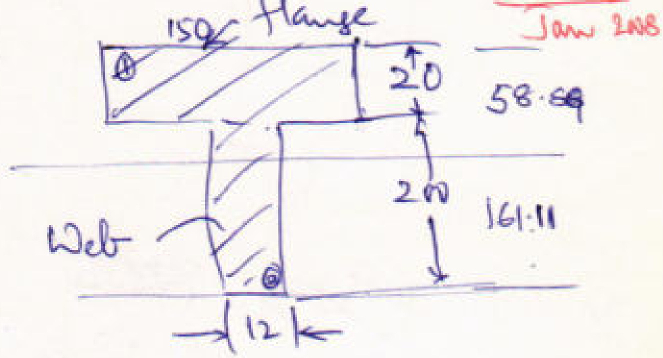
Hence there is no improvement in moment of resistance.

Write down the formulae employed to determine the flexural and shearing stresses in a beam subjected to the action of transverse loads in an axial plane of symmetry and explain all the notations involved. Aug 2001

A beam of T Section has a length of 2.5m and is subjected to a point load as shown in fig. Calculate the compressive bending stress and plot the stress distribution across the cross section of the beam. The maximum tensile stress is limited to 300 MPa. Calculate the value of W

10 marks
Jan 2008





$$\bar{y} = \frac{(150\times20)(210) + (200\times12)(100)}{(150\times20) + (200\times12)}$$

$$\bar{y} = 161.11$$

$$I = \left[\left(\frac{150\times20^3}{12}\right) + (150\times20)(48.89)^2\right] + \left[\left(\frac{200^3\times12}{12}\right) + (200\times12)(61.11)^2\right]$$

$$\boxed{I = 24.23\times10^6\ mm^4}$$

$$\frac{M}{I} = \frac{f}{y}$$

$$f_t = \frac{M}{I}\cdot y_t$$

$$M = \frac{f_t\cdot I}{y_t}$$

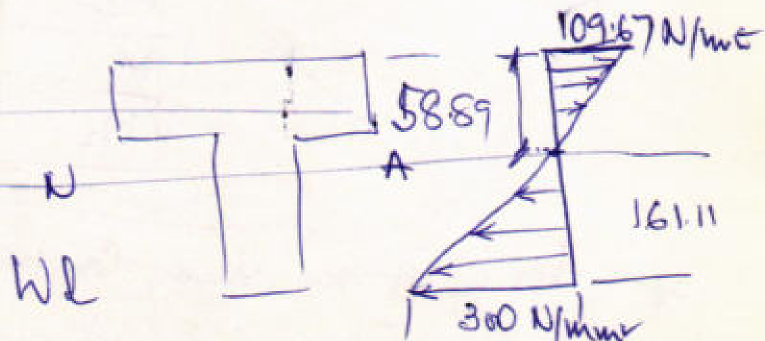
$$M = \frac{300\times24.23\times10^6}{161.11}$$

$$\boxed{M = 45.11\times10^6\ N\text{-}mm}$$

$$f_c = \frac{M}{I}\cdot y_c$$

$$f_c = \frac{45.11\times10^6\times58.89}{24.23\times10^6}$$

$$\boxed{f_c = 109.63\ N/mm^2}$$



For the Cantilever,

Maxm bending moment $= Wl$

$= W\times2.5$

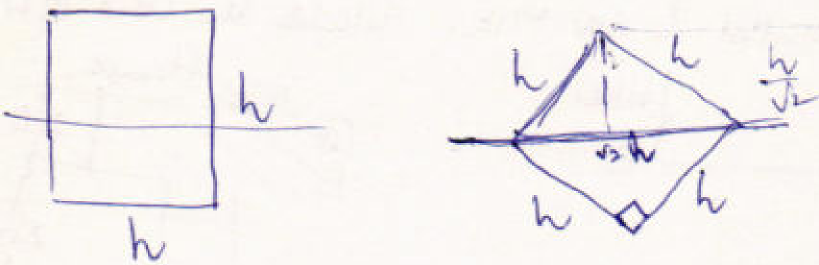
$= 2.5W$ kN-m

$= 2.5W\times10^6$ N-mm

Equating $2.5W\times10^6 = 45.11\times10^6$

$$\boxed{W = 18.04\ kN}$$

* A beam of square cross section is shown in fig. in two different positions. If the maximum stress induced for both positions is same, find the ratio of the bending moments (Bending takes place around the horizontal axis) July 08



$M = fZ$

$f_1 = \frac{M_1}{Z_1}$

$f_1 = M_1\left(\frac{I}{y_{max}}\right)$ $\quad \frac{f}{y} = \frac{M_1}{I_1}$

$= M_1\left(\frac{h^4}{12} \cdot \frac{2}{h}\right)$ $\quad f_1 = \frac{M_1}{I_1} \cdot y_1$

$\boxed{f_1 = M_1\left(\frac{h^3}{6}\right)}$

$M_2 = f_2 Z_2$

$f_2 = M_2 Z_2$

$\boxed{f_2 = M_2 \frac{\sqrt{2}h^3}{12}}$

$M = f_1 Z_1$

$f_1 = f_2$

$M_1\left(\frac{h^3}{6}\right) = M_2 \frac{\sqrt{2}h^3}{12}$

$\frac{M_1}{M_2} = \frac{\sqrt{2}}{12} \frac{h^3}{h^3} \cdot 6$

$\frac{M_1}{M_2} = \frac{1}{\sqrt{2}}$

$\boxed{\frac{M_1}{M_2} = 0.707}$

Correct

$\frac{b h^3}{12}$ $\quad b^2 = h^2 + h^2$

Base $b = \sqrt{2h^2}$

$b = \sqrt{2}h$

height $d = \frac{\sqrt{2}h}{2}$

$d = \frac{h}{\sqrt{2}}$

$Z_2 = \frac{I_2}{y_{max}}$

$I = \frac{(\sqrt{2}h)\left(\frac{h}{\sqrt{2}}\right)^3}{12}$

$I = \frac{h^4}{24}$

$Z_2 = \frac{\frac{h^4}{12}}{\frac{h}{\sqrt{2}}}$

$\boxed{Z_2 = \frac{\sqrt{2}}{12} h^3}$

$= \frac{\sqrt{2}}{12} h^3 = \frac{h^3}{6\sqrt{2}}$

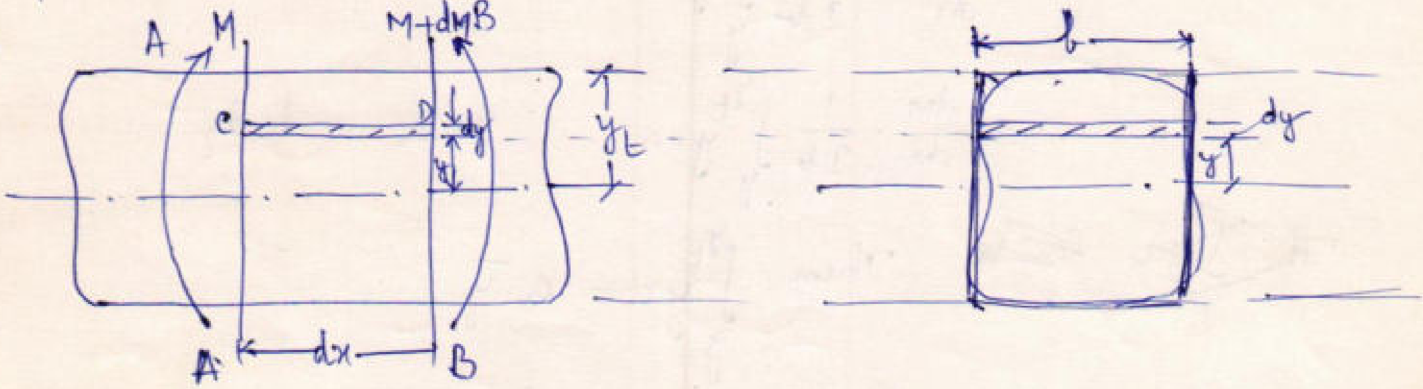
$f = \frac{M}{I} \cdot y$

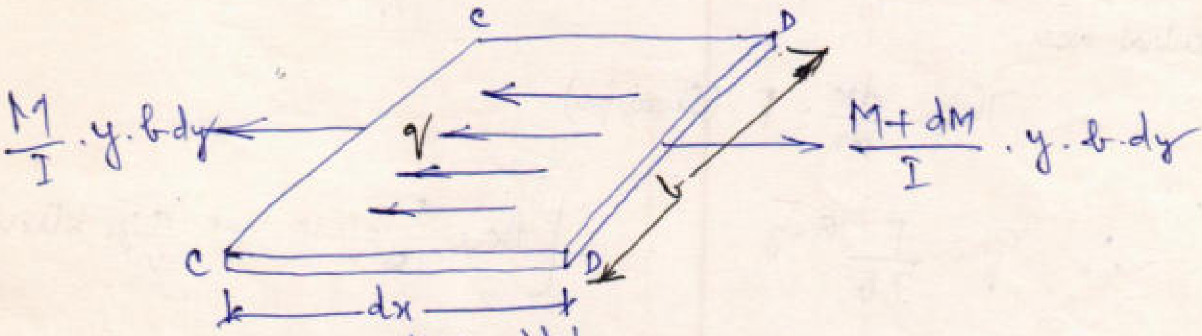
* For a given stress, compare the moment of resistance of a beam of square cross section when placed i) with its two sides horizontal ii) with its diagonal vertical.

Jan 08
Conaug

Shearing Stresses in Beams:

Consider an elemental length of beam between the sections A-A and B-B seperated by a distance 'dx'. Let the moments acting at A-A and B-B be M and M + dM.





Let CD be a fibre, of thickness 'dy' at a distance y from neutral axis.

Then bending stress 'f' on left hand side $= \frac{M}{I}.y$.

$\therefore$ The force on the left hand side $= \left(\frac{M}{I}.y\right)(b.dy)$

lly the force on the right hand side $= \left(\frac{M+dM}{I}.y\right)(b.dy)$

$\therefore$ Unbalanced force towards right in element

$$= \frac{M+dM}{I}.y\ bdy - \frac{M}{I}.y.bdy.$$

$$= \frac{dM}{I}.y.bdy.$$

There are no of such elements above section CD. Hence unbalanced horizontal force above section CD $= \int_{y}^{y_t} \frac{dM}{I}.y\ b\,dy$

This horizontal force is resisted by shear stress acting horizontally on plane at CD. Let 'q' be the intensity of shear stress.

Then Equating shear force to unbalanced horizontal force.

$$q.b.dx = \int_y^{y_t} \frac{dM}{I}.y.\,b\,dy$$

$$q = \frac{dM}{dx}\cdot\frac{1}{Ib}\int_y^{y_t} y.b.dy$$

$$= \frac{dM}{dx}\cdot\frac{1}{Ib}\int_y^{y_t} y.a \qquad \text{where } a = b.dy$$

~~The~~ $\cancel{\int_y^{y_t} ya}$ ~~can be~~ Then $\int_y^{y_t} y.a = a.\bar{y}$

where $a\bar{y}$ is the moment of the area above the section under consideration about neutral axis.

Then $\frac{dM}{dx} = F$ (Shear force)

$$\therefore\ q = \frac{F}{Ib}a.\bar{y} \qquad \left[\text{Here } \frac{F}{I} = \text{Const and } \frac{a.\bar{y}}{b} \text{ is the Variable}\right]$$

$q = \frac{F}{Ib}\cdot a.\bar{y}$ where q = Shear stress

F = Shear force

I = Moment of inertia of the entire section

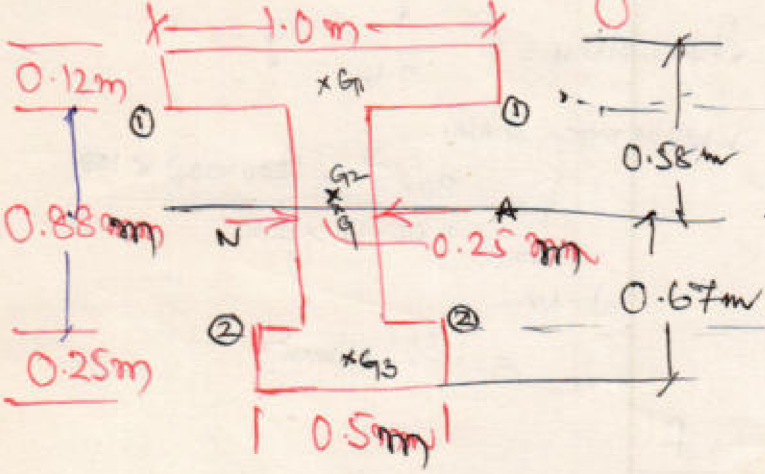
b = Breadth of section at the distance considered from NA

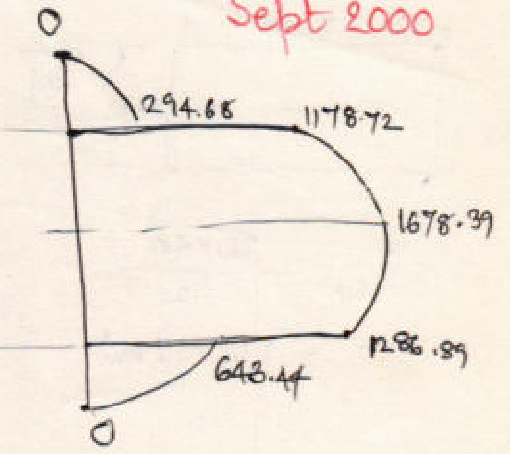
a = Area of the section above the distance considered from NA

$\bar{y}$ = Centroidal distance of the section above the considered distance from NA

A Cross section of a beam is as shown in fig. The Shear force on the section is 400kN. Estimate the shear stresses at various points and plot the shear stress distribution diagram. 9

Sept 2000





Soln:

Location of N Axis:

$$\bar{Y} = \frac{0.5\times0.25\times0.125 + 0.25\times0.88\times0.69 + 0.12\times1.0\times1.19}{0.5\times0.25 + 0.88\times0.25 + 0.12\times1}$$

$$= \frac{0.310225}{0.465} = 0.67 \text{ mts}$$

$$MI \text{ about NA} = I_{NA} = \left[\frac{1\times0.12^3}{12} + 0.12\times1\times0.52^2\right] + \left[\frac{0.25\times0.88^3}{12} + 0.88\times0.25\times0.02^2\right] + \left[\frac{0.5\times0.25^3}{12} + 0.25\times0.5\times0.545^2\right]$$

$$= 0.0847 \text{ m}^4$$

Shear Stress Values:

Shear stress at extreme fibre = 0

Shear stress at 1-1 (Just above 1-1) (bottom of top flange) $= \frac{F}{Ib}\cdot a\bar{y} = \frac{400\times}{0.0847\times1}\times0.12\times1\times0.52$

$= 294.68$ kN/m²

Shear stress at 11 (Just below 1-1) (top of web) $= \frac{400}{0.0847\times0.25}\times0.12\times1\times0.52 = 1178.72$ kN/m²

Shear stress at N-A $= \frac{400}{0.0847\times0.25}\times[0.88\times0.46\times0.23 + 1\times0.12\times0.52]$

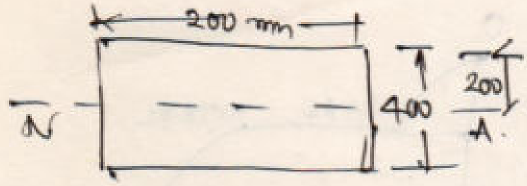
$= 1678.39$ kN/m²

Shear stress Just below 2-2 (top of lower flange) $= \frac{400}{0.0847\times0.5}[0.5\times0.25\times0.545] = 643.44$ kN/m²

Shear stress just above 2-2 (bottom of web) $= \frac{400}{0.0847\times0.25}[0.5\times0.25\times0.545] = 1286.89$ kN/m²

A rectangular beam 200mm wide and 400mm deep is subjected to a maxm shear force of 80kN. Find (i) Average shear stress (ii) Maximum shear stress

Sept 1999



Shear stress $= q = \frac{F}{Ib} \cdot a\bar{y}$.

q will be maxm at NA.

$$q_{max} = \frac{80\times10^3 \times (200\times200)\times100}{1.06\times10^9\times200}$$

at NA.

$$= 1.509\ N/mm^2.$$

$$I_{NA} = \frac{200\times400^3}{12}$$

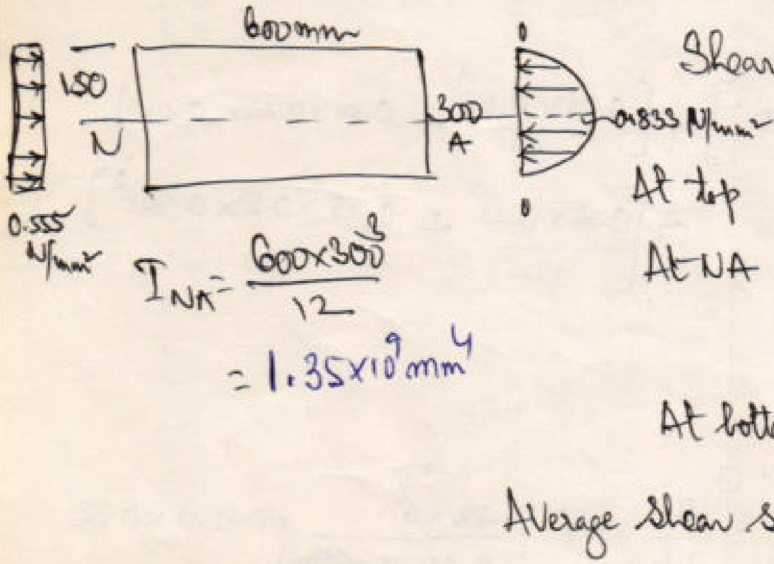
$$= 1.06\times10^9\ mm^4.$$

Average shear stress $= q_{Average} = \frac{F}{A}$

$$= \frac{80\times10^3}{200\times400} = 1\ N/mm^2$$

A rectangular section of 600mm wide and 300mm thick is subjected to vertical shear force of 100kN. Calculate the maxm and average shear stresses. Sketch the shear stress distribution in the section.

March 2000



Shear stress $= q = \frac{F}{Ib} \cdot a\bar{y}$

At top $\quad q = 0$

At NA $\quad q_{max} = \frac{100\times10^3 \times 600\times150\times75}{1.35\times10^9\times600}$

$$= 0.833\ N/mm^2.$$

At bottom $\quad q = 0$

$$I_{NA} = \frac{600\times300^3}{12}$$

$$= 1.35\times10^9\ mm^4$$

Average shear stress $= q_{av} = \frac{F}{A}$

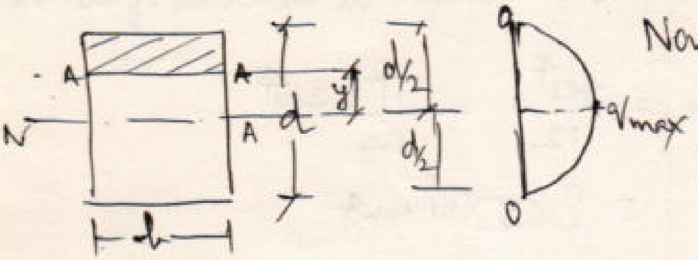
$$= \frac{100\times10^3}{600\times300} = 0.555\ N/mm^2.$$

S.T for a rectangular section subjected to shear force, the maximum shear stress is 1.5 times the average shear stress.

March 2000

Consider a rectangular section of width 'b' and depth 'd' subjected to SF 'F'.

Let A-A be the section at a distance 'y' from N.A.



Now $q = \frac{F}{I.b} . a\bar{y}$.

Here $I = \frac{bd^3}{12}$, $a = b(d/2 - y)$

$b = b$, $\bar{y} = (\frac{d}{2} - y)/2 + y$

$= \frac{d}{4} - y/2 + y$

$= \frac{d}{4} + y/2$

$= \frac{1}{2}(d/2 + y)$

Now
$$q = \frac{F \; \cancel{b}(d/2 - y)(d/2 + y)/2}{\frac{bd^3}{12} \times \cancel{b}}$$

$$= \frac{6F}{bd^3}\left(\frac{d^2}{4} - y^2\right)$$

[Shear stress variation is parabolic)

when $y = d/2$ $\quad q = 0$

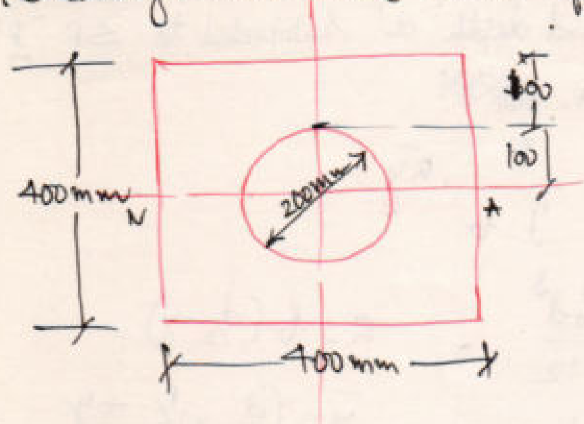
$y = 0$ $\quad q_{max} = \frac{6F}{4d^2} \frac{3}{2} \frac{F}{bd} = 1.5 \frac{F}{bd}$ —— (1)

$y = -d/2$ $\quad q = 0$

Here $q_{average} = \frac{F}{b.d}$

Hence $\boxed{q_{max} = 1.5 \; q_{average}}$

A wooden section of 400mm × 400mm has a central bore of 200mm dia as shown in fig. If it is used as a beam to resist a shear force of 20kN, find the shearing stress at the crown of the bore and at neutral axis. Sept 99



Soln:

a) To find shear stress at the crown of the bore.

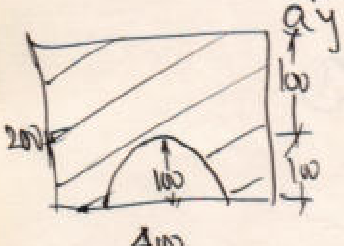
$$I = \frac{400^4}{12} - \frac{\pi \times 200^4}{64}$$
$$= 2.05 \times 10^9 \text{ mm}^4$$

Shear stress at crown of the circle $= \frac{F}{Ib} \cdot a\bar{y}$

$$= \frac{20 \times 10^3 \times 400 \times 100 \times 150}{2.05 \times 10^9 \times 400}$$

$$\boxed{q_{crown} = 0.146 \text{ N/mm}^2}$$

b) To find shear stress at N. Axis.



$a\bar{y} = a\bar{y}$ of rectangular portion $- a\bar{y}$ of semicircle

$$= 200 \times 400 \times 100 - \frac{\pi \times 200^2}{8} \times \frac{4 \times 100}{3\pi}$$

[CG of semicircle is at $\frac{4R}{3\pi}$ from base]

$$= 7333333.9 \text{ mm}^3$$

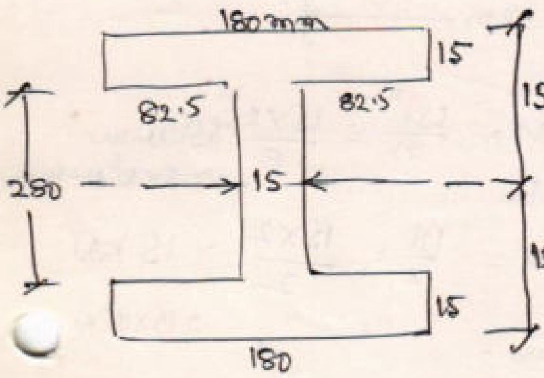
$$q_{N\text{-}Axis} = \frac{20 \times 10^3}{2.05 \times 10^9 \times 400} \times 7333333.9$$

$$\boxed{q_{NA} = 0.1789 \text{ N/mm}^2}$$

Calculate the Shear stress and bending stress values for an I section Consisting of 180mm X 15mm flanges and web of 280mm X 15mm subjected to BM of 120kN-m and SF of 60kN. Sketches the bending and shear stress distribution along the depth of the section

March 2001

Soln:



Symmetric section

$$I = \frac{1}{12}\left[180 \times 310^3 - 2 \times 82.5 \times 280^3\right]$$

$$= 1.45 \times 10^8 \ mm^4$$

To calculate Bending stress:-

$$M = \frac{I}{y} \cdot f$$

$$f = \frac{M \cdot y}{I}$$

$$f = \frac{120 \times 10^3 \times 10^3}{1.45 \times 10^8} \times y$$

$$f = 0.8274 \times y$$

when $y = 155$ $\quad f = 128.\cancel{280}\ N/mm^2$

$y = 140$ $\quad f = 115.84\ N/m^2$

$y = 0$ $\quad f = 0$

To Calculate Shear stress:

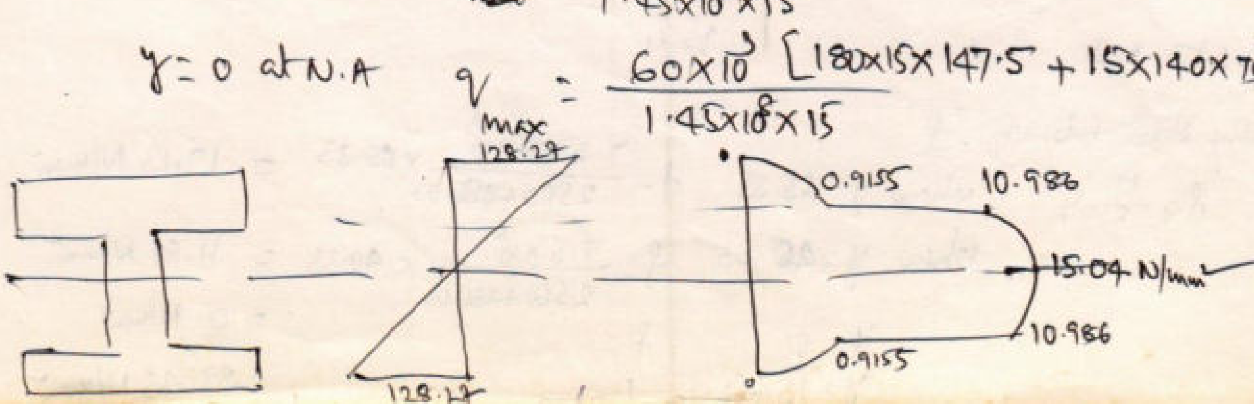
$$q = \frac{F}{Ib} \cdot a\bar{y}$$

when $y = 155$ $\quad q = 0$

$y = 140$ at bottom of flange $\quad q = \frac{60 \times 10^3}{1.45 \times 10^8 \times 180} \times 180 \times 15 \times 147.5 = 0.9155\ N/mm^2$ $\cancel{2.30 \times 10\ N/mm^2}$

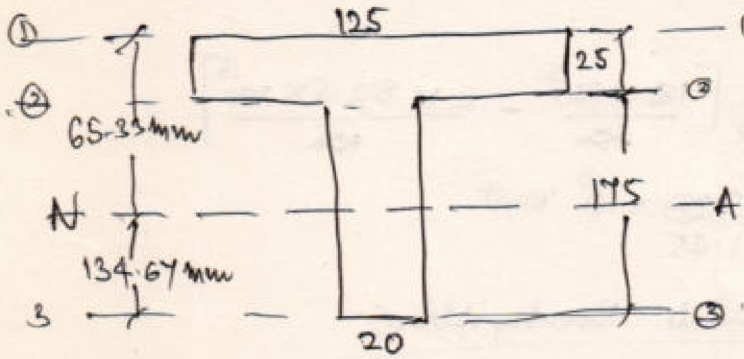
$y = 140$ (at Web top) $\quad q = \frac{60 \times 10^3 \times 180 \times 15 \times 147.5}{1.45 \times 10^8 \times 15} = 10.986\ N/mm^2$

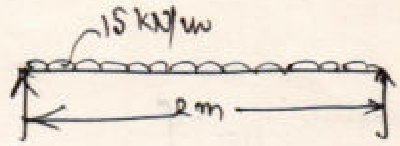
$y = 0$ at N.A $\quad q_{max} = \frac{60 \times 10^3 \left[180 \times 15 \times 147.5 + 15 \times 140 \times 70\right]}{1.45 \times 10^8 \times 15} = 15.04\ N/mm^2$



A Simply supported T beam has a span of 2m. The flange is 125mm X 25mm and Web is 175 X 20mm. The beam carries a UDL of 15 kN/m throughout. Calculate the bending stress and shear stress values for maximum values of BM and SF. Draw neat sketches showing bending stress and shear stress distribution diagrams across the section

Feb 2002





Max BM $= \frac{Wl^2}{8} = \frac{15\times 2^2}{8} = 7.5\,\text{kN-m}$
at centre $= 7.5\times 10^6\,\text{N-mm}$

Max SF $= \frac{Wl}{2} = \frac{15\times 2}{2} = 15\,\text{kN}$
$= 15\times 10^3\,\text{N}$

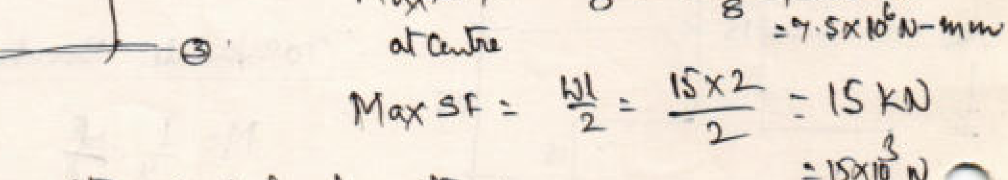
To Calculate ~~BM~~ ~~on~~ Shear stress and bending stress :-

~~I~~ Centroid $\bar{y} = \frac{125\times 25\times 12.5 + 175\times 20\times 112.5}{(125\times 25 + 20\times 175)}$

$= \frac{432812.5}{6625} = 65.33$ mm from top.

$I_{NA} = \left[\frac{125\times 25^3}{12} + 25\times 125\times 52.83^2\right] + \left[\frac{175^3\times 20}{12} + 20\times 175\times 47.17^2\right]$

$= 25604486.05\ \text{mm}^4$



Shear stress intensity:

At top 1-1 $q = 0.$

At 2-2 (Bottom of top flange) $q = \frac{15\times 10^3 \times (125\times 25)\times 52.83}{25604486.05\times 125}$ $= 0.77$ N/mm²

=

at 2-2 top of web $q =$ $= 4.837$ N/mm²

At N-A $q = \frac{15\times 10^3\times (125\times 25\times 52.83 + 40.33\times 20\times 20.165)}{25604486.05\times 20} = 5.31$ N/mm²

At 3-3 $q = 0$

Bending stress intensity : f

$f = \frac{M}{I}\cdot y$ when $y = 65.33$ $f = \frac{7.5\times 10^3\times 10^3}{25604486.05}\times 65.33 = 19.13$ N/mm²

when $y = 40.33$ $f = \frac{7.5\times 10^6}{25604486.05}\times 40.33 = 11.81$ N/mm²

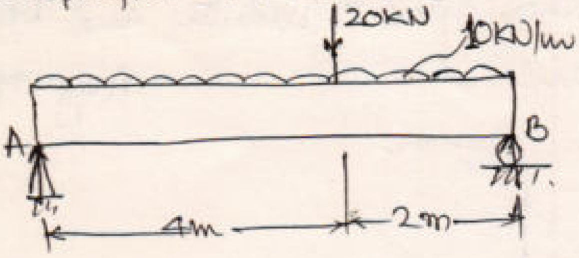
$y = 0$ $f =$ $= 0$ N/mm²

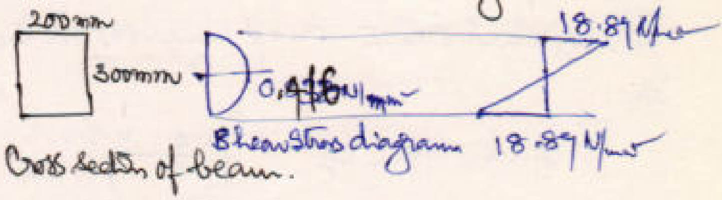
$y = 134.67$ mm $f =$ $= 39.43$ N/mm²

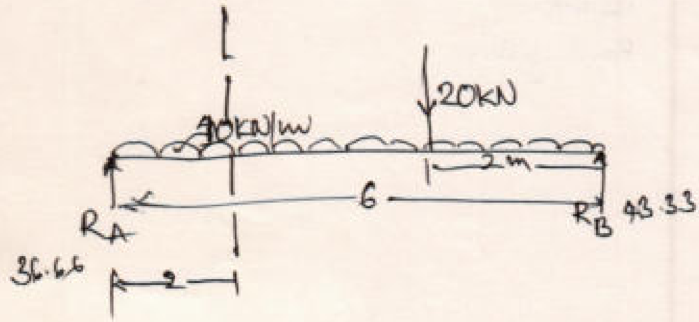
A Simply supported rectangular beam is loaded as shown in fig 8. Determine the maximum flexural stresses and maximum traverse shearing stress at a cross-section located 2m from the left support. Sketch the flexural and shearing stress distributions at the specified cross-section.

August 2001

Soln:







To find SF and BM at 2mts from left support

Taking $\Sigma M_A = 0$

$R_B \times 6 = 10 \times 6 \times 3 + 20 \times 4$

$R_B = \frac{260}{6} = 43.33$ KN.

$\Sigma V = 0 \quad R_A + R_B = 20 + 10 \times 6$

$R_A = 80 - 43.33 = 36.66$ KN

SF at 2m from A = $+36.66 - 10 \times 2 = +16.66$ KN.

BM at 2m from A = $+36.66 \times 2 - 10 \times 2 \times 1 = 56.66$ KN-m

$$I = \frac{200 \times 300^3}{12} = 450000000 \text{ mm}^4$$

Shear stress Calculations: $\quad q = \frac{F}{Ib} \cdot a \cdot \bar{y}$

q at extreme top fibre = 0

q at N.A $\quad q = \frac{16.66 \times 10^3 \times (200 \times 150) \times 75}{450000000 \times 200} = 0.416$ N/mm²

q at bottom fibre = 0

Bending Stress Calculations: $\quad f = \frac{M}{I} \cdot y$

f at extreme top fibre $\quad f = \frac{56.66 \times 10^3 \times 10^3 \times 150}{450000000} = 18.89$ N/mm²

f at → Netural-axis $\quad f = 0 \quad = 0$

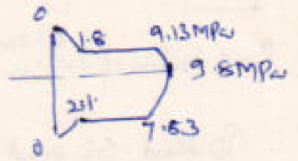
f at bottom fibre $\quad = \quad = 18.89$ N/mm²

Calculate shear stress and bending stress values for an unsymmetrical I section with top flange of 200 x 30 mm, bottom flange of 150 mm x 24 mm, web thickness of 40 mm and overall height of section 140 mm, when it carries a SF max of 42 kN and BM max of 69 kN-m. Draw neat sketches of shear stress distribution and bending stress distribution diagrams across the section. Aug 1999.

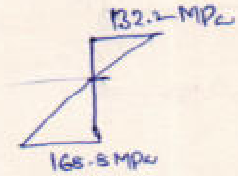
Soln:

$\bar{Y} = 61.5$ mm from top.

$I = 32.1 \times 10^6$ mm^4



Shear stress.

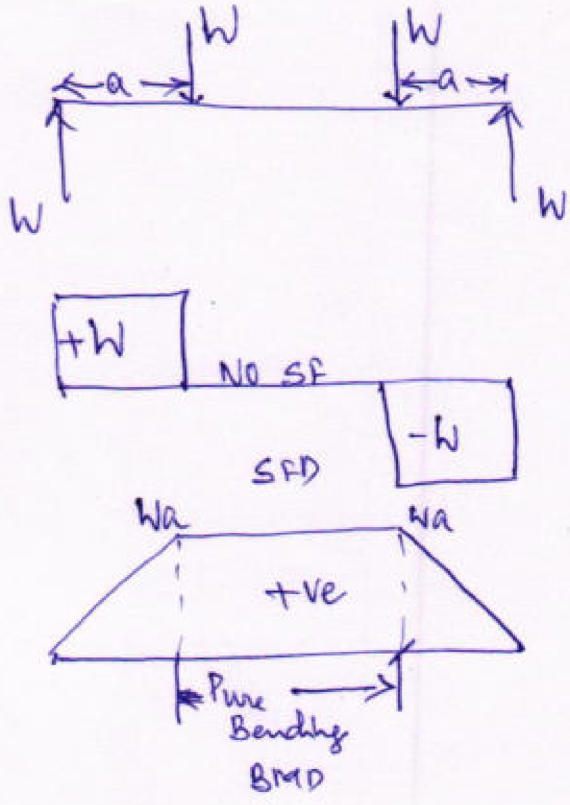


Bending stress.

Pure bending & Ordinary bending.

Pure bending is the bending caused in a beam due to only bending moment without shear force associated with it.

Ex:



Ordinary bending is the bending caused in a beam due to bending moment associated with shear force. Most of all the beams are subjected to ordinary bending.

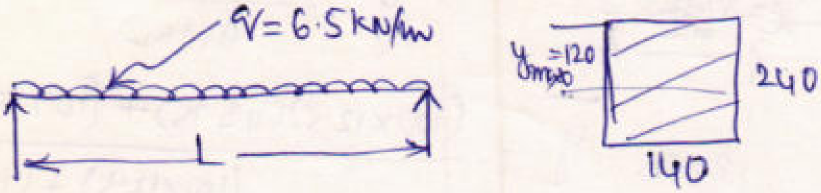
Modulus of Rupture:

When a beam is loaded upto failure, the stresses in a beam section can not be calculated using flexure formula. However, bending stress is some times calculated even at failure using flexure formula to compare strengths of beams of different materials & sections. The bending stress calculated at failure or rupture using bending equation is known as Modulus of rupture. The stresses calculated are the maximum values in the section due to failure load.

1. Determine the maximum allowable span length 'L' for a simple beam as shown in fig. The beam is of rectangular cross section 140mm × 240mm subjected to uniformly distributed load $q = 6.5$ kN/m and the allowable bending stress is 8.2 MPa

8 marks

June-July 2009



Soln

$$I = \frac{140 \times 240^3}{12} = 161.28 \times 10^6 \text{ mm}^4$$

$$f_{allowable} = 8.2 \text{ MPa} = 8.2 \text{ N/mm}^2 \qquad M = fZ$$

$$0.8125 \times (l^2) \times 10^6 = 8.2 \times 1.34 \times 10^6$$

$$\boxed{l = 3.67 \text{ m}}$$

$$Z = \frac{161.28 \times 10^6}{120}$$

$$= 1.34 \times 10^6 \text{ mm}^3$$

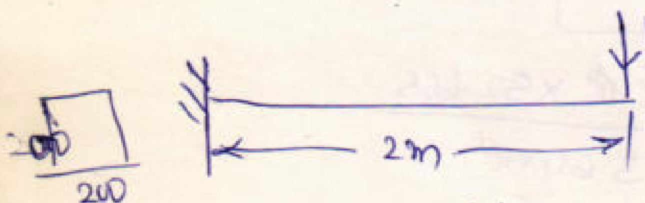
$$M = \frac{wl^2}{8} = \frac{6.5 \times l^2}{8}$$

$$= 0.8125 l^2 \text{ kN-m} \quad \& \quad 0.8125\, l^2 \times 10^6 \text{ N-mm}$$

$$= 0.8125 l^2 \text{ kN-m}$$

2. A cantilever of square section 200mm × 200mm, 2m long just fails in flexure when a load of 12kN is placed at its free end. A beam of same material and having a rectangular cross section 150mm wide and 300mm deep is simply supported over a span of 3m. Calculate the minimum central concentrated load required to break the beam

July 2009 | 10 marks



$$\text{Max BM} = WL$$
$$= 12 \times 2$$
$$= 24 \text{ kN-m}$$
$$= 24 \times 10^6 \text{ N-mm}$$

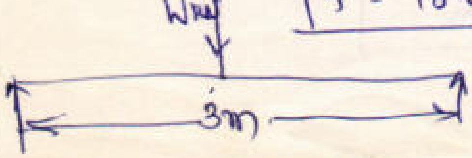
$$I = \frac{200^4}{12}$$

$$I = 1.33 \times 10^8 \text{ mm}^4$$

$$Z = \frac{1.33 \times 10^8}{100} = 1.33 \times 10^6 \text{ mm}^3$$

$$f = \frac{M}{Z} = \frac{24 \times 10^6}{1.33 \times 10^6}$$

$$\boxed{f = 18.04 \text{ N/mm}^2}$$



$$I = \frac{150 \times 300^3}{12} \qquad Z = \frac{337.5 \times 10^6}{150}$$

$$\boxed{I = 337.5 \times 10^6 \text{ mm}^4} \quad \boxed{Z = 2.25 \times 10^6}$$

$$\text{Max BM} = \frac{WL}{4} = \frac{W \times 3}{4} = 0.75W \text{ kN-m}$$

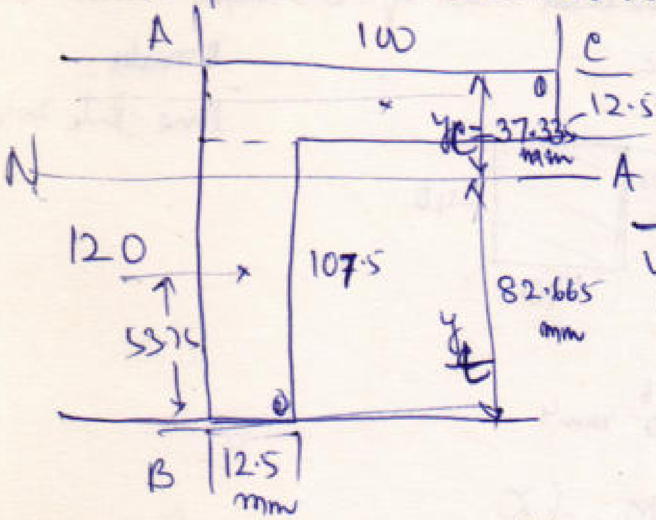
$$M = fZ$$

$$0.75W \times 10^6 = 18.04 \times 2.25 \times 10^6$$

$$\boxed{W = 54.12 \text{ kN}}$$

c An unequal angle section shown in fig is used as a simply supported bean over a span of 2m and uniformly distributed load of 10 kN/m inclusive of its own weight. Determine the maximum tensile and compressive stresses in the section

July 2008



$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2}$$

$$\bar{y} = \frac{(100 \times 12.5)(113.75) + (107.5 \times 12.5)(53.75)}{(100 \times 12.5) + (107.5 \times 12.5)}$$

$$= \frac{2.14 \times 10^5}{2593.75} = \cancel{156.50}\ 82.665\text{ mm}$$

$$y_c = 37.335 \qquad y_t = 82.665$$

$$I = \left[\left(\frac{100 \times 12.5^3}{12}\right) + \left((100 \times 12.5)(31.085^2)\right)\right] + \left[\left(\frac{12.5 \times 107.5^3}{12}\right) + (12.5 \times 107.5)(28.915)^2\right]$$

$$I = 3.64 \times 10^6\ \text{mm}^4$$

$$\text{Maxm BM} = \frac{WL^2}{8} = \frac{10 \times 2^2}{8} = 5\ \text{kN-m}$$

$$\text{Max BM} = 5 \times 10^6\ \text{N-mm}$$

$$\frac{M}{I} = \frac{f}{y}$$

$$f_c = \frac{M \cdot y_c}{I} = \frac{5 \times 10^6 \times 37.335}{3.64 \times 10^6}$$

Bending Compressive Stress

$$\boxed{f_c = 51.28\ \text{N/mm}^2}$$

$$f_t = \frac{M}{I} \cdot y_t = \frac{5 \times 10^6 \times 82.665}{3.64 \times 10^6}$$

Bending tensile Stress

$$\boxed{f_t = 113.55\ \text{N/mm}^2}$$

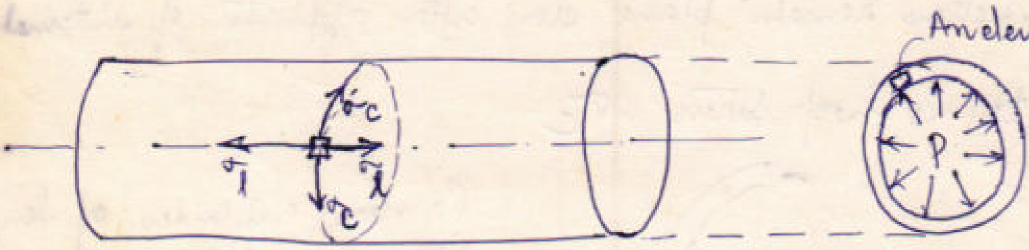
Thin AND THICK CYLILDERS

The Containers which are in the form of Cylinders and Spheres are used to store fluids under pressures. The Common examples are available in machineries and Chemical plants. They are Water tanks, steam engine cylinder, Compressed air storage tanks, steam boilers, ~~etc~~ pressure Cookers, etc. Cylinders are classified as thick and thin Cylinders. These are also known as pressure Vessels or Shells.

When the thickness of the cylinder is less than 1/10th to 1/15th of its radius, the Cylinder is Classified as thin cylinder.

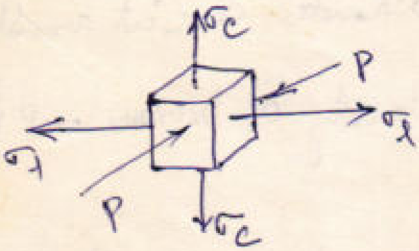
In thin cylinders, the stress in radial direction is neglected.

The hoop stress and longitudinal stress are uniformly distributed over the thickness



Thin cylinders
Tyres.
Gas storage tanks.

Thick cylinders
Steam boilers, etc

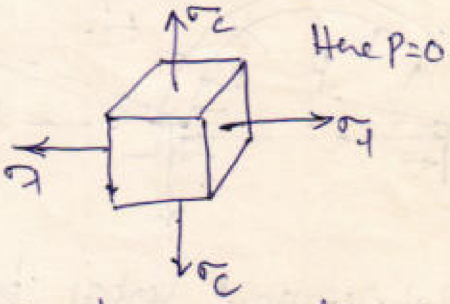


An element is normally subjected to pressure as shown in fig

σ_l = longitudinal stress

σ_c = Circumferential stress or hoopstress

P = Radial stress.



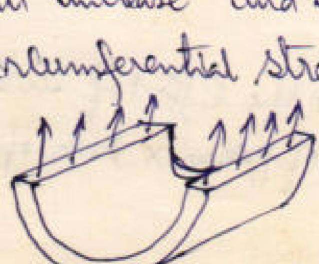
An element in a thin Cylinder subjected to σ_l and σ_c

σ_l = longitudinal stress

σ_c = Circumferential stress or hoop stress.

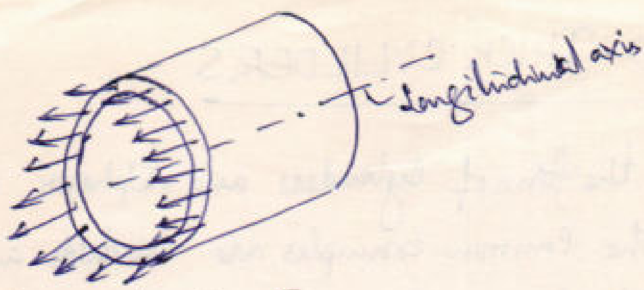
Stresses in the Walls of thin Cylinders:

Fluids and gases under pressure ~~will~~ exert forces in all directions. As a Consequence the Cylinder will always show a tendency to inflate (Expand). Then the diameter will increase and the Circumference will become stretched. This ~~gives~~ Creates Circumferential stress or hoop stress (Tensile).



Hoop stress.

If the ends of the Cylinder are closed, then the pressure at the ends will produce stresses in walls in a direction parallel to longitudinal axis of the Cylinder. These stresses are Called longitudinal stresses



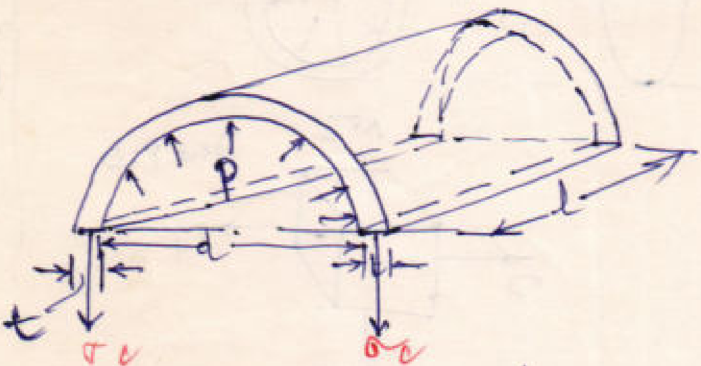
Longitudinal stress:

Expression for Circumferential stress (σ_c) or Hoop stress and longitudinal stress:

Following assumptions are made:

1. The material of the cylinder is homogeneous and isotropic
2. The radial stress is neglected.
3. The stresses are uniformly distributed.
4. The plane sections remain plane even after application of internal pressure

a. Circumferential or hoop stress σ_c



Consider a cylinder of length 'l' diameter 'd' cut as shown in fig.

Intensity of pressure = $P \; N/mm^2$

Total resultant vertical pressure = Intensity of pressure × Projected area of curved surface in horizontal plane

Resultant vertical pressure = $P \times d \times l$.

This force is resisted by circumferential stress or hoop stress σ_c which acts on an area $= 2\sigma_c t l$

Then

$$Pdl = \sigma_c 2tl$$

$$\sigma_c = \frac{Pd\cancel{l}}{2t\cancel{l}} = \frac{Pd}{2t}$$

b) Longitudinal stress: (σ_l)

The bursting force due to the effect of internal pressure on the end = Intensity of pressure × Area of application

$$= P \times \frac{\pi d^2}{4}$$

This is resisted by longitudinal stress σ_l acting over an area πdt

ie. $p\frac{\pi d^2}{4} = \sigma_l \pi dt$

$$\sigma_l = \frac{p \cdot \pi d^2}{4\pi dt} = \frac{Pd}{4t}$$

Thus Hoop stress is twice that of longitudinal stress.

The maximum shearing stress $= \frac{\sigma_c - \sigma_l}{2}$

$$= \frac{1}{2}\left[\frac{Pd}{2t} - \frac{Pd}{4t}\right]$$

$$\boxed{q_{max} = \frac{Pd}{8t}}$$

S.T in the case of thin cylindrical shell subjected to internal fluid pressure, the volumetric strain is equal to the sum of twice the hoop strain and longitudinal strain.

Soln:

The two principal stresses which are acting at any point in a thin cylindrical shell are

i) Circumferential stress $\sigma_c = \frac{Pd}{2t}$
ii) Longitudinal stress $\sigma_l = \frac{Pd}{4t}$

} These two stresses are acting at right angles to each other.

Let e_c be the circumferential strain,
e_l be the longitudinal strain,
$\frac{1}{m}$ be the poisson's ratio.

Circumferential strain $= e_c = \frac{\sigma_c}{E} - \frac{1}{m}\left(\frac{\sigma_l}{E}\right) = \frac{1}{E}\left[\sigma_c - \frac{1}{m}\sigma_l\right] = \frac{1}{E}\left[\frac{Pd}{2t} - \frac{1}{m}\frac{Pd}{4t}\right]$

Since circumference is proportional to dia, the above strain can be written as

diametral strain $= \frac{\delta d}{d} = e_c$

$$\therefore \frac{\delta d}{d} = \frac{\sigma_c}{E} - \frac{1}{m}\frac{\sigma_l}{E} = \frac{Pd}{2tE}\left[1 - \frac{1}{2m}\right]$$

Longitudinal strain $= e_l = \frac{\sigma_l}{E} - \frac{1}{m}\frac{\sigma_c}{E} = \frac{1}{E}\left[\sigma_l - \frac{\sigma_c}{m}\right] = \frac{1}{E}\left[\frac{Pd}{4t} - \frac{Pd}{2tm}\right]$

$$= \frac{Pd}{2tE}\left[\frac{1}{2} - \frac{1}{m}\right]$$

If V is the volume of thin cylindrical shell, then

$$V = \frac{\pi d^2}{4} L.$$

$$\log V = \log\frac{\pi}{4} + 2\log d + \log L$$

Taking differential on either sides,

$$\frac{dv}{V} = 2\frac{\delta d}{d} + \frac{\delta l}{l}$$

$$\boxed{e_V = 2e_c + e_l}$$

Volumetric strain is equal to twice the hoop strain and one times the longitudinal strain

$$e_V = 2\left[\frac{Pd}{2tE}\left[1-\frac{1}{2m}\right]\right] + \frac{Pd}{2tE}\left[\frac{1}{2}-\frac{1}{m}\right]$$

$$= \frac{Pd}{2tE}\left[2-\frac{1}{m}+\frac{1}{2}-\frac{1}{m}\right] = \frac{Pd}{2tE}\left[\frac{5}{2}-\frac{2}{m}\right]$$

$$\text{or} = \frac{Pd}{4tE}\left[5-\frac{4}{m}\right]$$

A cylindrical shell is 3m long and is having 1m internal diameter and 15mm thickness. Calculate the maximum intensity of shear stress induced and also the changes in dimensions of the shell if it is subjected to an internal fluid pressure of 1.5 N/mm². Take $E = 2\times10^5$ N/mm² and $\mu = 0.3$

Soln. $L = 3m = 3000$ mm

$d = 1m = 1000$ mm

$t = 15$ mm

Hoop stress. $\sigma_c = \frac{Pd}{2t} = \frac{1.5\times1000}{2\times15} = 50$ N/mm²

Longitudinal stress $\sigma_l = \frac{Pd}{4t} = \frac{1.5\times1000}{4\times15} = 25$ N/mm²

$$q_{max} = \frac{f_c - f_l}{2} = \frac{50-25}{2} = 12.5 \text{ N/mm}^2.$$

Now diametral or diametrical strain or circumferential strain $= \frac{\delta d}{d} = e_c = \frac{\sigma_c}{E} - \frac{1}{m}\left[\frac{\sigma_l}{E}\right]$

$$= \frac{1}{E}\left[\sigma_c - \mu\sigma_l\right]$$

$$= \frac{1}{2\times10^5}\left[50 - 0.3\times25\right]$$

$$\boxed{e_c = 2.125\times10^{-4}}$$

Change in diameter $= \delta d = 2.125\times10^{-4}\times d = 2.125\times10^{-4}\times1000$

$$= 0.2125 \text{ mm}$$

Longitudinal strain $= e_1 = \frac{\sigma_1}{E} - \frac{1}{m}\frac{\sigma_c}{E}$

$$= \frac{1}{E}\left[\sigma_1 - \mu \sigma_c\right]$$

$$= \frac{1}{2\times10^5}\left[25 - 0.3\times50\right]$$

$$\boxed{e_l = \frac{\delta l}{l} = 5\times10^{-5}}$$

$$\boxed{\delta l = 5\times10^{-5}\times3000 = 0.15\,mm}$$

Change in volume $= \frac{\delta v}{v} = 2e_c + \cancel{e}e_1$

$$= 2\times2.125\times10^{-4} + 5\times10^{-5}$$

$$= 4.75\times10^{-4}$$

$$\delta v = 4.75\times10^{-4}\times v = 4.75\times10^{-4}\times\frac{\pi}{4}\times1000^2\times3000$$

$$= 1119192.4\,mm^3.$$

A thin cylindrical shell, 2m long has 200mm dia and thickness of metal 10mm. It is filled completely with a fluid at atmospheric pressure. If an additional 25000mm³ fluid is pumped in, find the hoop stress developed and pressure developed. Find also the changes in dia and length. Take $E = 2\times10^5\,N/mm^2$. and $\mu = 0.3$.

Soln: Let the pressure developed be 'p'.

Then $\sigma_c = \frac{P.d}{2t} = \frac{P\times200}{2\times10} = 10P$

$$\sigma_1 = \frac{P.d}{4t} = \frac{P\times200}{4\times10} = 5P.$$

Diametrical strain = Circumferential strain.

$$e_c = \frac{\delta d}{d} = \frac{\sigma_c}{E} - \mu\frac{\sigma_1}{E}$$

$$= \frac{1}{E}\left[10P - 0.3\times5P\right]$$

$$e_c = 8.5P/E$$

Longitudinal strain $e_l = \frac{\delta l}{l} = \frac{1}{E}\left[\sigma_1 - \mu\sigma_c\right]$

$$= \frac{1}{E}\left[5P - 0.3\times10P\right]$$

$$= \frac{2P}{E}$$

Volumetric strain $= 2e_c + e_l$

$$= \frac{2\times 8.5P}{E} + \frac{2P}{E}.$$

$$\frac{\delta v}{v} = \frac{19P}{E}$$

$$P = \frac{\delta v \times E}{19 \times V}$$

$$= \frac{2500 \times 2\times 10^5}{19 \times \frac{\pi \times 200^2}{4} \times 2000}$$

$$\boxed{P = 4.188\ N/mm^2}$$

σ_c Hoop stress $= \frac{P.d}{2t} = \frac{4.188 \times 200}{2\times 10} = 41.88 N/mm^2.$

$$\sigma_l = 5P = 5\times 4.188 = 20.94 N/mm^2$$

$$\frac{\delta d}{d} = \frac{8.5P}{E}$$

$$\delta d = \frac{8.5 \times 200 \times 4.188}{2\times 10^5} = \begin{matrix}0.0356 \\ 0.356\,mm.\end{matrix}$$

$$\frac{\delta l}{l} = \frac{2P}{E}$$

$$\delta l = \frac{2P}{E} \times l = \frac{2\times 4.188}{2\times 10^5} \times 2000 = 0.08376\ mm.$$

Thick Cylinders:

March 2011. ④

When the thickness of cylinders is more than $\frac{1}{10}^{th}$ of ~~diameter~~ Radius, the shells or cylinders are termed as thick cylinder.

In the analysis of thick cylinders, the radial stresses neglected in the analysis of ~~thin~~ cylinders are considered. The hoop stress which is considered as uniform in thin cylinders does not hold good.

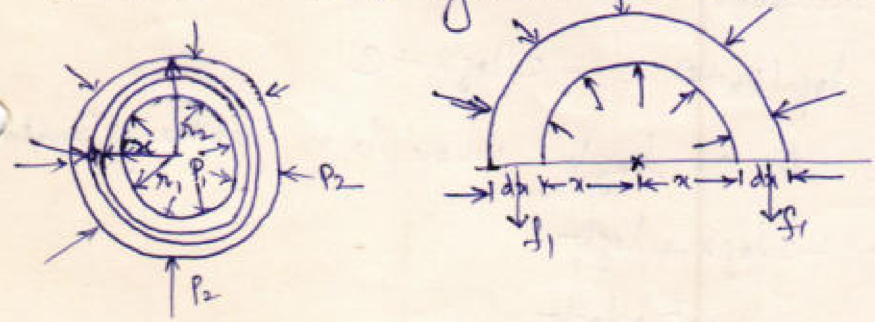
LAME'S Equation:

The following assumptions are made in the analysis of thick cylinders.

1) The material of the cylinder is homogeneous and isotropic
2) The material of the cylinder obeys Hooke's law
3) plane sections of the cylinder remains plane even after the application of internal and external pressure.
4) All the fibres of the cylinders will expand or contract independently.
(variation of hoop stress and radial stresses ~~are~~ occurs across the cross section).
5) The longitudinal strain is uniform across the cross section of the cylinder.

The theory is based on the assumption by "Lame."

Consider the thick cylinder as shown in fig.



Let r_1 = Inner radius
r_2 = outer radius
P_1 = Internal radial pressure
P_2 = outer radial pressure.
x =

Consider a semicircular ring element with internal radius x and thickness dx

Let the internal pressure on it be P_x and external pressure $P_x + dP_x$.

Let 'L' be the length of the element.

Then bursting force $= P_x(2x.L) - (P_x+\delta P_x)\,2(x+\delta x)L$. — ①

If f_c is the hoop stress, the resisting force $= f_c 2dx.L = 2f_c dxL$ — ②

$$f_c \cancel{2}\delta x \cancel{L} = P_x \cancel{2}x\cancel{L} - (P_x+\delta P_x)\cancel{2}(x+\delta x)\cancel{L}$$

$$f_c\delta x = \cancel{P_x x} - \cancel{P_x x} - P_x.dx - x.\delta P_x - \delta x.\delta P_x \quad \text{(Neglected)}$$

$$f_c dx = -P_x dx - x.\delta P_x. \quad \div \text{ by } \delta x.$$

$$f_c = -P_x - x.\delta P_x/\delta x.$$

Then $\sigma_c + P_x + x\frac{\delta P_x}{dx} = 0$ ——— (3)

Let σ_1 be the longitudinal stress. Then

Longitudinal strain $= e_1 = \frac{\sigma_1}{E} - \mu\frac{\sigma_c}{E} + \mu\frac{P_x}{E}$ [Since P_x is compressive]

$$e_1 = \frac{\sigma_1}{E} - \frac{\mu}{E}[\sigma_c - P_x]$$

In Lame's theory this longitudinal strain is assumed constant throughout. Since σ_1 is constant, then $\sigma_c - P_x$ should be constant.

Let it be equal to $2a$. Thus $\sigma_c - P_x = 2a$

$\sigma_c = P_x + 2a$. — (4)

Substituting it in Eqn (3),

$$P_x + 2a + P_x + x\frac{\delta P_x}{\delta x} = 0$$

$$x\frac{\delta P_x}{\delta x} = -2[P_x + a]$$

$$\frac{\delta P_x}{(P_x + a)} = -2\frac{dx}{x}.$$

Integrating both sides,

$$\log(P_x + a) = -2\log x + C$$

Here C is constant of integration. Let $C = \log b$ where b is arbitrary constant.

$$\log(P_x + a) = -2\log x + \log b$$

$$= -\log x^2 + \log b$$

$$\log(P_x + a) = \log\frac{b}{x^2}$$

$$P_x + a = b/x^2 \qquad P_x = b/x^2 - a$$

P_x = Compressive
σ_c = Tensile

Substituting this Eqn (4),

$$\sigma_c = \frac{b}{x^2} + a$$

Thus the Lame's Eqn for thick cylinders are

$$\left.\begin{aligned} P_x &= \frac{b}{x^2} - a. \\ \sigma_c &= \frac{b}{x^2} + a \end{aligned}\right\}$$ where a and b are arbitrary constants.

$\boxed{P_x = \sigma_x}$

σ_c = Hoop stress / Circumferential stress

(5)

A pipe 400mm internal dia and 100mm thick Contains fluid at a pressure of $80 N/mm^2$. Find the maxm and minimum hoop stress across the section. Also Sketch the radial and hoop stress distribution across the section.

$r_1 = \frac{400}{2} = 200mm$

$r_2 = 200+100 = 300mm$

$P_1 = 80 N/mm^2$

$P_2 = 0$

From Lame's eqn

$$P_x = \frac{b}{x^2} - a$$

at $x = 200mm$ $P_x = 80 N/mm^2$

$$80 = \frac{b}{200^2} - a. \quad -①$$

also at $x = 300mm$ $P_x = 0$

$$0 = \frac{b}{300^2} - a. \quad -②$$

$$80 = \frac{b}{200^2} - \frac{b}{300^2}$$

$$\boxed{b = 576 \times 10^4}$$

From eqn ②

$$a = \frac{576 \times 10^4}{300^2} = 64.$$

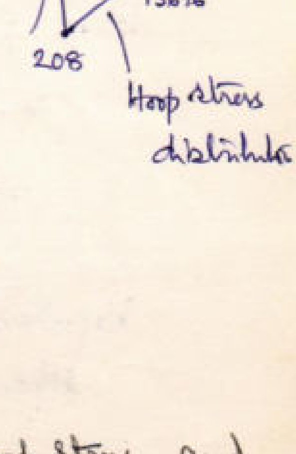
Now

Radial stress developed: $P_x = \frac{576 \times 10^4}{x^2} - 64$

At $x = 200mm$ $P_x = 80 N/mm^2$

$x = 250mm$ $P_x = \frac{576 \times 10^4}{250^2} - 64 = 28.16 N/mm^2$

$x = 300mm$ $P_x = 0$



Also

Hoop stress developed:

$$\sigma_c = \frac{b}{x^2} + a$$

At $x = 200$ $\sigma_c = \frac{576 \times 10^4}{200^2} + 64 = 208 N/mm^2$

$x = 250$ $\sigma_c = \quad = 156.16 N/mm^2$

$x = 300$ $\sigma_c = \quad = 128 N/mm^2$

Nov.

Differentiate between thin and thick cylinders. Also explain hoop stress and longitudinal stress in connection with thin cylinders. Draw neat sketches and write the expression.

Feb 2002

A thick ~~spherical~~ cylinder ~~shell~~ of 40cm internal dia is subjected to an internal fluid pressure of 15N/mm². Determine the necessary thickness of the shell if the permissible stress in the shell is 30N/mm². Draw the radial and hoop stress distribution diagrams across the wall section.

Aug 99

Soln:

$r_1 = 200\text{mm}$

$P_1 = 15\ \text{N/mm}^2$

$\sigma_{max} = 30\text{N/mm}^2$.

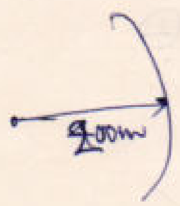
From Lame's Eqn

$$P_x = \frac{b}{x^2} - a$$

$$\sigma_c = \frac{b}{x^2} + a.$$

Now

$$15 = \frac{b}{200^2} - a \qquad -\ (1)$$



Also, hoop tension at the inner edge is max; stress ie at least value of x ie when $x = 200$

$$30 = \frac{b}{200^2} + a \qquad -\ (2)$$

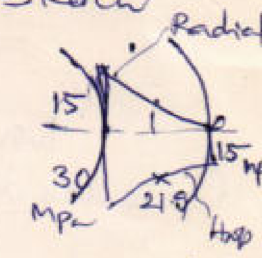
Adding (1) and (2)

$$45 = 2\frac{b}{200^2}$$

$$b = \frac{45 \times 200^2}{2} = 900000$$

and $a = 7.5$.

Sketch



Radial stress distribution

When $x = 200$ $\qquad P_x = 15\ \text{N/mm}^2$

$x = 250$ $\qquad P_x = \frac{9\times10^5}{250^2} - 7.5 = 6.9\ \text{N/mm}^2$

$x = 346.41$ $\qquad P_x = \frac{9\times10^5}{(346.41)^2} - 7.5 = 0\ \text{N/mm}^2$

Hoop stress distribution

when $x = 200$ $\qquad \sigma_c = \frac{9\times10^5}{200^2} + 7.5 = 30\text{N/mm}^2$

$x = 250$ $\qquad \sigma_c = \frac{9\times10^5}{250^2} + 7.5 = 21.9\ \text{MPa}$

$x = 346.41$ $\qquad \sigma_c = \qquad = 15\ \text{MPa}$

To Calculate thickness

Since external pressure is 0, at r_2 then

$$0 = \frac{b}{r_2^2} - a$$

$$= \frac{900000}{r_2^2} - 7.5$$

$$r_2 = \sqrt{\frac{900000}{7.5}}$$

$$\boxed{r_2 = 346.41}$$

Thickness $= r_2 - r_1$

$= 346.41 - 200$

$= 146.41$ mm.

A thick cylinder of 250 mm internal dia and 350 mm outer dia contains a fluid at a pressure of 12 N/mm². Determine the hoop stresses and radial stresses and draw a neat sketch showing the stress distribution across the wall thickness.

Aug 99

Soln

$r_1 = \frac{250}{2} = 125\,mm$

$r_2 = \frac{350}{2} = 175\,mm$

$P = 12\,N/mm^2$

From Lame's eqn.

$$P_x = \frac{b}{x^2} - a$$

$$\sigma_c = \frac{b}{x^2} + a.$$

Then

$$12 = \frac{b}{125^2} - a \qquad \text{——— (1)}$$

and

$$0 = \frac{b}{175^2} - a \qquad \text{——— (2)}$$

Solving (1) and (2),

$$12 = b\left[\frac{1}{125^2} - \frac{1}{175^2}\right] \qquad b = 382811.74$$

then $a = 12.5$

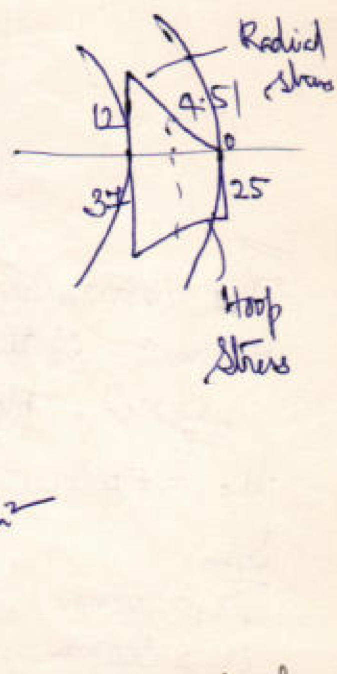
Radial Stress

$$P_x = \frac{b}{x^2} - a$$

At $x = 125$ $\quad P_x = 12\ N/mm^2$

$x = 150$ $\quad P_x = 4.513\,N/mm^2 = \frac{382811.74}{(150)^2} - 12.5$

$x = 175$ $\quad P_x = 0$



Hoop Stress

$$\sigma_c = \frac{b}{x^2} + a$$

At $x = 125$ $\quad \sigma_c = \frac{382811.74}{125^2} + 12.5 = 37\ N/mm^2$

$x = 175$ $\quad \sigma_c = \frac{b}{175^2} + 12.5 = 25\ N/mm^2$

n.a.

A cylinder of internal dia 3m and of thickness 6 cms contains a gas. If the tensile stress in the material is not to exceed 90 N/mm², find the internal pressure of gas.

$r_1 = 1500\,mm$

$t = 60\,mm$

$r_2 = 1560\,mm$,

$P_2 = 0$

Hoop stress at inner edge is max.

$$\sigma_c = \frac{b}{x^2} + a$$

$$90 = \frac{b}{1500^2} + a \qquad \text{—— (1)}$$

$$P_x = \frac{b}{x^2} - a \quad \text{i.e.} \quad 0 = \frac{b}{1560^2} - a \qquad \text{—— (2)}$$

Then $90 = b\left[\frac{1}{1500^2} + \frac{1}{1560^2}\right]$

$b = 1.0522 \times 10^8$

$a = 43.235$

then $P_x = \frac{1.0522 \times 10^8}{1500^2} - 43.235$

$P_{gas} = 3.528\ N/mm^2$.

Find the thickness of metal necessary for a cylindrical shell of internal dia 160mm to withstand an internal fluid pressure of 8N/mm². The max hoop stress is not to increase 35 N/mm²

March 2000

Soln

$r_1 = 80mm$

$P_1 = 8N/mm^2$

$\sigma_{c_{max}} = 35N/mm^2$

From Lame's eqn

$$P_x = \frac{b}{x^2} - a$$

$$\sigma_c = \frac{b}{x^2} + a.$$

Now

$$8 = \frac{b}{80^2} - a. \quad \text{——} (1)$$

Hoop stress is max at inner radius and

$$35 = \frac{b}{80^2} + a \quad \text{——} (2)$$

Add (1) and (2)

$$b = 137600$$
$$a = 13.5$$

External radial pressure is zero at $x = r_2$.

Then

$$0 = \frac{137600}{r_2^2} - 13.5$$

$$r_2 = 101 \text{ mm}$$

$$\text{Thickness} = r_2 - r_1 = 101 - 80 = 21 \text{ mm}.$$

The maximum stress permitted in a thick cylinder radii 200mm and 300mm is 16N/mm². If the internal pressure is 12N/mm², what external pressure can be applied. plot curves showing the variation of hoop and radial stresses through the material.

March 2001.

Sept 2020

12 Marks

Soln

$r_1 = 200mm$

$r_2 = 300mm$

$P_1 = 12N/mm^2$

$\sigma_{c_1} = 16N/mm^2$

Using Lame's Eqns.

$$P_x = \frac{b}{x^2} - a$$

$$\text{At } x = 200mm, \; 12 = \frac{b}{200^2} - a \quad \text{——} (1)$$

$$\sigma_c = \frac{b}{x^2} + a$$

ie

$$\text{At } x = 200 \quad 16 = \frac{b}{200^2} + a \quad \text{——} (2)$$

Solving (1) and (2), then

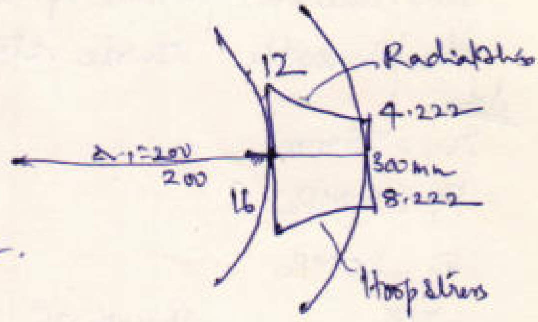
$$28 = \frac{2b}{200^2}. \text{ Then } b = \frac{28 \times 200^2}{2} = 56 \times 10^4$$
$$a = 2$$

Then $\sigma_c = \frac{56\times10^4}{x^2} + 2$

At $x = 300$ $\sigma_c = \frac{56\times10^4}{300^2} + 2 = 8.222\ N/mm^2$.

At $x = 300$ $P_x = \frac{56\times10^4}{x^2} - 2$

$= \frac{56\times10^4}{300^2} - 2 = 4.222\ N/mm^2$.



A thick cylinder of internal dia 160mm is subjected to an internal pressure of intensity 40MPa. Determine the minimum thickness required if the maximum tensile stress is not to exceed 90MPa.

Aug 2001.

Soln:

$r_1 = 80mm$
$P_1 = 40MPa$
$\sigma_c = 90MPa$

From Lame's Equations.

$P_x = \frac{b}{x^2} - a$

$40 = \frac{b}{80^2} - a$ —①

and

$90 = \frac{b}{80^2} + a$ —②

Adding ① and ②, $130 = \frac{2b}{80^2}$ $b = \frac{130\times80^2}{2} = 416000$

$a = \quad = 25$

To find the thickness

Ext press is 0 at r_2

$0 = \frac{416\times10^3}{r_2^2} - 25$

$\frac{416\times10^3}{r_2^2} = 25$

$r_2 = \sqrt{\frac{416\times10^3}{25}} = 129\ mm$

Thickness $= r_2 - r_1 = 129 - 80 = 49\ mm$

What are the difference between thin and thick cylinders? Aug/Sept 2000

= A thick metallic cylindrical shell of 150mm internal dia is required to withstand an internal pressure of 8 MPa. Find the necessary thickness of the shell, if the permissible tensile stress in the section is 20 MPa. Sept 2000

Soln

$r_1 = 75$ mm

$P_1 = 8$ MPa

$\sigma_{c\,max} = 20$ MPa

From Lame's equation,

$$P_x = \frac{b}{x^2} - a \qquad \sigma_c = \frac{b}{x^2} + a$$

At $x = r_1 = 75$

$$8 = \frac{b}{75^2} - a \quad (1) \qquad 20 = \frac{b}{75^2} + a \quad (2)$$

Adding (1) + (2)

$$28 = \frac{2b}{75^2} \qquad b = \frac{28 \times 75^2}{2} = 78750.$$

$$a = \frac{78750}{75^2} - 8 = 6$$

To find the thickness of the shell:

Ext pressure is 0 at $x = r_2$

$$0 = \frac{78750}{r_2^2} - 6$$

$$r_2 = \sqrt{\frac{78750}{6}} = 114.56\text{ mm}$$

$$\text{Thickness} = r_2 - r_1 = 114.56 - 75 = 39.56\text{ mm.}$$

= A pipe of external diameter 600mm and of wall thickness 125mm carries a fluid at pressure of 11.2 N/mm². Determine the hoop stresses and radial pressures developed. Sketch the hoop stress and radial pressure distribution diagrams across the section. Feb 2002

$r_2 = 300$ m

$r_1 = 300 - 125 = 175$ mm

$P_1 = 11.2\ N/mm^2$.

From Lame's eqn

$$P_x = \frac{b}{x^2} - a$$

At $x = 175$ mm (r_1) $\quad 11.2 = \frac{b}{175^2} - a \quad (1)$

At $x = 300$ mm (r_2) $\quad 0 = \frac{b}{300^2} - a \quad (2)$

Adding (1) and (2), we get

$$11.2 = b\left[\frac{1}{175^2} - \frac{1}{300^2}\right]$$

$$b = 519917$$

a

$$b = 519917 \qquad a = 5.78$$

Radial stress distribution: $\left(P_1 = \frac{b}{x^2} - a\right)$

at $r_2 = 175$ $\quad P = 11.2\ N/mm^2$

$r_1 = 300$ $\quad P = 0$

$r_{mid} = 250$ $\quad P = 2.534\ N/mm^2$

Hoop stress distribution: $\sigma_c = \frac{b}{x^2} + a$

at $r_2 = 175$ $\quad \sigma_c = 22.4\ N/mm^2$

$r_1 = 300$ $\quad \sigma_c = 11.53\ N/mm^2$

$r_{mid} = 250$ $\quad \sigma_c = 14.07\ N/mm^2$

A thin Cylindrical Shell 1.2 m in diameter and 3m long has a metal wall thickness of 12 mm. It is subjected to an internal fluid pressure of 3.2 MPa. Find the Circumferential stress and longitudinal stress in the wall. Determine the Change in length, diameter and Volume of the Cylinder $E = 210 GPa$ and $\mu = 0.3$.

10 marks

July 2008

$L = 3m = 3000 mm$

$d = 1.2m = 1200 mm$

$t = 12 mm$

$P = 3.2 MPa$

$= 3.2 N/mm^2$

Hoop Stress $= \sigma_c = \dfrac{P.d}{2t}$

$= \dfrac{3.2 \times 1200}{2 \times 12} = 160 N/mm^2.$

Longitudinal stress $= \sigma_l = \dfrac{P.d}{4t} = \dfrac{3.2 \times 1200}{4 \times 12} = 80 N/mm^2$

Now Diametral strain or

Diametrical strain or

Circumferential strain $= \dfrac{\delta d}{d} = \dfrac{\sigma_c}{E} - \dfrac{1}{m}\dfrac{\sigma_l}{E}$

$t < \frac{1}{10} \times 600$

$12 < 60$

Hence thin Cylinder

$= \dfrac{1}{E}\left[\sigma_c - \mu \sigma_l\right]$

$= \dfrac{1}{210 \times 10^3}\left[160 - 0.3 \times 80\right]$

$= 6.47 \times 10^{-4}$

Change in diameter $\delta d = 6.47 \times 10^{-4} \times 1200$

$\boxed{\delta d = 0.777 mm}$

Longitudinal strain $= \dfrac{\delta l}{l} = \left[\dfrac{\sigma_l}{E} - \mu \dfrac{\sigma_c}{E}\right]$

$= \dfrac{1}{E}\left[\sigma_l - \mu \sigma_c\right]$

$= \dfrac{1}{210 \times 10^3}\left[80 - 0.3 \times 160\right]$

$= 1.52 \times 10^{-4}$

$\delta l = 1.52 \times 10^{-4} \times 3000$

$\boxed{\delta l = 0.457 mm}$

Change in volume $= \delta v = (2 e_c + e_l) V$

$= (2 \times 6.47 \times 10^{-4} + 1.52 \times 10^{-4}) \dfrac{\pi \times 1200^2}{4} \times 3200 = 5.2 \times 10^{6} mm^3$

Determine the maximum and minimum hoop stress across the section of a pipe 400mm internal diameter and 100 mm thick, when the pipe contains a fluid at a pressure of 8 N/mm². Also sketch the radial pressure distribution and hoop stress distribution across the section.

08 Marks
June-July 2019.

$r_1 = \frac{d_1}{2} = \frac{400}{2} = 200\,mm$

$t = 100\,mm$

$r_2 = 200 + 100 = 300\,mm$

$P = 8\,N/mm^2$

Lame's Equations

$P_x = \frac{b}{x^2} - a$

$\sigma_c = \frac{b}{x^2} + a$

Now

$8 = \frac{b}{200^2} - a \quad \text{——} (1)$

Radial Pressure →

(At $r_2 = 300\,mm$, $P = 0$)

$0 = \frac{b}{300^2} - a \quad \text{——} (2)$

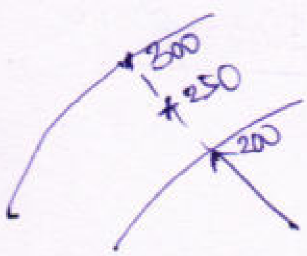
Adding

$8 = b\left[\frac{1}{200^2} - \frac{1}{300^2}\right]$

$\boxed{b = 5.76\times10^5}$

$a = \frac{5.76\times10^5}{200^2} - 8$

$\boxed{a = 6.4}$



Radial stress distribution: P.

P @ $x = 200\,mm$ $P = 8\,N/mm^2$ $= 8\,N/mm^2$

$x = 250\,mm$ $P = \frac{5.76\times10^5}{250^2} - 6.4 = 2.816\,N/mm^2$

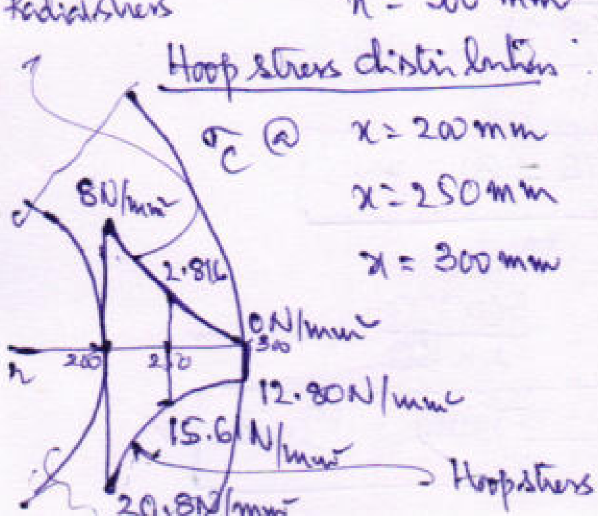
$x = 300\,mm$ $P = 0$ $= 0$

Hoop stress distribution:

σ_c @ $x = 200\,mm$ $\sigma_c = \frac{5.76\times10^5}{200^2} + 6.4 = 20.8\,N/mm^2$

$x = 250\,mm$ $\sigma_c = \frac{5.76\times10^5}{250^2} + 6.4 = 15.61\,N/mm^2$

$x = 300\,mm$ $\sigma_c = \frac{5.76\times10^5}{300^2} + 6.4 = 12.80\,N/mm^2$



Find the thickness of the metal necessary for a steel cylindrical shell of internal dia 150 mm to withstand an internal pressure of 50 N/mm². The maximum hoop stress in the section not to exceed 150 N/mm². If the thickness is found using the cylinder analysis, what is the percentage error?

12 Marks

Dec-Jan 2017
16

$r_1 = \frac{150}{2} = 75 \text{ mm}$

$P = 50 \text{ N/mm}^2$

$\sigma_c = 150 \text{ N/mm}^2$

Lame's equations

$$P_x = \frac{b}{x^2} - a$$

$$\sigma_c = \frac{b}{x^2} + a$$

Now

$$50 = \frac{b}{75^2} - a \quad \text{———— (1)}$$

Hoop tension $\Rightarrow$ $$150 = \frac{b}{75^2} + a \quad \text{———— (2)}$$

Adding (1) and (2) $$200 = \frac{2b}{75^2} \quad \therefore \; b = \frac{200 \times 75^2}{2}$$

$$\boxed{b = 5.62 \times 10^5}$$

Then $$a = \frac{5.62 \times 10^5}{75^2} - 50$$

$$\boxed{a = 49.91}$$

To find thickness:

Radial pressure $\Rightarrow$ $$0 = \frac{5.62 \times 10^5}{r_2^2} - 49.91$$
(at $r_2 = 0$)

$$r_2 = \sqrt{\frac{5.62 \times 10^5}{49.91}}$$

$$\boxed{r_2 = 106.11}$$

$$\boxed{\text{Thickness} = t = r_2 - r_1 = 106.11 - 75 = 31.11 \text{ mm}}$$

II Case: If thin cylinder analysis is used, then

$$\sigma_c = \frac{Pd}{2t} \rightarrow t = \frac{Pd}{2\sigma_c} = \frac{50 \times 150}{2 \times 150} = 25 \text{ mm}$$

$$\boxed{\text{Then percentage error} = \% \text{ error} = \left(\frac{31.11 - 25}{31.11}\right) \times 100 = 19.63}$$

Bending Moment and Shear forces in beams

A beam is normally a horizontal member subjected to transverse loads. It can also be inclined and can have axial loads acting on them.

A beam is statically determinate if its reaction components (unknowns) can be determined by using equations of static equilibrium only. They are

a) Cantilever beams
b) S.S. Beams
c) Overhanging beams.

These beams are usually subjected to the following types of transverse loads:

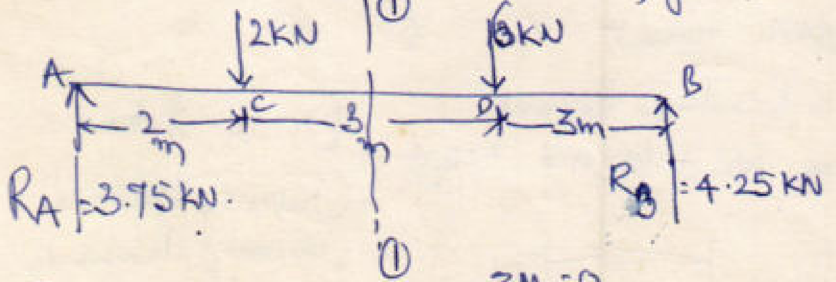
a) Concentrated loads.
b) UDL
c) UVL
d) Externally applied moments.

Concept of Shear force and Bending moment:

Shear force:

Shear force at a section in a beam is obtained as the algebraic sum of all the forces including reactions acting normal to the axis of the beam either to the left or right of the section.

Consider a beam shown in the fig



Taking moment about A, $\Sigma M_A = 0$

$$R_B \times 8 = 6 \times 5 + 2 \times 2$$

$$R_B = \frac{34}{8} = 4.25 \text{ KN}$$

$\Sigma V = 0$

$$R_A + R_B = 2 + 6$$

$$R_A = 8 - R_B$$

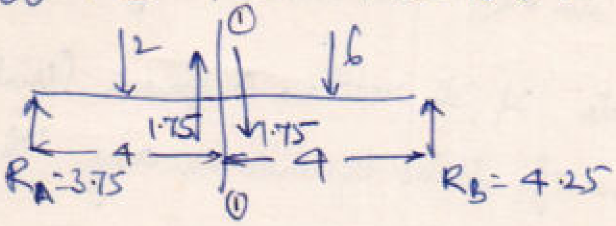
$$= 8 - 4.25 = 3.75 \text{ KN}$$

Consider a section ①-① as shown in fig.
Considering only left side

$SF_{①-①} = 3.75 - 2 = 1.75\ kN \uparrow$ ✓

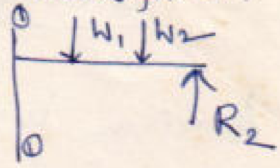
Considering right side only

$SF_{①①} = 6 - 4.25 = 1.75\ kN \downarrow$ ✓



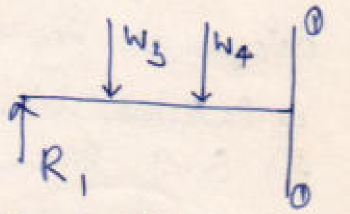
RUN

Sign Convention for SF:



SF at 1-1

$SF_{1-1} = W_1 + W_2 - R_2$



$SF_{①-①} = R_1 - W_3 - W_4$

From the above, it is very clear the shear force of equal magnitude acts on either side of a section for equilibrium.

Bending moment:

Bending moment is defined as the algebraic sum of moment either to the left or to the right of the section.

Considering moment at section ①-①, at a distance 4 units from A

Sign convention: Anticlockwise moment to the right hand side and clockwise moment to the left hand side is taken as ~~positive~~ ~~negative~~

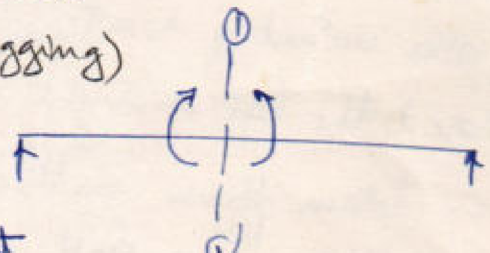
$M_{①①} = + R_A \times 4 - 2 \times 2$

$= +11\ kN\text{-}m$ (Sagging)

$M_{①①} = 4.25 \times 4 - 6 \times 1$

$= 11$ (Sagging)

Thus at section ①-①,



We have equal and opposite moment acting which causes the bending moment.

Here sagging BM is taken as +ve and hogging BM is taken as -ve.

i.e. (Concavity upwards) (Convexity upwards) Concavity downwards.

Clockwise moment on LHS of section and anticlockwise moment on RHS of section is considered +ve (sagging)

The values of BM and SF varies for different sections over the length of the beam.

The diagrams showing or representing the values of BM and SF at all the sections along the length of the beam are called BMD and SFD respectively.

SFD and BMD for Standard Cases:

1. Cantilever a) a Concentrated load at free end.
b) UDL over entire span
c) UVL over entire span.

Cantilever of length 'l' subjected to Concentrated load at free end:

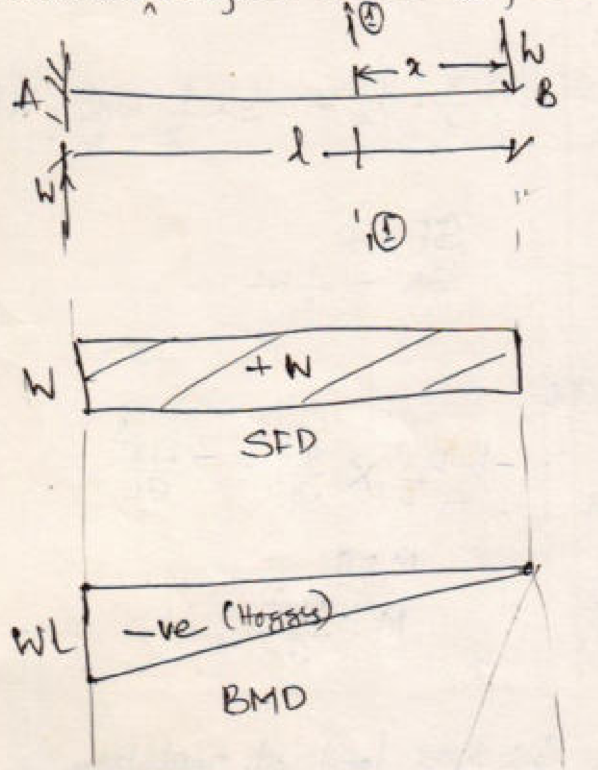


Fig shows a Cantilever fixed at A and free at 'B' Carrying the load at the free end.

Consider a section at a distance 'x' from 'B'.

Then SF at ①① = +W

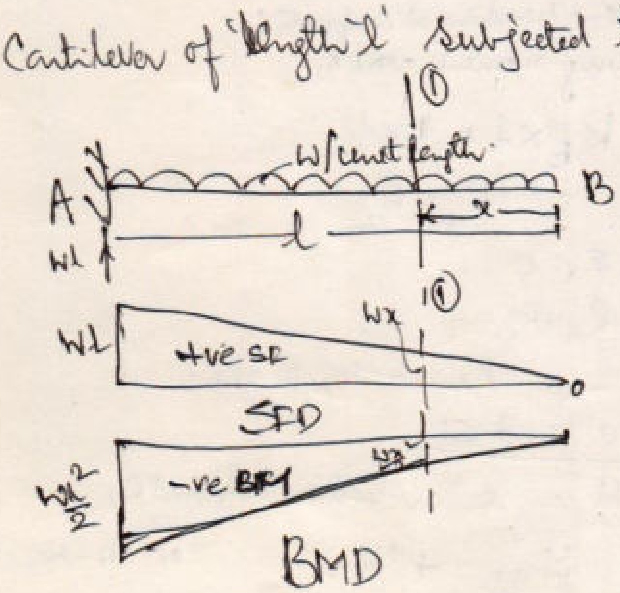
Here SF is Constant throughout the length between A and B.

Then BM at ①① = $-Wx$

when $x=0$ $M_{①①} = 0$

$x = l$ $M_{①①} = -Wl$

Cantilever of length 'l' subjected to uniformly distributed load over the entire span.



Consider a section at distance 'x' from 'B'

Then SF at ①-① = $+wx$

Here SF is varying quantity.

when $x=0$ $SF_{①-①} = 0$

$x = l$ $SF_{1-1} = +wl$ (Linear Variation)

BM at ①-① = $-wx \cdot x/2$

when $x=0$ $M_{1-1} = 0$

$x = l$ $M_{1-1} = -\frac{wl^2}{2}$ (Variation of BM is a/c to parabolic law)

3. Cantilever of length 'l' carrying uniformly varying load over the entire span

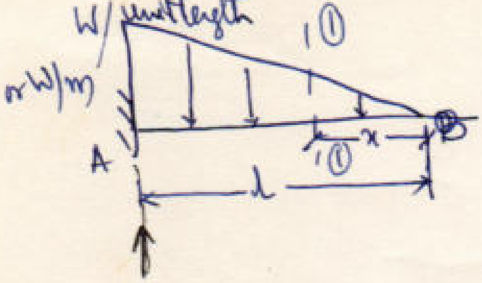
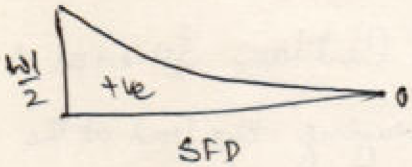


Fig shows a Cantilever with UVL i.e. 0 at free end to W/unit length at the fixed end.

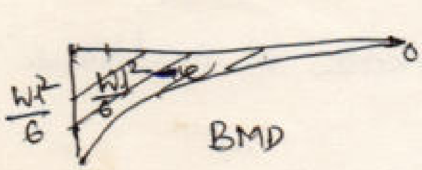
Consider a section at a distance 'x' from free end. The intensity of load at section 1-1 is $W \cdot \frac{x}{l}$.

$$SF_{at\ 1-1} = W \cdot \frac{x}{l} \cdot \frac{1}{2} x = +\frac{1}{2} \frac{Wx^2}{L} \quad \text{(Parabolic Variation)}$$



At $x=0$, $SF = 0$

At $x = l$, $SF = \frac{1}{2} WL$



BM at 1-1

$$M_{11} = -\frac{1}{2}\frac{Wx^2}{L} \times \frac{x}{3} = -\frac{Wx^3}{6L}$$

$R_A = \frac{1}{2} \cdot Wl$

$-M + \frac{Wl^2}{6} = 0$

$M = \frac{Wl^2}{6}$

At $x=0$, $M = 0$

At $x = l$, $M = -\frac{Wl^2}{6}$

Simply Supported beam Subjected to a Concentrated load at midspan.

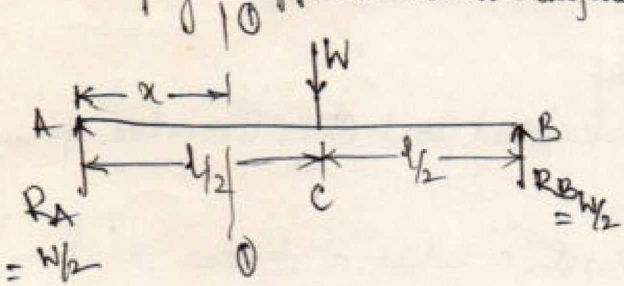


Fig shows a SSB subjected to load W at midspan.
To find reaction at supports:
Taking moment about A,

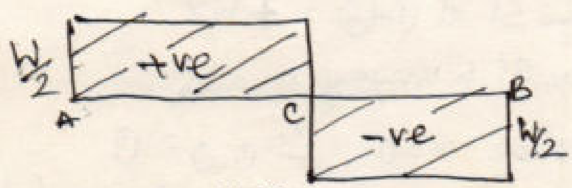
$$R_B \times l = W \times l/2$$

$$R_B = W/2$$

$\Sigma V = 0$

$R_A + R_B = W$

$R_A = W - W/2 = W/2$.

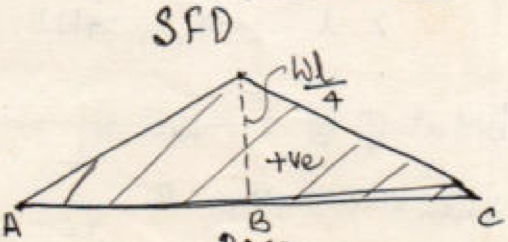


<u>To find SF:</u>

At any section between A and C,

$SF_{11} = +W/2$. or $W - W/2 = W/2$

At C SF changes from $\frac{W}{2}$ to $W/2$.



To find BM:

$M_{11} = \frac{W}{2} \cdot x$ (Sagging moment)

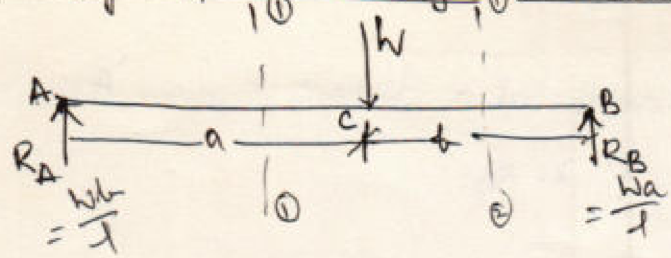
At $x=0$ $M_{11} = 0$

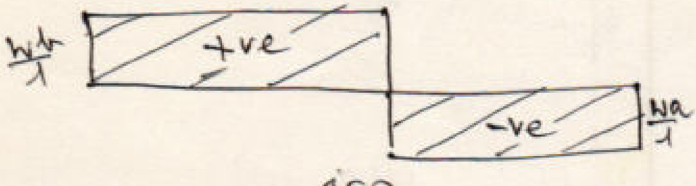
$x = l/2$, $M_{11} = \frac{W}{2} \cdot l/2 = Wl/4$

* Note: Maximum BM occurs where the SF changes its sign normally.

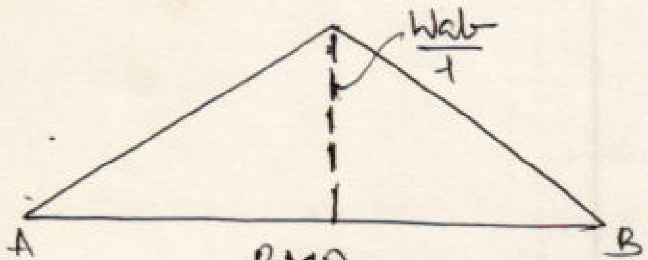
III^{ly} the BM decreases uniformly from $\frac{Wl}{4}$ at C to zero at B.

Simply supported beam subjected to concentrated load acting eccentrically on midspan.





SFD



BMD

Note: Max^m BM occurs when the SF changes its sign from +ve to -ve.

To find reaction at supports

$R_B \times l = W \times a$

$R_B = \frac{Wa}{l}$.

$\Sigma V = 0$

$R_A + R_B = W$

$R_A = W - R_B$

$= W - \frac{Wa}{l}$

$= \frac{Wl - Wa}{l} = \frac{W(l-a)}{l} = \frac{Wb}{l}$

To find SF:

At any section between A and C,

$SF_{①①} = + \frac{Wb}{l}$.

At any section between C and B,

$SF_{②②} = - \frac{Wa}{l}$.

To find M:

At any section between A and C at a distance 'x' from A,

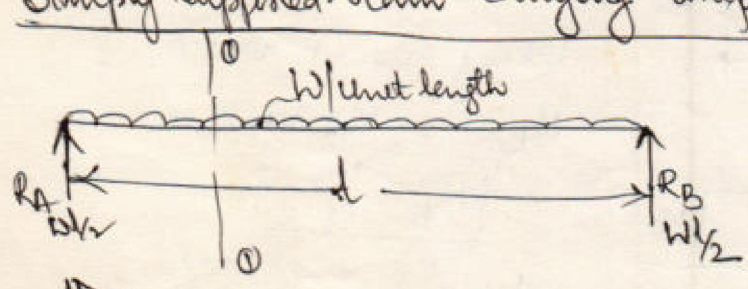
$M_{①①} = \frac{Wb}{l} \cdot x$

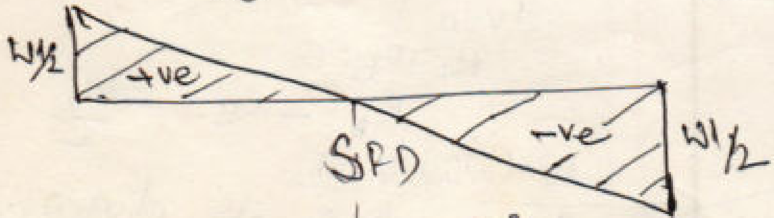
when $x = 0$ $M = 0$

$x = a$ $M = + \frac{Wab}{l}$

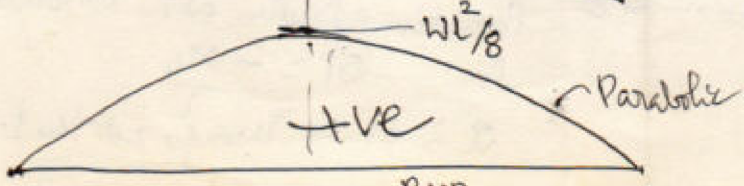
The BM increases uniformly from 0 to $\frac{Wab}{l}$. $|||^{ly}$ the BM will decrease uniformly from $\frac{Wab}{l}$

Simply supported beam carrying uniformly distributed load over the whole span.





SFD



BMD

To find reaction at supports.

$R_B \times l = W \times l \times l/2$

$R_B = Wl/2$

$\Sigma V = 0$

$R_A + R_B = Wl$

$R_A = Wl - Wl/2$

$= Wl/2$

(3A)

To find ~~Moment~~: SF:

At any section between A and B at a distance 'x' from 'A'

SF at ① ① $= + Wl/2 - W.x.$

When $x=0$ $\quad SF = Wl/2$

$x = l/2 \quad SF = Wl/2 - W.\,l/2$

$= 0$

$x = l \quad SF = Wl/2 - W.l = -Wl/2$

To find moment:

At the same section as above,

$$M_{1-1} = \frac{Wl}{2}.x - W.x.x/2$$

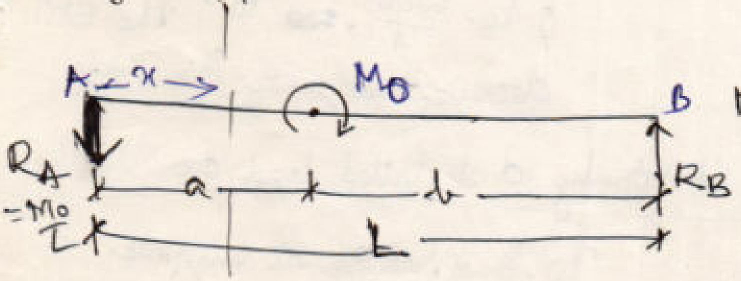
When $x=0$ $\quad M=0.$

$x = l/2 \quad = Wl/2 . l/2 - W . l/2 . l/4$

$$= \frac{2Wl^2 - Wl^2}{8} = \frac{+Wl^2}{8}$$ (Parabolic variation)

$x = l \quad M=0$

Simply supported beam subjected to external moment M_0 at $x=a$ from left support

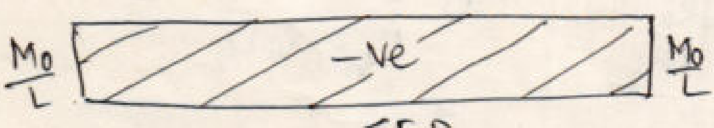


Let the SSB AB be subjected to clockwise moment M_0 at a point at a distance 'a' from support A as shown in fig.

Taking moment about A,

$R_B \times l = M_0$

$$\boxed{R_B = \frac{M_0}{l}}$$



SFD

$\Sigma V = 0$

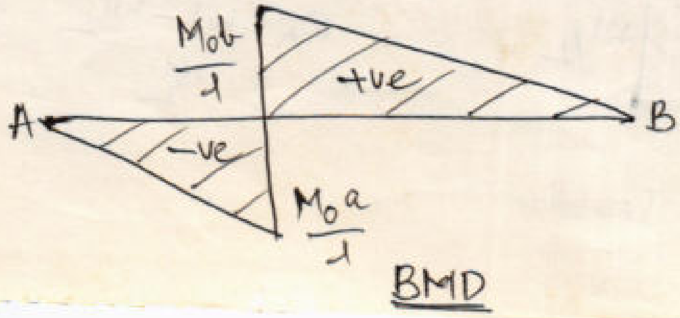
$R_A + R_B = 0$

$R_A = -R_B = -\frac{M_0}{L}$

To Calculate SF:-

Consider a section at a distance x' from A, $\quad SF = -\frac{M_0}{L}$

It is same throughout the length.



BMD

To Calculate moment.

At section ①-① 'x' distance from A,

$$M_{①-①} = -\frac{M_0}{L}.x$$

When $x = 0$ $\quad M = 0$

$x = a$ $\quad M_{①-①} = -\frac{M_0 a}{l}$.

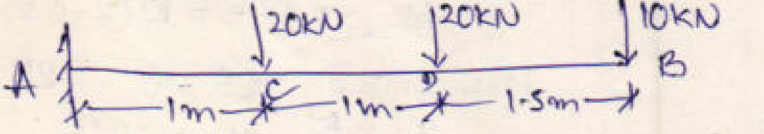
Consider a section a distance "x" from B

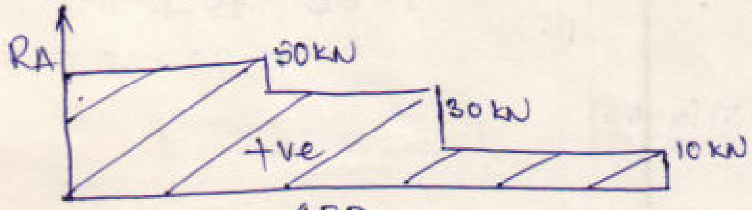
$$M_{①-①} = +\frac{M_0}{l}.x$$

When $x = 0$ $\quad M_{①-①} = 0$

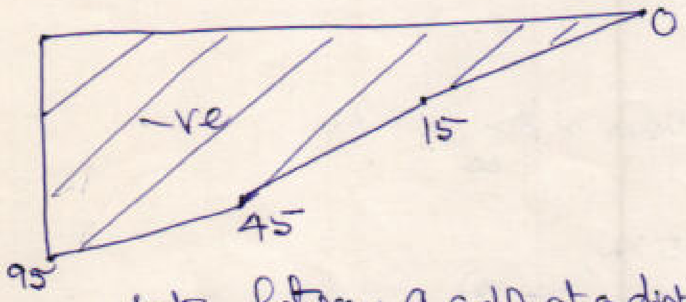
$x = b$ $\quad M_{①-①} = \frac{M_0 . b}{l}$

Draw the BMD and SFD for the Cantilever beam





SFD



$\Sigma V = 0$

$R_A = 20 + 20 + 10 = 50$ kN

To Calculate SF:

At any section between D and B at a distance x from B,

SF at D $= +10$ kN

~~At any section between C and D,~~

~~SF at ②-② = +30 kN~~

M at 1-1 $= -10.x$

When $x = 0$ $\quad M_{1-1} = 0$

$x = 1.5$ $\quad M_{1-1} = -15$ kN-m

At any section between C and D at a distance x from B.

SF at$_{2-2}$ $= 10 + 20 = +30$ kN

M at 2-2 $= -10x - 20(x - 1.5)$

When $x = 1.5$ $\quad M_{2-2} = -15$ kN-m

$x = 2.5$ $\quad M_{2-2} = -10 \times 2.5 - 20(1)$

$= -25 - 20 = -45$ kN-m.

At any section between A and C at a distance x from B

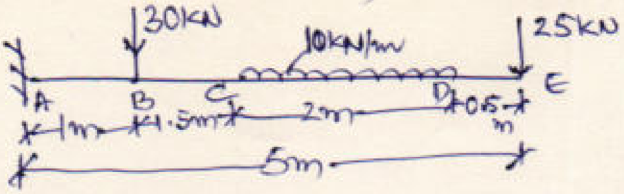
SF at 3-3 $= +50$ kN

M at 3-3 $= -10x - 20(x - 1.5) - 20(x - 2.5)$

When $x = 2.5$ $\quad M_{3-3} = -25 - 20 = -45$ kN-m

$x = 3.5$ $\quad M_{3-3} = -35 - 40 - 20 = -95$ kN-m.

Draw SFD and BMD for the cantilever shown in fig



Reaction at A $= +25 + 10\times 2 + 30$

$\boxed{R_A = 75\ KN}$

To calculate SF :

Consider a section 1-1 at a distance 'x' from E

$SF_{1-1} = +25\ KN.$

$M_{1-1} = -25 \times x$

when $x = 0$ $M = 0$

$x = 0.5$ $M = -25\times 0.5$

$= 12.5\ KN\text{-}m$

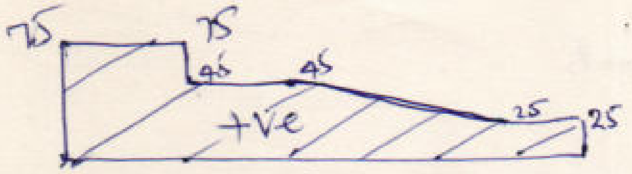
Consider a section 2-2 at a distance 'x' from free end.

$SF_{2-2} = +25 + 10(x - 0.5)$

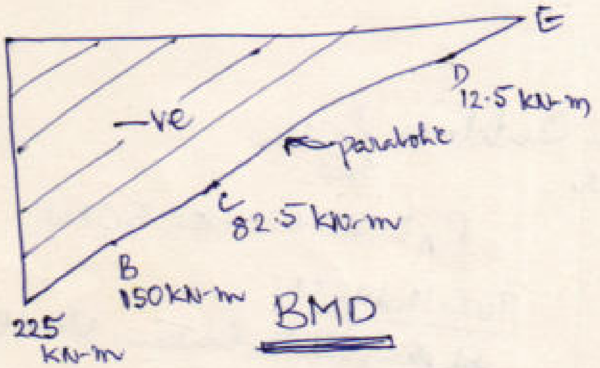
when $x = 0.5$ $= 25 + 0 = 25\ KN$

$x = 2.5$ $= 25 + 10(2)$

$= 25 + 20 = 45\ KN$



SFD



BMD

$M_{2-2} = -25x - 10(x-0.5)\frac{(x-0.5)}{2}$

when $x = 0.5$ $= -25\times 0.5 - 0$

$x = 2.5$ $= -25\times 2.5 - 10(2)\frac{(2)}{2}$

$= 82.5\ KN\text{-}m$

Consider a section 3-3 at a distance 'x' from free end.

$SF_{3-3} = +25 + 10\times 2$

$= 25 + 20 = 45\ KN$

$M_{3-3} = -25\cdot x - 10\times 2\times(x - 1.5)$

when $x = 2.5$ $= -25\times 2.5 - 10\times 2\times 1$

$= -82.5\ KN\text{-}m$

$x = 4$ $M_{3-3} = -25\times 4 - 10\times 2(2.5)$

$= -150\ KN\text{-}m$

Consider a section 4-4 at a distance 'x' from free end.

$SF_{4-4} = 25 + 10\times 2 + 30 = +75\ KN$

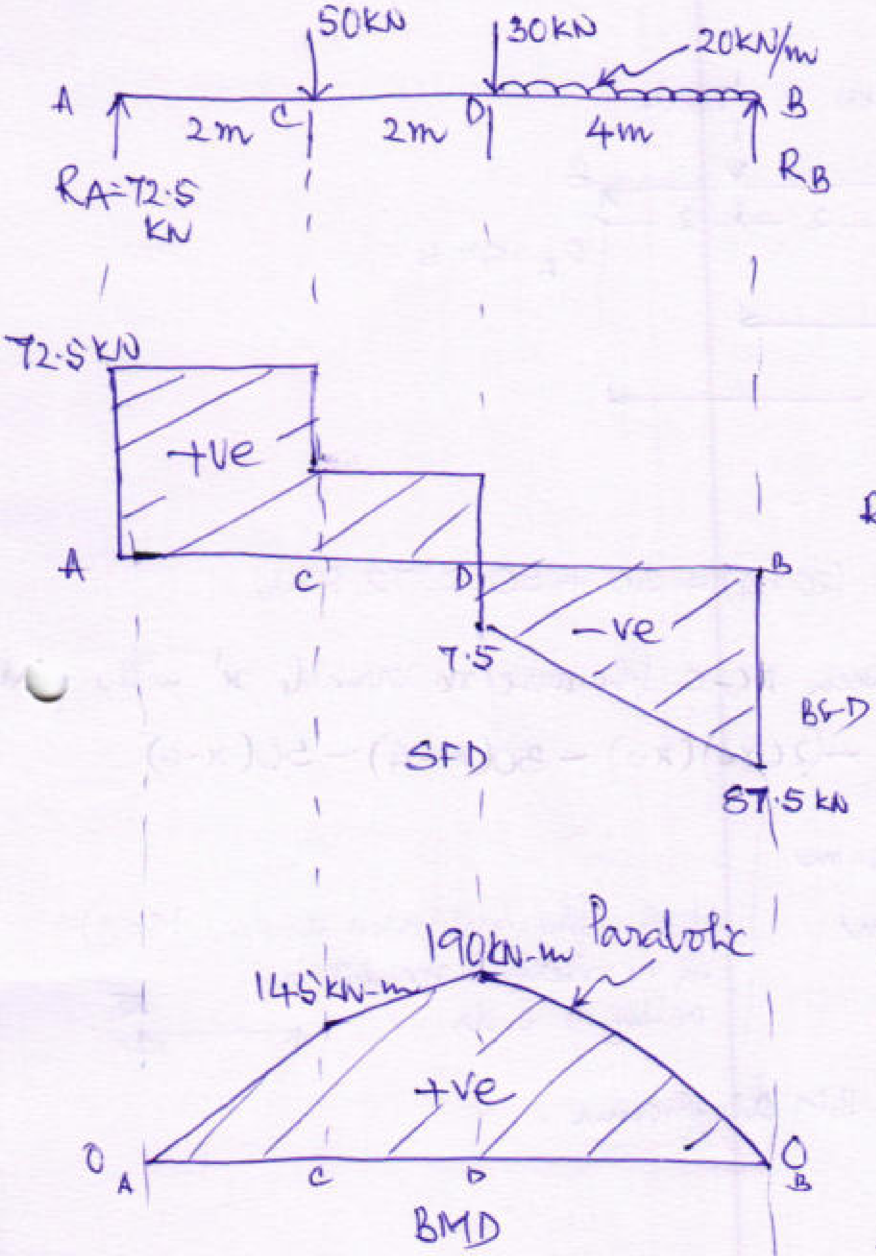
$M_{4-4} = -25x - 10\times 2\times(x - 1.5) - 30(x - 4)$

when $x = 4$ $M_{4-4} = -150\ KN\text{-}m$

$x = 5$ $M = -25\times 5 - 10\times 2\times(3.5) - 30(1)$

$= -125 - 70 - 30 = 225\ KN\text{-}m.$

Draw SFD and BMD for SSB shown in figure



Soln:

Taking moment about A,

$\Sigma M_A = 0$

$-R_B \times 8 + (20 \times 4) \times 6 + 30 \times 4 + 50 \times 2 = 0$

$\boxed{R_B = 87.5\,kN}$

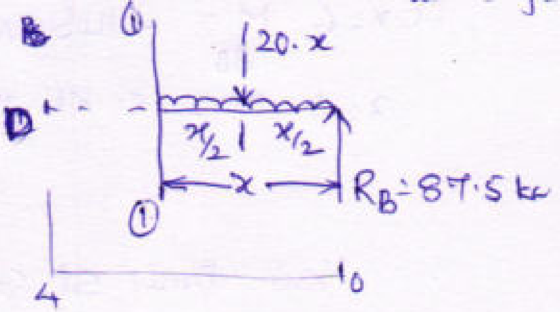
$\Sigma V = 0$

$R_A + 87.5 - 50 - 30 - (20 \times 4) = 0$

$\boxed{R_A = 72.5\,kN}$

Calculation of SF & BM

BSD Consider a section ①-① between B and D at a distance 'x' from B.



$SF_{①-①} = -87.5 + 20.x$

At $x = 0$ $SF = -87.5\,kN$

$x = 4$ $SF = -7.5\,kN$

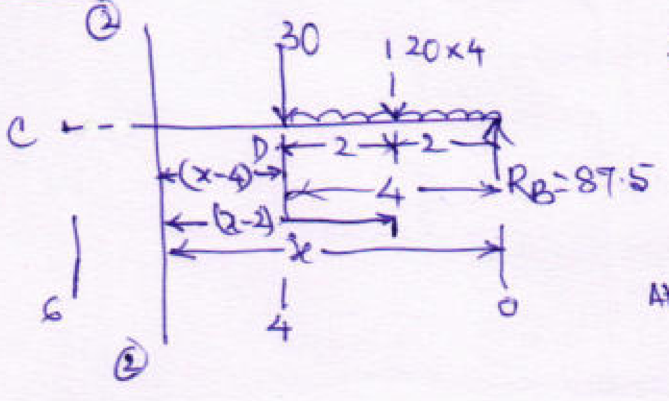
$M_{①-①} = +87.5.x - 20.x.\frac{x}{2}$

At $x = 0$ $M = 0$

$x = 4$ $M = 190\,kN\text{-}m$

(Parabolic Variation)

C&D: Consider a section ②-② between D and C at a distance 'x' from B



$SF_{②②} = -87.5 + (20 \times 4) + 30 = +22.5\,kN$

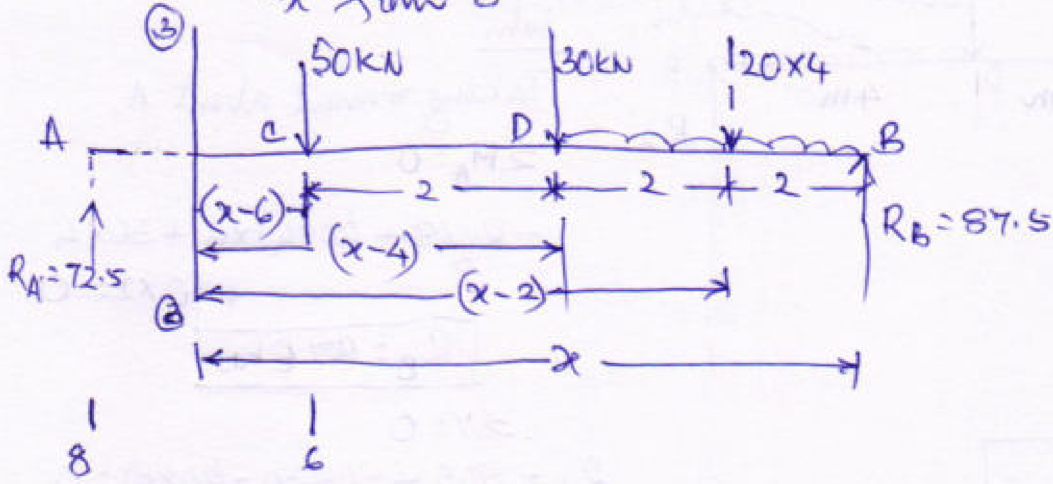
SF is constant between C & D (because no variable distance x in the equation).

$M_{②②} = +87.5x - (20 \times 4)(x-2) - 30(x-4)$

At $x = 4$ $M = +190\,kN\text{-}m$

$x = 6$ $M = +145\,kN\text{-}m$

A & C: Consider a section ⑤-⑤ between A & C at a distance 'x' from B



$$SF_{⑤-⑤} = -87.5 + (20\times4) + 30 + 50 = 72.5\,kN$$

SF is constant between A & C (because no variable 'x' in the equation)

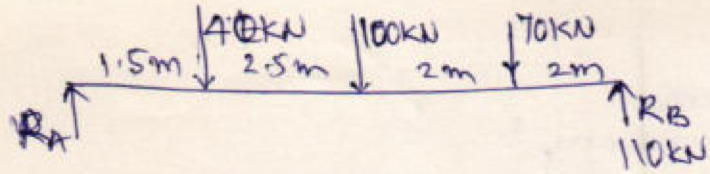
$$M_{⑤-⑤} = +87.5x - (20\times4)(x-2) - 30(x-4) - 50(x-6)$$

At $x = 6$ $M = +145\ kN\text{-}m$

$x = 8$ $M = 0\ kN\text{-}m$ (At simply supported ends, $M=0$) if no external moment is acting at ends

Now Draw SF & BM diagram.

Draw SFD and BMD for the beam shown in fig.



Soln:

Taking moment about A,

$R_B \times 8 = 40 \times 1.5 + 100 \times 4 + 70 \times 6$

$R_B = 110\,KN$

$R_A + R_B = 40 + 100 + 70$

$R_A = 100\,KN$

To Calculate SF:

Consider a section at a distance 'x' from B.

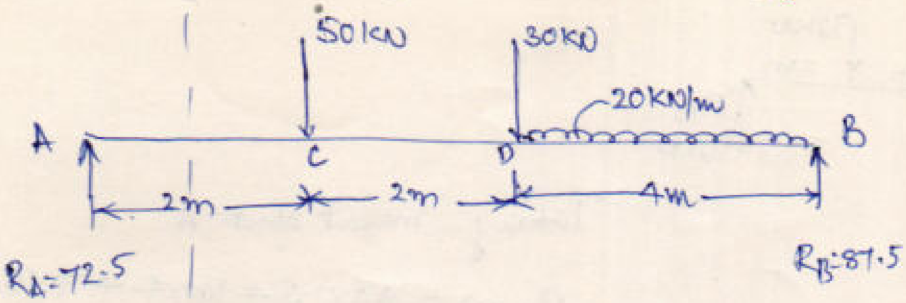
$SF_{1-1} = -110\,KN$

$M_{1-1} = +110 \times x$

when $x=0$ $M = 0$

$x=2$ $M_{1-1} = +220\,KNm$ (Sagging)

Draw SFD and BMD for SSB beam in fig. Aug 1999



Soln:

Taking moment about A,

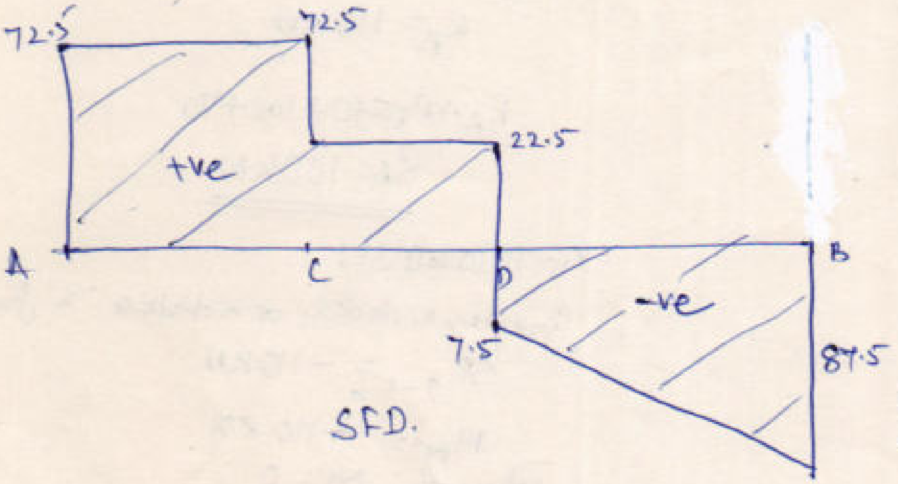
$R_B \times 8 = 20 \times 4 \times 6 + 30 \times 4 + 50 \times 2$

$R_B = \frac{700}{8} = 87.5$ kN

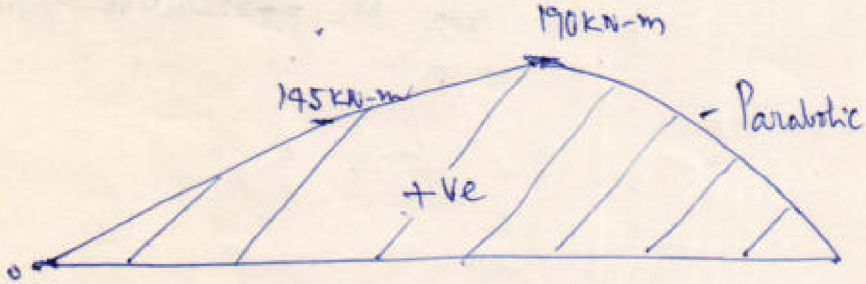
$R_A + R_B = 160$

$R_A = 160 - 87.5$

$R_A = 72.5$ kN



SFD.



BMD

Calculation of SF and BM:- Between A and C

$SF_{1-1} = +72.5$ kN (Between A and C)

$M_{1-1} = 72.5 \times 2$

when $x = 0$ $M = 0$

$x = 2$ $M = 145$ kN-m.

Between C and D

$SF_{2-2} = 72.5 - 50 = 22.5$ kN

$M_{2-2} = 72.5 \cdot x - 50 \times (x - 2)$

when $x = 2$ $= 72.5 \times 2 - 0 = 145$ kN-m

$x = 4$ $=$ $= 190$ kN-m

Between D and B,

~~$SF_{3-3} = 72.5x - 50(x-2) - 30(x-4) - 20(x-4) \cdot \frac{(x-4)}{2}$~~

~~when $x = 4$ $= 190$ kN-m $=$~~

~~$x = 8$ $= 72.5 \times 8 - 50(6) - 30(4) - 20 \times 4 \times 2$~~

~~$= 0$ kN-m.~~

when $SF_{3-3} = 72.5 - 50 - 30 - 20(x-4)$

when $x = 4$ $SF_{3-3} = 72.5 - 50 - 30 - 0 = -7.5$ kN

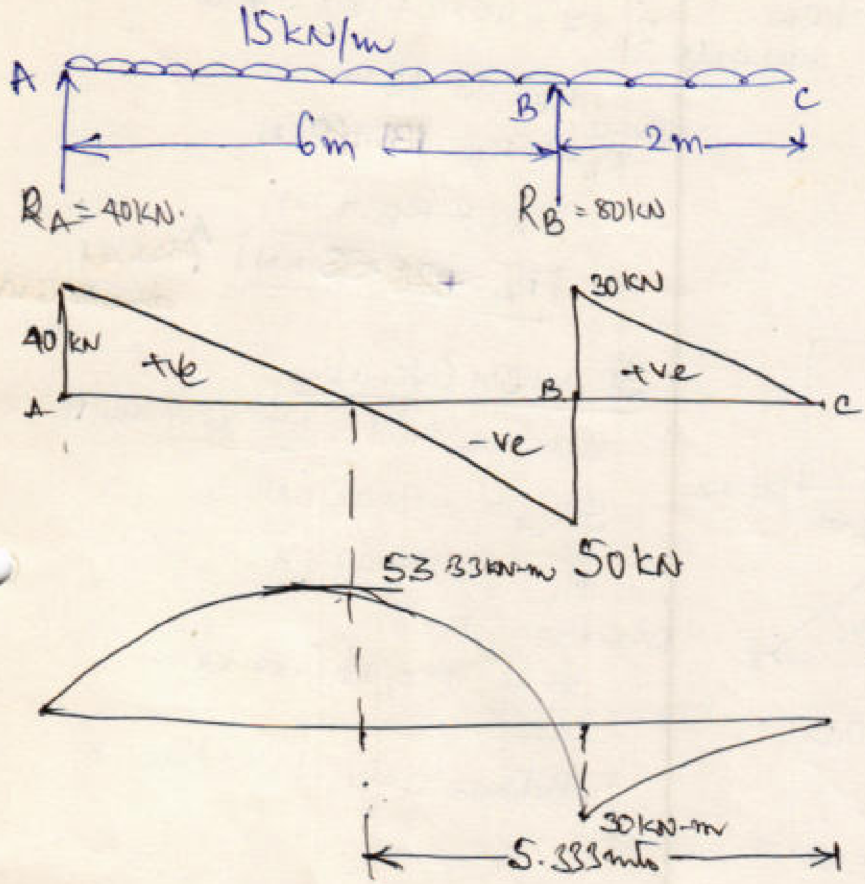
$x = 8$ $SF_{3-3} = 72.5 - 50 - 30 - 20(4) = -87.5$ kN

$M_{3-3} = 87.5 \times x - 20 \times x \cdot x/2$

when $x = 0$ $M_{3-3} = 0$

$x = 4$ $M_{3-3} = 87.5 \times 4 - 20 \times 4 \times 2 = 190$ kN-m

Beams with overhang at one end and carrying a uniformly distributed load over the whole length.



Soln:

$\Sigma M_A = 0$

Taking moment about A,

$R_B \times 6 = 15 \times 8 \times 4$

$R_B = \frac{15 \times 8 \times 4}{6} = 80kN.$

ΣV

$R_A + R_B = 15 \times 8$

$R_A = 120 - 80 = 40kN.$

SF and BM Calculation

* Between B & C at x from C.

$SF_{1-1} = +15 \cdot x$

when x=0 SF = 0

x = 2 SF = +30kN

$M_{1-1} = -15 \cdot x \cdot x/2$

x=0 = 0

x=2 $M_H = -30 kN\text{-}m$

* Between A and B at x from C

$SF_{2-2} = 15 \cdot x - 80$

when x=2 SF = 30-80 = -50kN

x=8 SF = 120-80 = +40kN.

$M_{2-2} = -15 \cdot x \cdot x/2 + 80(x-2)$

x=2 $M_{2-2} = 30 - 0 = 30 kN\text{-}m$

x=8 $M_{2-2} = -15 \cdot 8 \cdot 4 + 80(6)$

$= +0 kN\text{-}m.$

Between A and B, SF changes its sign and is equal to zero at distance

$\therefore 15x - 80 = 0$

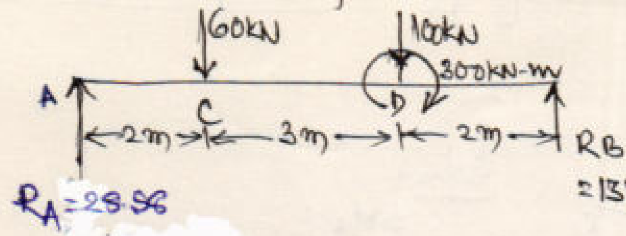
$x = \frac{80}{15} = 5.333$ mts

M_{2-2} when x = 5.333 mts,

$M_{2-2} = -15 \cdot \frac{(5.33)^2}{2} + 80(3.333)$

$= 53.33 \ kN\text{-}m$

Draw SFD and BMD for the beam shown in fig.



To find reaction

$R_B \times 7 = 60\times2 + 100\times5 + 300$

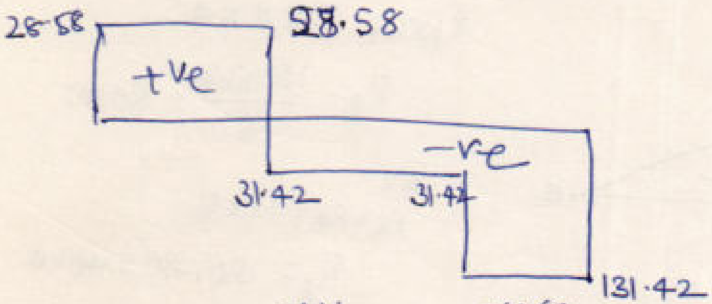
$R_B = 131.42$ kN.

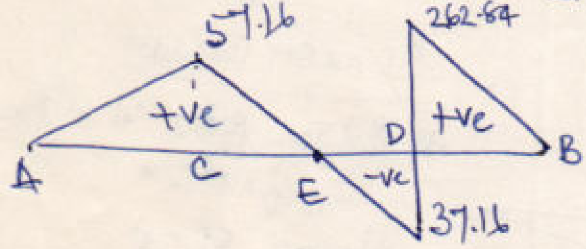
$\Sigma V=0$

$R_A + R_B = 100+60$ kN

$R_A = -R_B + 160$

$\boxed{R_A = +28.58 \text{ kN}}$ ↑





SF and BM Calculation

Between D and B at "x" from B.

$SF_{1-1} = -131.42$ kN.

$M_{1-1} = 131.42 \times x$

when $x=0$ $M=0$

$x=2$ $M=+262.84$ kN-m

Between C and D at 'x' from B

$SF_{2-2} = -131.42+100$

$= -31.42$ kN.

$M_{2-2} = 131.42\times x - 100\times(x-2) - 300$

$x=2$ $M_{2-2} = 262.84 - 300 = -37.16$ kN-m.

$x=5$ $M_{2-2} = 131.42\times5 - 100\times3 - 300$

$= +57.16$ kN-m.

LHS

Between A and C at 'x' from A.

$SF_{3-3} = +28.58$ kN

$M_{3-3} = +28.58 . x$

$x=0$ $M=0$

$x=2$ $M = 57.16$ kN-m

RHS AC

$SF = -131.42+60+100 = +28.58$ kN

SF const between A & C

$M_{3-3} = +131.42 . x - 100(x-2) - 300 - 60(x-5)$

$x=5$; $M = 57.16$ kN-m

$x=7$ $M = 0$ kN-m

Point E is called Point of Contraflexure where BM is zero and BM changes it sign from +ve to -ve

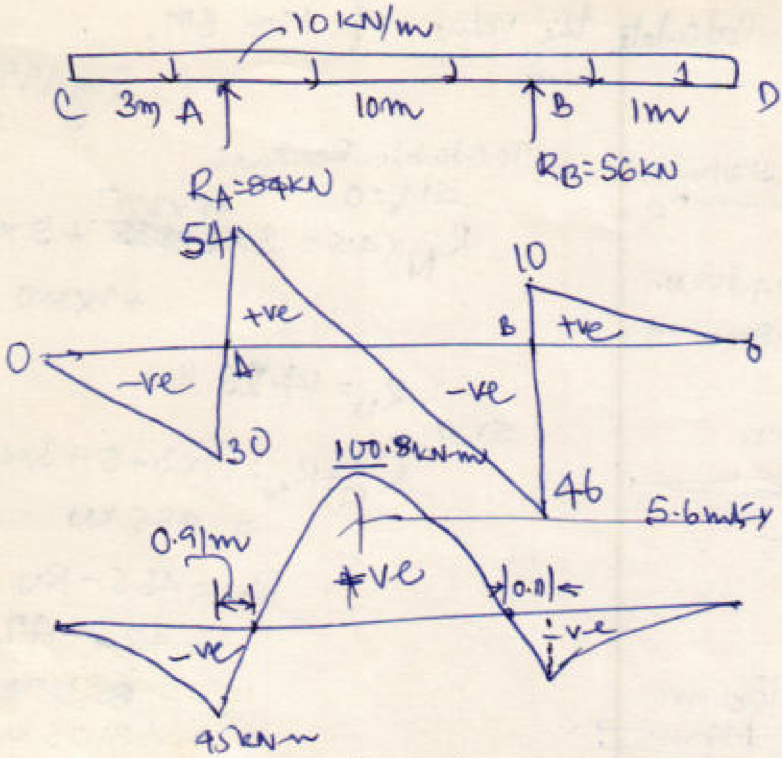
Then $131.42x - 100(x-2) - 300 = 0$

$131.42x - 100x + 200 - 300 = 0$

$31.42x = 100$

$x = \frac{100}{31.42} = 3.18$ mts from 'B'

Draw SFD and BMD for the beam shown in fig.



Soln: Taking moment about A,

$R_B \times 10 = 10 \times 14 \times 4$

$\boxed{R_B = 56\,kN}$

$\Sigma V = 0$

$R_A + R_B = 10 \times 14$

$R_A = -56 + 140$

$\boxed{R_A = 84\,kN}$

BM and SF Calculation

Between B and D at 'x' from 'D'

$SF_{1-1} = +10 \cdot x$

$x = 0 \quad SF = 0$

$x = 1 \quad SF = 10\,kN.$

$M_{1-1} = -10 \cdot x \cdot x/2$

$x = 0, \quad M = 0$

$x = 1 \quad M = -5\,kN\text{-}m$

Between AB at a distance 'x' from D.

$SF_{1-1} = 10 \cdot x - 56.$

$x = 1m \quad SF = -46\,kN \qquad M_{1-1} = -10 \cdot x \cdot x/2 + 56(x-1)$

$x = 11m \quad SF = +54\,kN. \qquad x = 1 \quad M_{1-1} = -5\,kN\,m$

$x = 11 \quad M_{1-1} = -45\,kN\text{-}m$

Between C and A at a distance 'x' from C.

$SF_{11} = -10 \cdot x$

$x = 0 \quad SF = 0$

$x = 3 \quad SF = -30\,kN$

$M_{1-1} = 10x \cdot x/2$

$x = 0 \quad M = 0$

$x = 3 \quad M = 45\,kN\text{-}m$

Between A and B,

$SF = 0$

$10x - 56 = 0$

$x = 5.6$ mts from D

M_{1-1} at 5.6 mts

$M_{1-1} = -10 \cdot (5.6)\frac{(5.6)}{2} + 56(4.6)$

$= +100.8\,kN\text{-}m$

Distance of point of Contraflexure.

$$-10\frac{x^2}{2} + 56(x-1) = 0$$

$$-5x^2 + 56x - 56 = 0$$

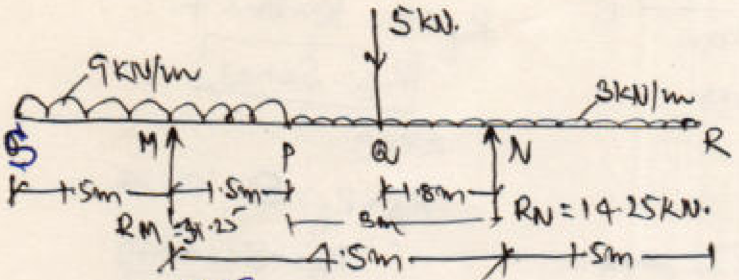
$$x^2 - 11.2x + 11.2 = 0$$

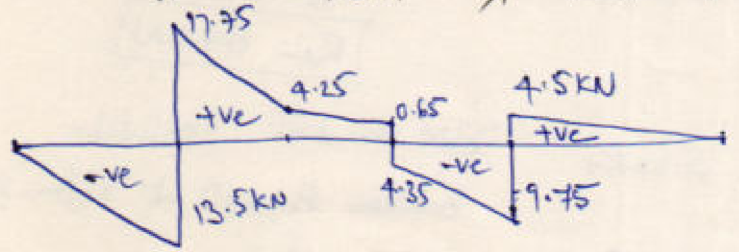
$$x = \frac{-(-11.2) \pm \sqrt{(11.2)^2 - 4(1)(+11.2)}}{2(1)}$$

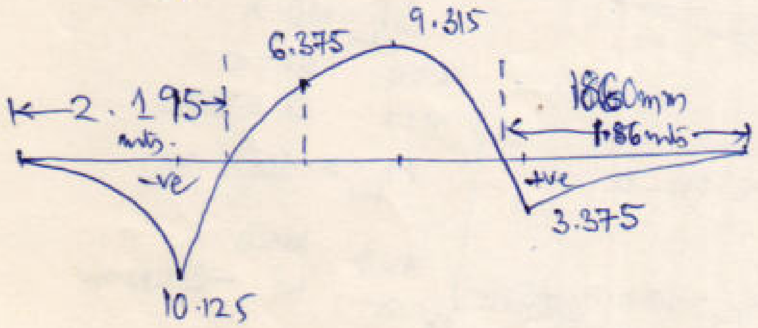
$= 10.09$ or 1.11 from D

Draw SFD and BMD. Indicate the values of all salient points. Locate point of Contraflexure, if any. Further Calculate the value of Max BM.

Aug 1999







To Calculate Reactions:

$\Sigma M_M = 0$

$R_N \times 4.5 = 3 \times \cancel{5} \times \cancel{2.25}$ (45 × 3.75) $+ 5 \times 2.7$

$+ 9 \times 3 \times 0$

$R_N = 14.25$ kN

$\Sigma V = 0$

$R_M + R_N = 9 \times 3 + 5 + 3 \times 4.5$

$= 45.5$ kN

$R_M = 45.5 - R_N$

$= 45.5 - 14.25$ kN

$= 31.25$ kN

To Calculate SF and BM (N and R)

$SF_{1-1} = +3 \times x$

when $x=0$ SF = 0

$x = 1.5$ SF = +4.5 kN.

$M_{1-1} = -3 \cdot x \cdot x/2$

$x=0$ M = 0

$x = 1.5$ $M = -3 \frac{(1.5)^2}{2} = -3.375$ kN-m

Between (Q and N)

$SF_{2-2} = +3 \cdot x - 14.25$

$x=1.5$ SF = −9.75 kN

$x = 3.3$ SF = −4.35 kN.

$M_{2-2} = -3 \cdot x \cdot x/2 + 14.25(x-1.5)$

$x=1.5$ M = −3.375 kN-m

$x = 3.3$ M = +9.315 kN-m

Between P and Q

$SF_{3-3} = 3 \cdot x - 14.25 + 5$

$x = 3.3$ SF = +0.65 kN

$x = 4.5$ SF = +4.25 kN

$M_{3-3} = -3 \cdot x \cdot x/2 + 14.25(x-1.5) - 5(x-3.3)$

When $x=3.3$ M = +9.315 kN-m

$x = 4.5$ M = +6.375 kN-m

Between P and M

$SF_{4-4} = -9 \cdot x + 31.25$

When $x = 1.5$ SF $= -9 \times 1.5 + 31.25$

$= +17.75$ kN

$x = 3.0$ SF $= -9 \times 3 + 31.25$

$= +4.25$ kN.

$M_{4-4} = -9 \cdot x \cdot x/2 + 31.25(x-1.5)$

$x=1.5$ $= -10.125$ kN-m.

$x = 3.0$ $= +6.375$ kN-m

Between S and M

$SF_{S-S} = -9 \cdot x$

when $x=0$ $\quad SF = 0$

$x = 1.5$ $\quad SF = -13.5$ kN

$M_{S-S} = -9 \cdot x \cdot x/2$

$x=0$ $\quad M = 0$

$x=1.5$ $\quad M = -9 \cdot \frac{(1.5)^2}{2} = -10.125$ kN-m

TO find the point of Contraflexure:

Equate the BM eqn to zero, i.e.

$$-\frac{3x^2}{2} + 14.25(x-1.5) = 0$$

$$-3x^2 + 28.5(x-1.5) = 0$$

$$x^2 - 9.5(x-1.5) = 0$$

$$x^2 - 9.5x + 14.25 = 0$$

$$\frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$= \frac{-(-9.5) \pm \sqrt{(9.5)^2 - 4(1)(14.25)}}{2(1)}$$

$$= \frac{9.5 \pm 5.76}{2}$$

$= 1.86$ mts or 10.51 mts

and also

$$-\frac{9x^2}{2} + 31.25(x-1.5) = 0$$

$$-9x^2 + 62.5(x-1.5) = 0$$

$$x^2 - 6.94(x-1.5) = 0$$

$$x^2 - 6.94x + 10.42 = 0$$

$$\frac{-(-6.94) \pm \sqrt{(-6.94)^2 - 4(1)(10.42)}}{2(1)}$$

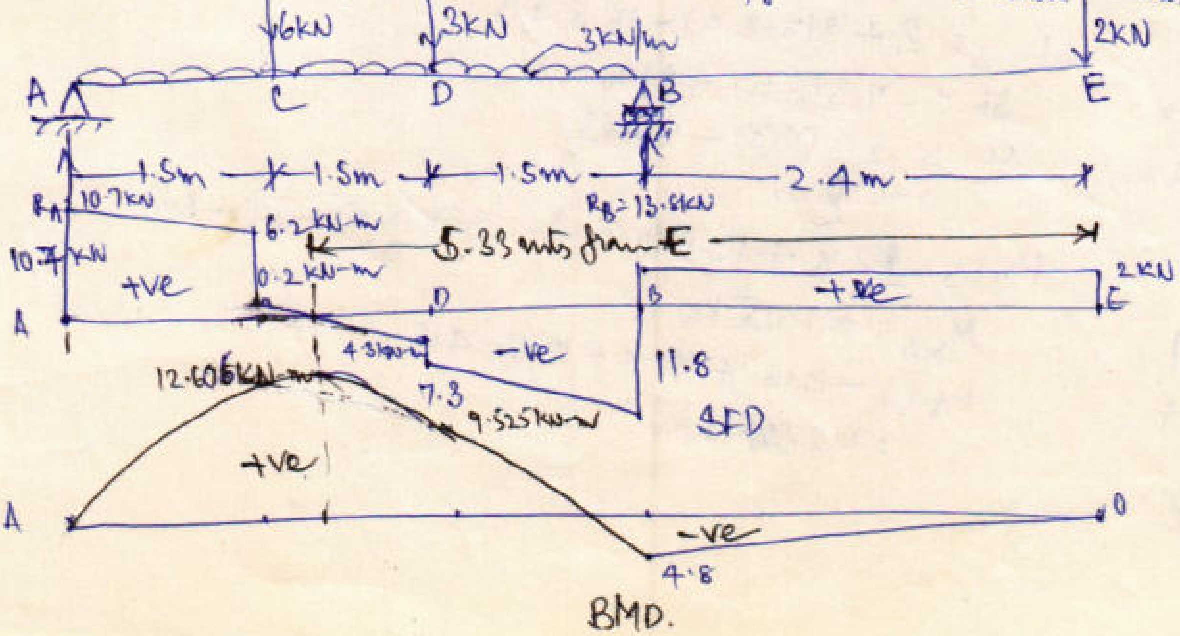
$$= \frac{6.94 \pm 2.55}{2}$$

$= 2.195$ or 4.745 m

The point of Contraflexure is at 1.86 mts from R
2.195 mts from S.

Draw BMD and SFD for the beam shown in fig. Mark the salient values on the fig.

March 2000



Reactions at A and B $\Sigma M_A = 0$

$$R_B \times 4.5 = 6 \times 1.5 + 3 \times 3 + 3 \times 4.5 \times 2.25 + 2 \times 6.9 \text{ mts}$$

$$R_B = 13.8\text{kN}.$$

$\Sigma V = 0$

$$R_A + R_B = 6 + 3 + \cancel{13.5} + 2$$

$$R_A = -13.8 + 24.5 = 10.7\text{kN}.$$

To find SF and BM:

Between B and E. Consider a section at a distance 'x' from E.

$$SF_{1-1} = +2\text{kN}$$

$$M_{1-1} = -2 \cdot x$$

When $x = 0$ $M = 0$

$x = 2.4$ $M = -2 \times 2.4 = -4.8\text{kN-m}$

Between B and D. Consider a section at a distance x from E

$$SF_{2-2} = +2 - 13.8 + 3.(x - 2.4)$$

When $x = 2.4$ $SF = 2 - 13.8 + 0$

$= -11.8\text{kN}$

$x = 3.9$ $SF = 2 - 13.8 + 3(1.5) = -7.3\text{kN}$

$$M_{2-2} = -2x + 13.8 \times (x - 2.4) - 3.\frac{(x - 2.4)^2}{2}$$

$x = 2.4$ $M_{2-2} = -4.8\text{ kN-m}$

$x = 3.9$ $M_{2-2} = -2(3.9) + 13.8(1.5) - 3 \times \frac{1.5^2}{2}$

$= +9.525\text{ kN-m}$

Between C and D. Consider a section at a distance 'x' from E.

$$SF_{3-3} = 2 + 3(x - 2.4) - 13.8 + 3$$

$x = 3.9\text{m}$ $SF = -7.3 + 3 = -4.3\text{kN}.$

$x = 5.4\text{m}$ $SF = 2 + (3)(3) - 13.8 + 3$

$= +0.2\text{ kN}$

$$M_{3-3} = -2x + 13.8(x - 2.4) - \frac{3(x - 2.4)^2}{2} - 3(x - 3.9)$$

$x = 3.9$ $M_{3-3} = +9.525\text{ kN-m}$

$x = 5.4$ $M_{3-3} = -10.8 + 41.4 - 13.5 - 4.5$

$= +12.6\text{ kN-m}$

B.

Between A and C. Consider a section at a distance 'x' from A.

$SF_{4-4} = +10.7 - 3.x$

$x=0$ $\quad SF = 10.7\,kN$

$x=1.5$ $\quad SF = 10.7 - 4.5 = 6.2\,kN$

$M_{4-4} = +10.7.x - 3.x.x/2$

$x=0$ $\quad M=0$

$x=1.5$ $\quad M = 10.7\times1.5 - 3\frac{(1.5)^2}{2}$

$= +12.6\,kN\text{-}m$

Between C and D, SF=0.

$2+3x-7.2-13.8+3=0$

$3x=16$

$x=\frac{16}{3}=5.333$ mts

Max BM.

$M = -2(5.33)+13.8(2.93) - \frac{3(2.93)^2}{2} - 3(1.43)$

$\boxed{M_{max} = 12.606\text{ mts}}$

To find the location of Contraflexure.

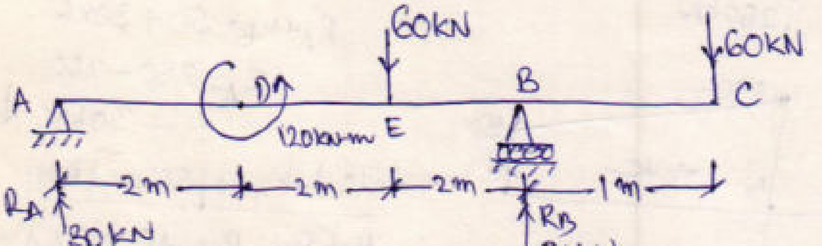
Equate the BM equation between D and B to zero

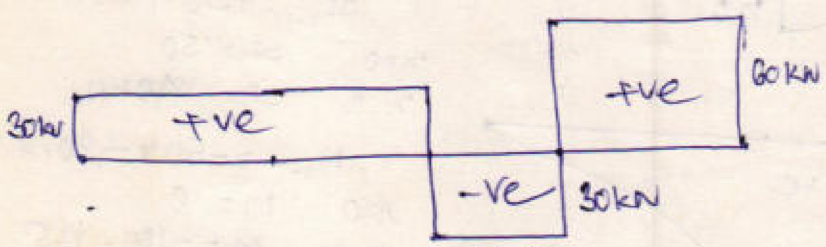
$-2x + 13.8(x-2.4) - \frac{3(x-2.4)^2}{2} = 0$

$-4x + 27.6(x-2.4) - 3(x-2.4)^2 = 0$

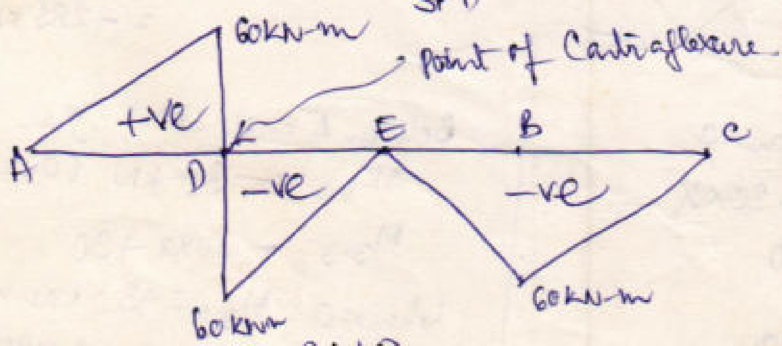
$-4x + 27.6x - 27.6\times2.4 - 3(x^2+(2.4)^2-2(2.4)x) = 0$

Draw BMD and SFD for the beam and mark the Salient values on the diagrams. Aug 2000





SFD



BMD

Soln: $\Sigma M_A=0$

$R_B\times6 = 60\times7+60\times4-120$

$R_B = 90\,KN$

$\Sigma V=0$

$R_A+R_B=120$

$R_A = 120-90$

$= 30\,kN$

Calculate SF and BM values.

Between B and C at x' from C.

$SF_{1-1} = +60\,kN$ (Constant).

$M_{1-1} = -60x$

when $x=0$ $\quad M=0$

$x=1$ $\quad M = -60\,kN\text{-}m$

Between E and B at x' from C

$SF_{2-2} = +60-90 = -30\,kN$. (Const)

$M_{2-2} = -60x + 90(x-1)$

when $x=1$ $\quad M_{2-2} = -60\,kN\text{-}m$

$x=3$ $\quad M_{2-2} = -0\,kN\text{-}m$

Between D and E at 'x' from C

$SF_{3-3} = +60 - 90 + 60 = +30\ KN$ Const

$M_{3-3} = -60 \cdot x + 90(x-1) - 60(x-3)$

When $x=3$ $M = 0$

$x=5$ $M = -300 + 360 - 120 = -60\ KN\text{-}m$

Between A and D at 'x' from A

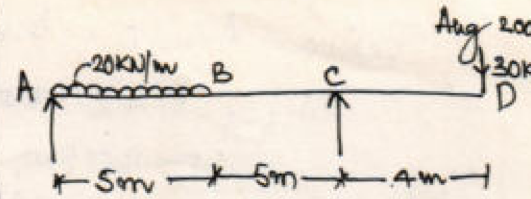
$SF_{4-4} = +30\ KN$

$M_{4-4} = +30 \times x$

When $x=0$ $M = 0$

$x=2$ $M = +60\ KN\text{-}m$

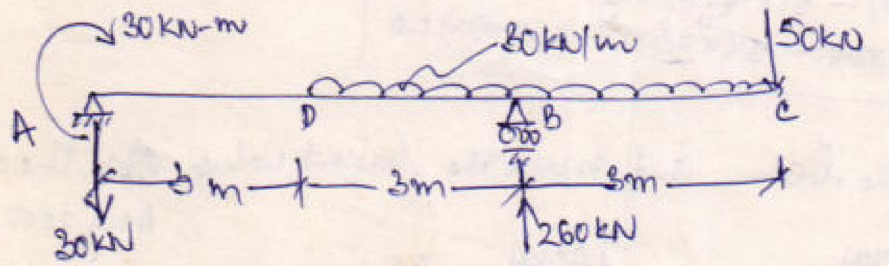
Aug 2001



Soln given in Page 12.

Draw SFD and BMD for the beam and locate the position of point of Contraflexure.

March 2001



Soln:

$\Sigma M_A = 0$

$R_B \times 6 = 50 \times 9 + 30 \times 6 \times 6 + 30$

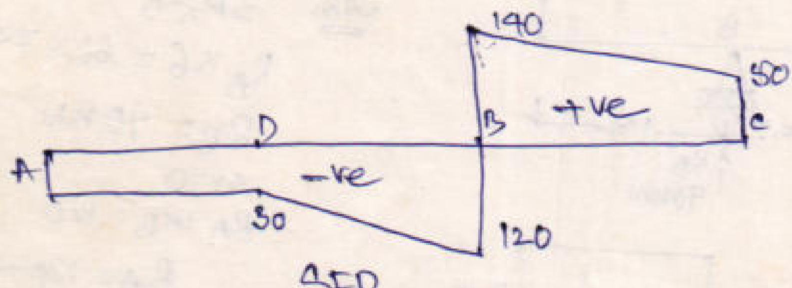
$R_B = 260\ KN \uparrow$

$\Sigma V = 0$

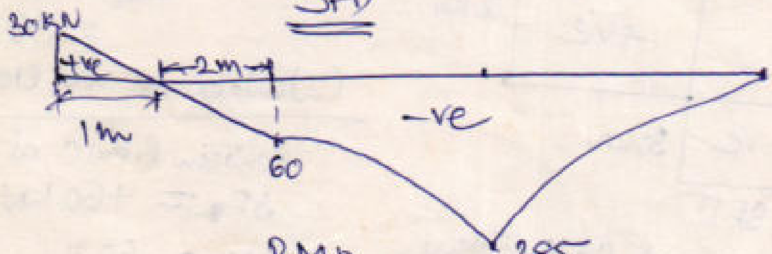
$R_A + R_B = 50 + 30 \times 6$

$R_A = 230 - 260$

$= -30\ KN \downarrow$



SFD



BMD

Calculation of SF and BM.

Between B and C at x from C

$SF_{1-1} = +50 + 30 \times x$

$x=0$ $SF = 50$

$x=3$ $SF = 140\ KN$.

$M_{1-1} = -50x - 30 \cdot x \cdot x/2$

$x=0$ $M = 0$

$x=3$ $M = -150 - 135$

$= -285\ KN\text{-}m$.

Between D and B at 'x' from C

$SF_{2-2} = +50 - 260 + 30 \times x$

When $x=3$ $SF = +50 - 260 + 90$

$= -120\ KN$

$x=6$ $SF = +50 - 260 + 180$

$= -30\ KN$

$M_{2-2} = -50 \times x + 260(x-3) - 30 \cdot x \cdot x/2$

$x=3$ $M = -285\ KN\text{-}m$

$x=6$ $M = -60\ KN\text{-}m$.

Between A and D at 'x' from A.

$SF_{3-3} = -30\ KN$ (Const)

$M_{3-3} = -30 \times x + 30$

When $x=0$ $M_{3-3} = +30\ KN\text{-}m$

$x=3$ $M_{3-3} = -90 + 30$

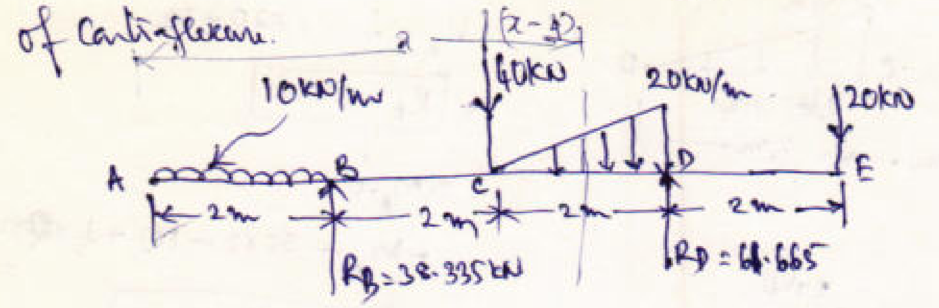
$= -60\ KN\text{-}m$

To find the point of Contraflexure equate BM eqn between A and D to zero

$-30x + 30 = 0$

$30x = 30$ $x = 1$ mt from A

Draw the SF and BMDs for a overhanging beam shown in fig and locate the points of Contraflexure. (15)



Soln

$\Sigma M_B = 0$

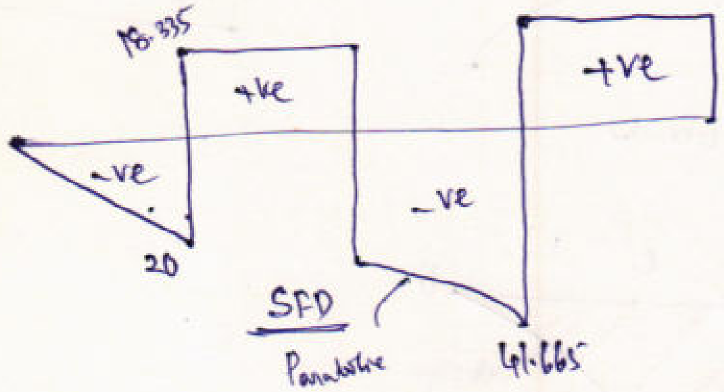
$-R_D \times 4 + 20 \times 6 + (\frac{1}{2} \times 2 \times 20)(\frac{2}{3} \times 2 + 2)$

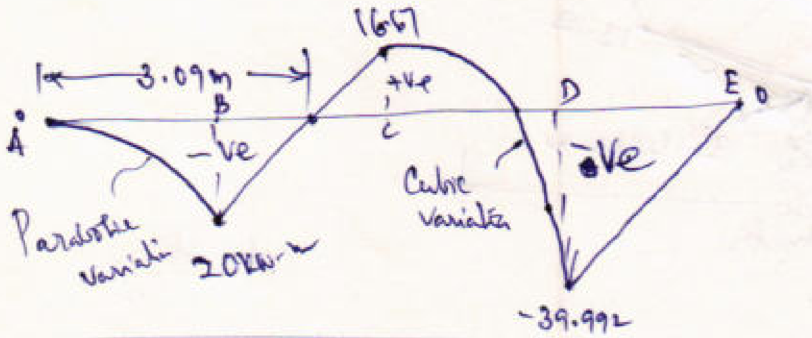
$+ 40 \times 2 - 10 \times 2 \times 1 = 0$

$R_D = 61.665$ kN

$R_A = 10 \times 2 + 40 + \frac{1}{2} \times 2 \times 20 + 20 - 61.665$

$= 38.335$ kN





$\frac{20}{2} = \frac{y}{x}$

$y = 10(x)$

AB

SF at 0-0 $= -(10x)$

$x = 0$ SF $= -0$

$x = 2$ SF $= -20$

BM at 0-0 $= -(10 \cdot \frac{x^2}{2})$

$x = 0$ M $= 0$

$x = 2$ M $= -20$ kN-m

B & C

SF at Q-Q $= -(10 \times 2) + 38.335$

SF $= +18.335$

BM at 0-0 $= -(10 \times 2)(x-1) + 38.335(x-2)$

$x = 2$ M $= -20$ kN-m

$x = 4$ M $= +16.67$ kN-m

C & D

SF at (3)-(3) $= -(10 \times 2) + 38.335 - 40 - (\frac{1}{2} \times (x-4) \times 10(x-4))$

Intensity of loading at $x = \frac{20(x-4)}{2} = 10(x-4)$

when $x = 4$ SF $= -20 + 38.335 - 40 - 0 = -21.665$ kN

$x = 6$ SF $= -41.665$ kN

BM $= (-10 \times 2)(x-1) + 38.335(x-2) - 40(x-4) - (\frac{1}{2} \times 10(x-4) \times (x-4))\frac{(x-4)}{3}$

$x = 4$ M $= +16.67$ kN-m | $x = 6$, M $= -39.992$

D and E

SF $= +20$ kN

M $= -20 \cdot x$

$x = 0$ M $= 0$

$x = 2$ M $= -40$

Points of Contraflexure between B & C and C & D

$-(10 \times 2)(x-1) + 38.335(x-2) = 0$

$-20x + 20 + 38.335x - 76.67 = 0$

$18.335x = 56.67$

$\boxed{x = 3.09 \text{ m}}$

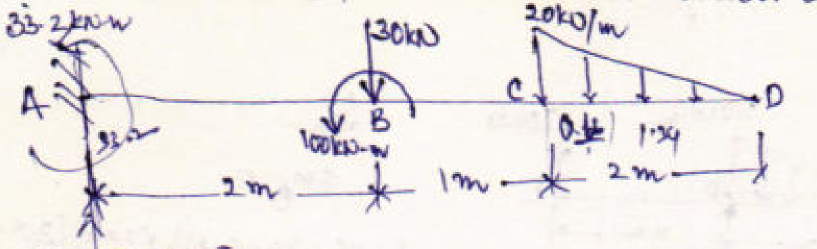
C & D

$-20x + 20 + 38.335x - 76.67 - 40x + 160 - 1.66(x-4)^3 = 0$

$-21.675x - 1.66(x-4)^3 + 103.33 = 0$

$\boxed{x = 4.731 \text{ m}}$

Draw SFD and BMD for the beam loaded as shown in fig



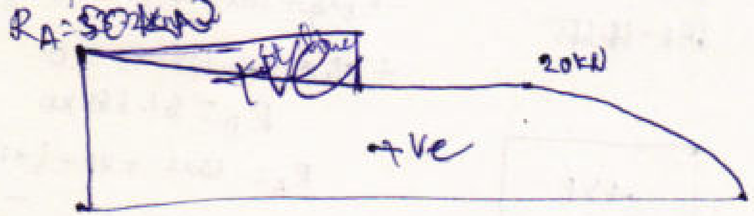
$R_A = \frac{1}{2} \times 2 \times 20 + 30$

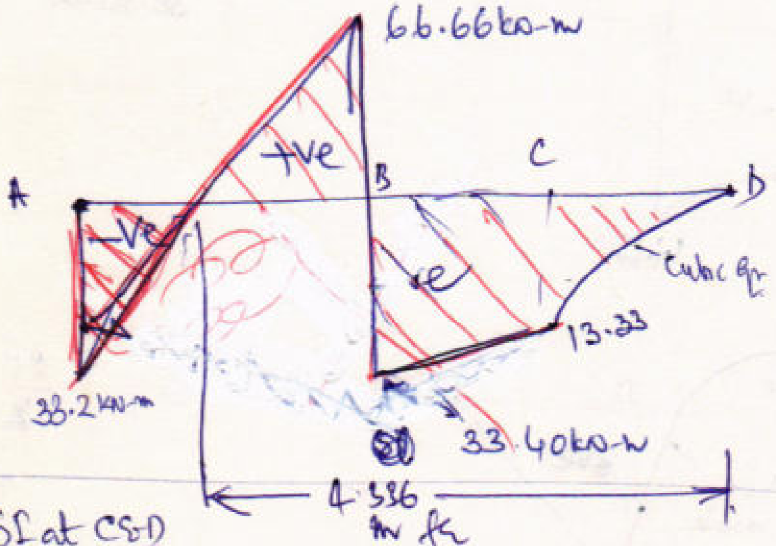
$\boxed{R_A = 50\,kN}$

$\Sigma M_A = 0$

$-M_A + 30 \times 2 - 100 + \frac{1}{2} \times 2 \times 20 \times 3.$
$= 0$

$\boxed{M_A = 33.2\,kN\text{-}m}$





$\frac{h}{20} = \frac{x}{2}$

$h = 10.x$

SF at C&D

$SF = +\frac{1}{2} \times x \times 10.x$

$x = 0 \quad SF = 0$

$x = 2 \quad SF = +20\,kN$

$M = -\left(\frac{1}{2} \times x \times 10.x\right)\left(\frac{x}{3}\right)$

$x = 0 \quad M = 0$

$x = 2 \quad M = -13.33\,kN\text{-}m$

B&C

$SF = +\left(\frac{1}{2} \times 2 \times 20\right)$

$SF = 20\,kN$

$M = \left(\frac{1}{2} \times 2 \times 20\right)(x - 1.34)$

$x = 2 \quad M = -13.33\,kN\text{-}m$

$x = 3 \quad M = \cancel{-49.5}\,kN\text{-}m$ 7.50 $-33.4\,kN\text{-}m$

A&B

$SF = +\left(\frac{1}{2} \times 2 \times 20\right) + 30$

$SF = 50\,kN$

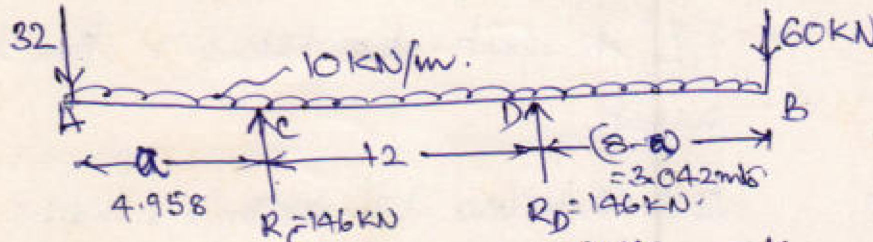
$M = \left(\frac{1}{2} \times 2 \times 20\right)(x - 1.34) - 30(x - 3) + 100$

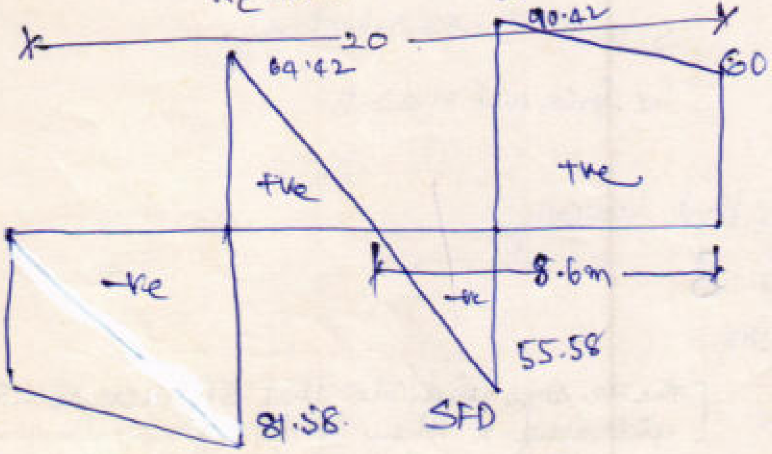
$x = 3 \quad M = \cancel{80.2\,kN\text{-}m} + 66.6\,kN$

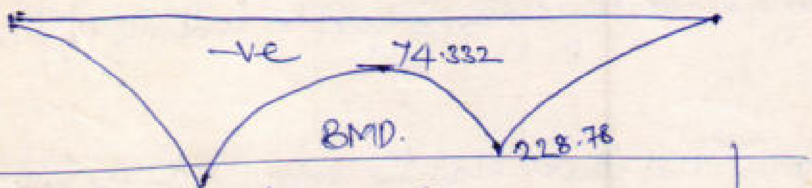
$x = 5 \quad M = -33.2\,kN\text{-}m$

A beam AB 20m long supported on two intermediate props 12m apart carries a UDL of 10KN/m together with concentrated loads of 32 KN at the left end A and 60KN at the right end B. The props are so located that the reaction is same at each support. Determine the position of the props and draw BM and SF diagrams.

Feb 2002







Soln:

$\Sigma V=0$

$R_C+R_D = 10\times20+32+60$

$2R_C = 292$

$R_D = R_C = 146KN$

Let the props be positioned at distances as shown in fig.

Taking moment A, $\Sigma M_A=0$

$60\times20 + 10\times20\times10 = 146\times a + 146(12+a)$

$2\times146\,a + 1752 = 3200$

$a = 4.958$ mts

Calculation of SF and BM.

Between B and D at a distance x from B.

$SF_{1-1} = +60+10.x$

when $x=0$ $SF = +60$ KN

$x=3.042$ $SF = 90.42$ KN

$M_{1-1} = -60x - 10.x.x/2$

$x=0$ $M=0$

$x=3.042$ $M = -228.78$ KN-mts

Between C and D at a distance 'x' from B.

$SF_{2-2} = +60+10.x-146$

when $x=3.042$ $SF = 60+30.42-146$

$= -55.58$ KN

$x=15.042$ $SF = +64.42$KN

SF=0 at $60+10x-146=0$

$10x=86$

$x=8.6$M.

$M_{2-2} = -60.x - 10.x.x/2 + 146(x-3.042)$

$x=3.042$ $M_{2-2} = -228.78$ KN-m.

$x=15.042$ $M_{2-2} = -281.56$ KN-m

$x=8.6$m $M_{2-2} = -74.332$ KN-m

Between A and C at a distance 'x' from A.

$SF_{3-3} = -32-10x$

$x=4.958$ $SF = -81.58$ KN

$x=0$ $SF = -32$ KN

$M_{3-3} = -32.x - 10.x.x/2$

$x=0$ $M=0$

$x=4.958$ $M = -281.56$ KN-m

Relation between load intensity, SF and BM

Aug 2001, March 2001
Sept 2000
Sept 1999

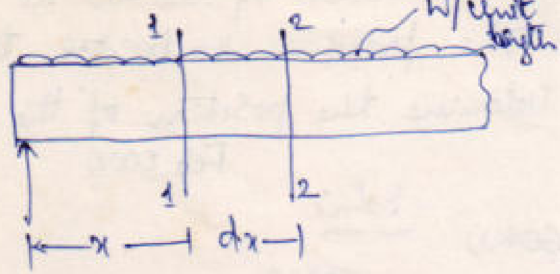
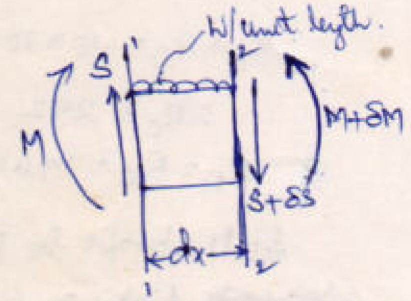


Fig shows a beam subjected to external loading. Consider the Equilibrium of the portion of the beam between Sections 1-1 and 2-2, dx apart at a distance 'x' from left Support.



Let the Shear force at the sections 1-1 and 2-2 be S and S+δS respectively.

The forces and moments

Resolving the forces on this part Vertically,

$$S + dS + W dx = S$$

$$dS = -W\delta x$$

$$\frac{dS}{dx} = -W$$ [The -ve sign indicates that SF decreases with increasing x when loaded uniformly downwards]

i.e the rate of change of Shearforce is equal to ~~rate of change~~ intensity of loading.

or Slope of Shear force diagram at any section is equal to intensity of loading at that Section.

Taking moments of the forces and Couples about the section 2-2, we have

$$M + dM = M + S.dx - W.dx.\frac{dx}{2}$$ $\left\{ M + S.dx - W.dx\frac{dx}{2} - (M + dM) = 0 \right.$

Ignoring higher powers of small quantities,

$$dM = S.dx$$

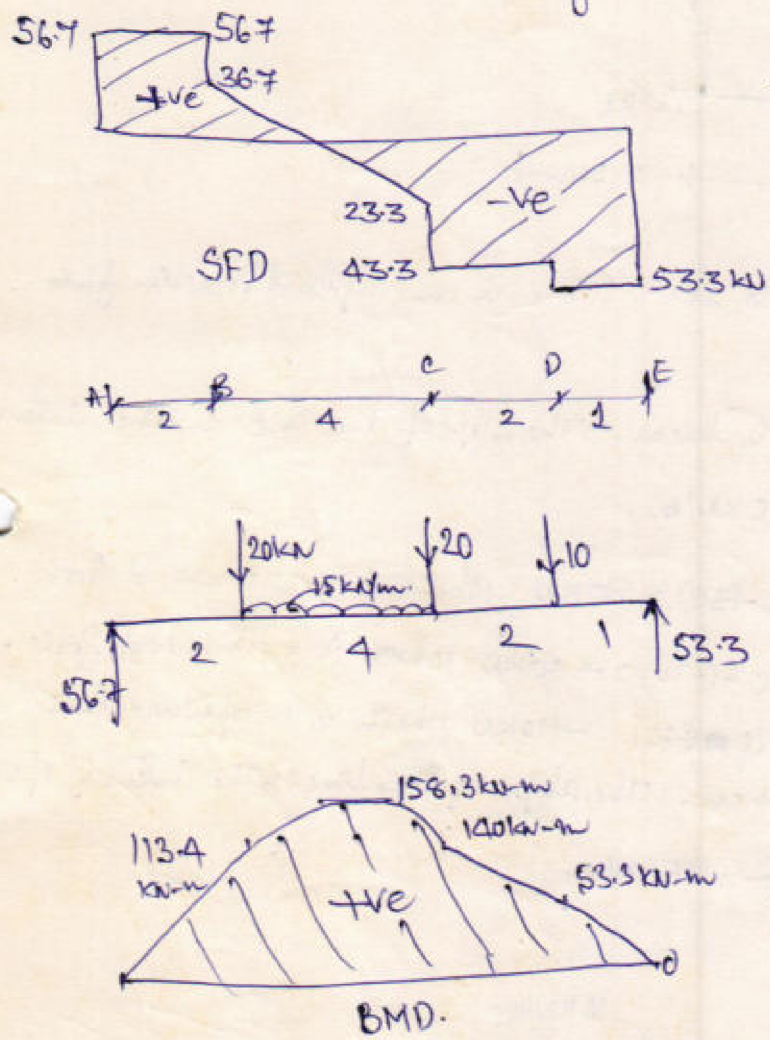
$$\frac{dM}{dx} = S$$

i.e the rate of change of bending moment is equal to shear force. or

Slope of bending moment diagram at any section is equal shearforce at that section.

Obtaining loading pattern using SFD and BMD

A shear force diagram for a beam, simply supported at its ends is as shown in fig. Find the forces acting on the beam and draw the BMD.



Soln:

point A: SF increases from 0 to 56.7 kN. Therefore an upward reaction force of 56.7 kN is acting at point A.

point B: There is a sudden reduction in SF from 56.7 to 36.7 kN ie. a downward force of 56.7 − 36.7 = 20 kN is acting at point B.

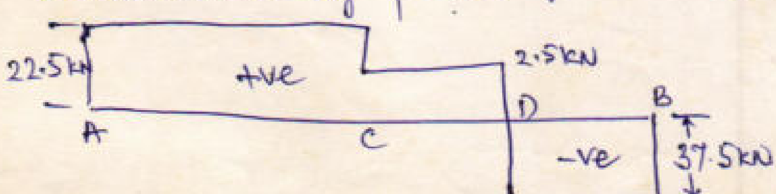
point BC: There is a linear variation of SF from 36.7 to −23.3 kN over a length of 4 m. ie UDL of $\frac{36.7-(-23.3)}{4}$ = 15 kN/m is acting on portion BC.

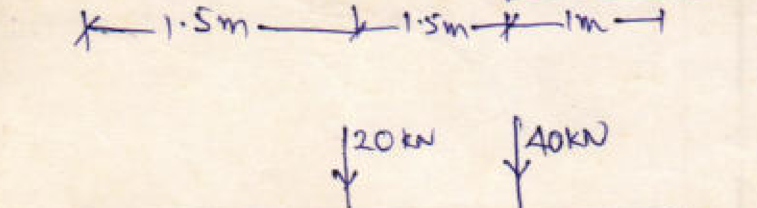
point C: Sudden change in SF from −23.3 kN to −43.3 kN indicates a downward force of −23.3 − (−43.3) = 20 kN acting at C.

point D:
A downward force of −43.3 − (−53.3) = 10 kN is acting at D.

point E: The SF varies from −53.3 kN to zero. Therefore an upward reaction of 0 − (−53.3) = 53.3 kN is acting at point E

Obtain the loading pattern of the beam by making use of SFD





point A: SF increases from 0 to 22.5 kN. There is an upward reaction force of 22.5 kN acting at point A

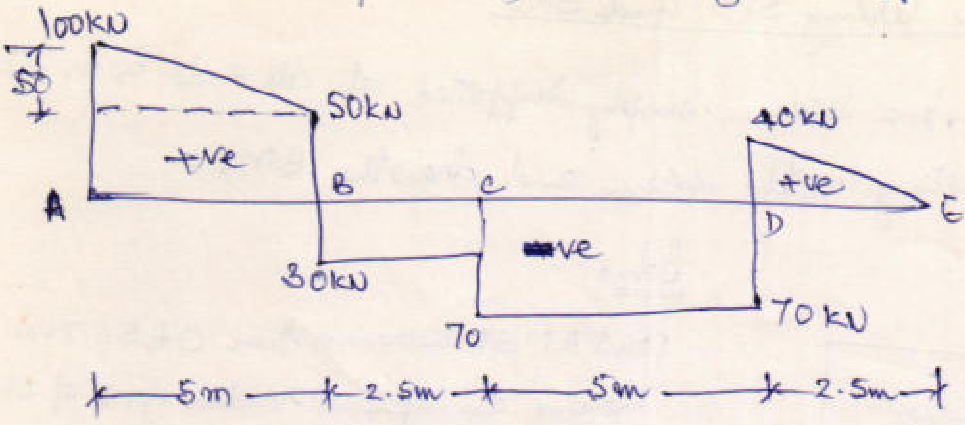
point C: There is a downward force of 22.5 − 2.5 = 20 kN at C.

point D: There is a sudden change in SF from +2.5 kN − (37.5) = 40 kN at D.

point B: from ~~0~~ −37.5 to zero

0 − (−37.5) = 37.5 kN at point B acting upward

Obtain a loading pattern from SFD given in fig



Soln: At point A, SF changes from 0 to 100. There is an upward reaction force of 100KN acting at A.

In the region AB, the SF variation is linear. The slope of this line is the intensity of Udl. $W = \frac{50}{5} = 10 KN/m$.

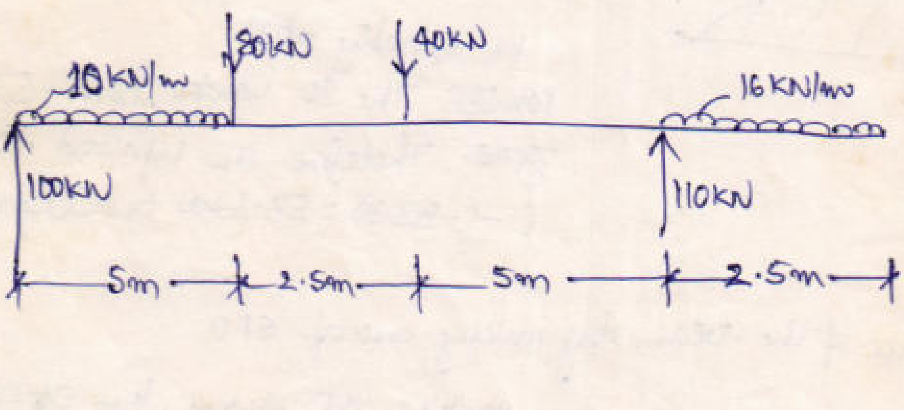
At point B, SF changes from +50-(30) = 80KN which is a downward force.

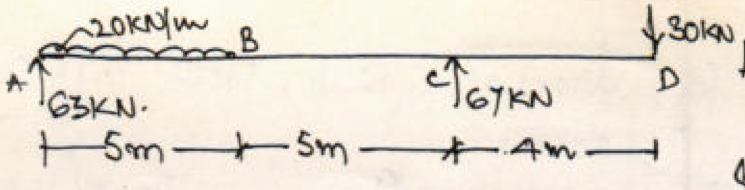
At point C, SF changes from -30-(-70) = +40KN which is a downward force.

At point D, SF changes from -70 (40) = -110KN which is a upward force.

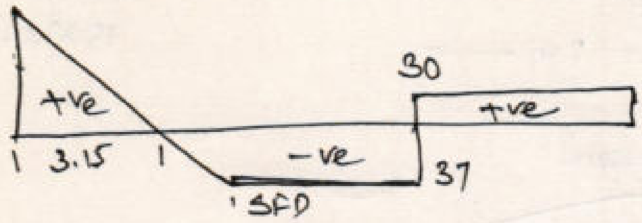
At point E & D, SF variation is linear. The slope of this line is the intensity of Udl

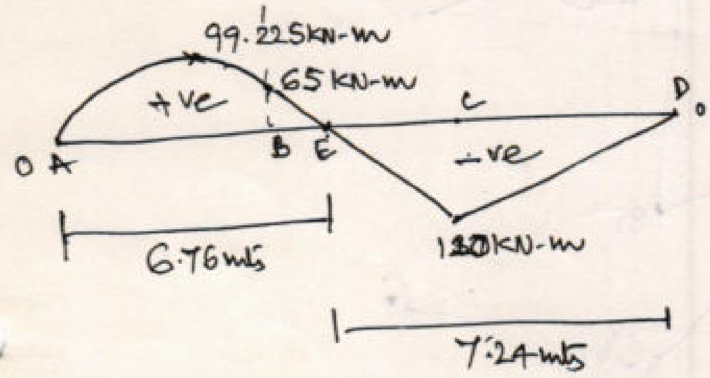
$$W = \frac{40}{2.5} = 16 KN/m,$$





Aug 2001





Soln. $\Sigma M_A = 0$

$R_C \times 10 = 30 \times 14 + 20 \times 5 \times 2.5$

$R_C = \frac{670}{10} = 67\,KN.$

$\Sigma y = 0$

$R_A + R_B = 20 \times 5 + 30$

$R_A = 130 - 67 = 63\,KN$

Calculation of BM and SF:

Between C and D at 'x' from D.

$SF_{1-1} = +30\,KN.$ (Const)

$M_{1-1} = -30 \times x$

$x = 0 \quad M_{1-1} = 0$

$x = 4 \quad M = -120\,KN\text{-}m$

Between A and B at 'x' from A.

$SF_{3-3} = +63 - 20 \cdot x$

$x = 0 \quad SF = 63$

$x = 5 \quad SF = 63 - 100$
$= -37\,KN.$

$SF = 0$ at $x \cdot 20 = 63$

$x = \frac{63}{20} = 3.15\,mts$

$M_{3-3} = +63 \cdot x - 20 \cdot x \cdot x/2.$

$x = 0 \quad M = 0$

$x = 5 \quad M = 65\,KN\text{-}m$

$x = 3.15 \quad M = 99.23\,KN\text{-}m$

Between B and C at 'x' from D.

$SF_{2-2} = +30 - 67 = -37\,KN$ (Const)

$M_{2-2} = -30 \cdot x + 67 \times (x - 4)$

$x = 4 \quad M = -120\,KN\text{-}m$

$x = 9 \quad M = -270 + 67(5)$
$= 65\,KN\text{-}m.$

To find the point of Contraflexure

Equating BM eqn to zero.

$-30x + 67x - 268 = 0$

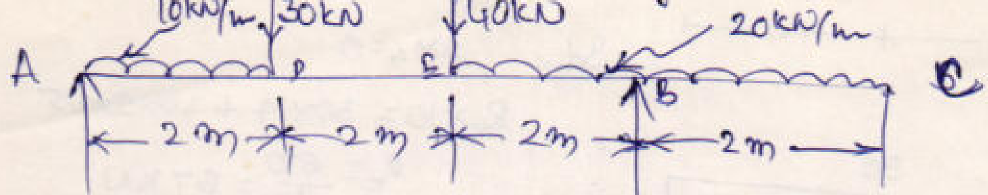
$37x = 268$

$\boxed{x = 7.24 \text{ from } D}$

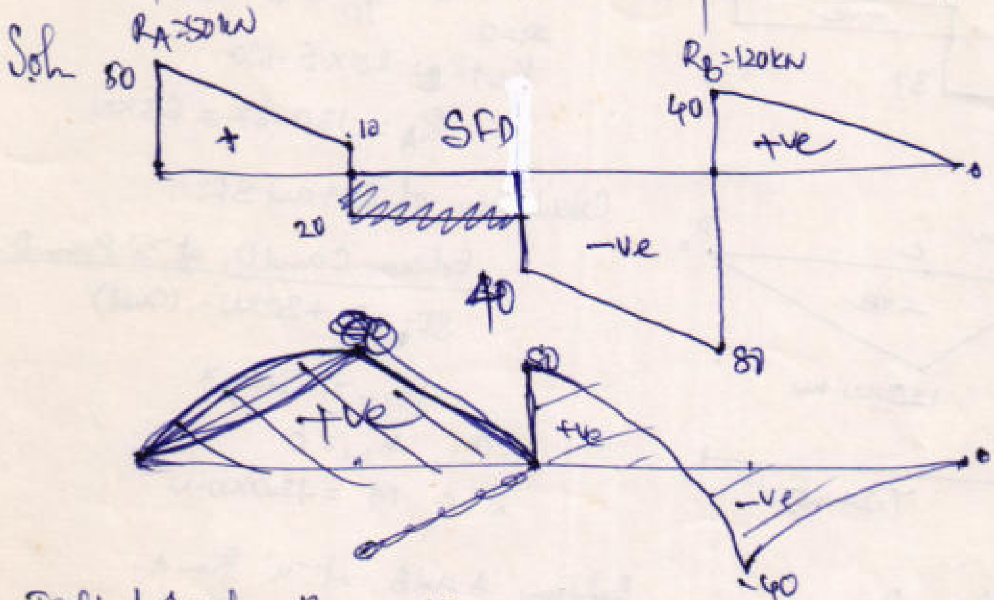
or 6.76 from A

5 For the beam shown in the fig, draw SF & BMD. Locate Point of Contraflexure if any

Dec 08 / Jan 09
15 Marks



Soln



To find reactions R_A and R_B $\quad \Sigma M_A = 0$

$$-R_B \times 6 + (20\times4)\times6 + 40\times4 + 30\times2 + (10\times2)(1) = 0$$

$$\boxed{R_B = 120}$$

$$\Sigma V = 0$$

$$R_A + R_B = (10\times2) + 30 + 40 + 80$$

$$\boxed{R_A = 50\text{ kN}}$$

Calculation of SF

Between A and D

SF at 1-1 $= +50 - 10.x$

When $x=0$ $\quad$ SF $= 50$

$x=2$ $\quad$ SF $= +10$

Between D and E

SF at 2-2 $= +50 - (10\times2) - 30$

$x=2$ $\quad$ SF $= -10$

$x=4$ $\quad$ SF $= -10$

Between E and B

SF at 3-3 $= +50 - (10\times2) - 30 - 40 - 20(x-4)$

When $x=4$ $\quad$ SF $= -60$ kN

$x=6$ $\quad$ SF $= +50 - 20 - 30 - 40 - 20\times2$
$= -80$ kN

Between B and C

SF at 4-4 $= +20.x$

When $x=0$ $\quad$ SF $= 0$

$x=2$ $\quad$ SF $= +40$ kN

Calculation of BM

Between B and C

BM at 1-1 $= -W.\frac{x^2}{2}$

$x=0$ $\quad$ M $= 0$

$x=2$ $\quad$ M $= \frac{20.2^2}{2} = -40$ kN-m

Between E and B

BM at 2-2 $= -\frac{Wx^2}{2} + 120(x-2)$

$x=2$ $\quad$ BM $= -40$ kN-m

$x=4$ $\quad$ BM $= -\frac{20\times4^2}{2} + 120\times2 = 80$ kN-m

Between D and E

$x=4$ $\quad$ BM $= -20\times4\times(x-2) + 120(x-2)$
$- 40(x-4)$

$x=6$ $\quad$ M $= 0$

M $= 80$ kN

Between A and D

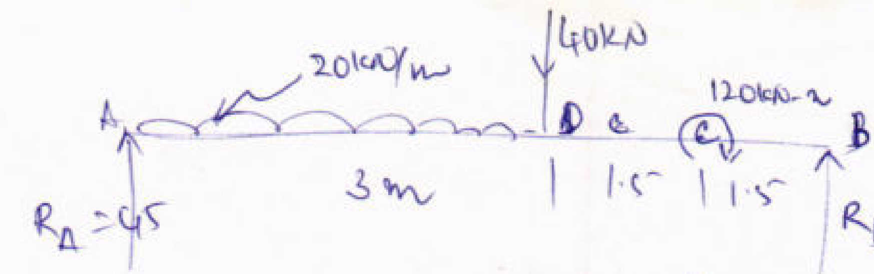
$x=6$ $\quad$ BM $= -20\times4\times(x-2) + 120(x-2)$

$x=8$ $\quad$ $- 10\frac{(x-6)^2}{2} - 40(x-4) - 30(x-6)$

M $= 80$

M $= 0$

(13)



June July 2009
16 marks

$6 \times R_B = 120 + 40 \times 3 + 20 \overset{\times 3}{} \times 1.5$

$R_B = 55$ kN
$R_A = 45$ kN

SF & BM calculation:

C & B SF = −55
BM = 55 × x
when x = 0 M = 0
x = 1.5 M = 82.5 kN-m

D & C SF = −55
BM: 55.x − 120
when x = 1.5 = −37.5 kN-m
x = 3 BM = 45 kN-m

A and D
SF = −55 + 40 + 20.(x − 3)
x = 3 SF = −15 kN-m
x = 6 SF = +45 kN

BM BM = 55.x − 120 − 40(x−3) − 20.$\frac{(x-3)^2}{2}$
x = 3 = 45 kN-m
x = 6 BM = 0

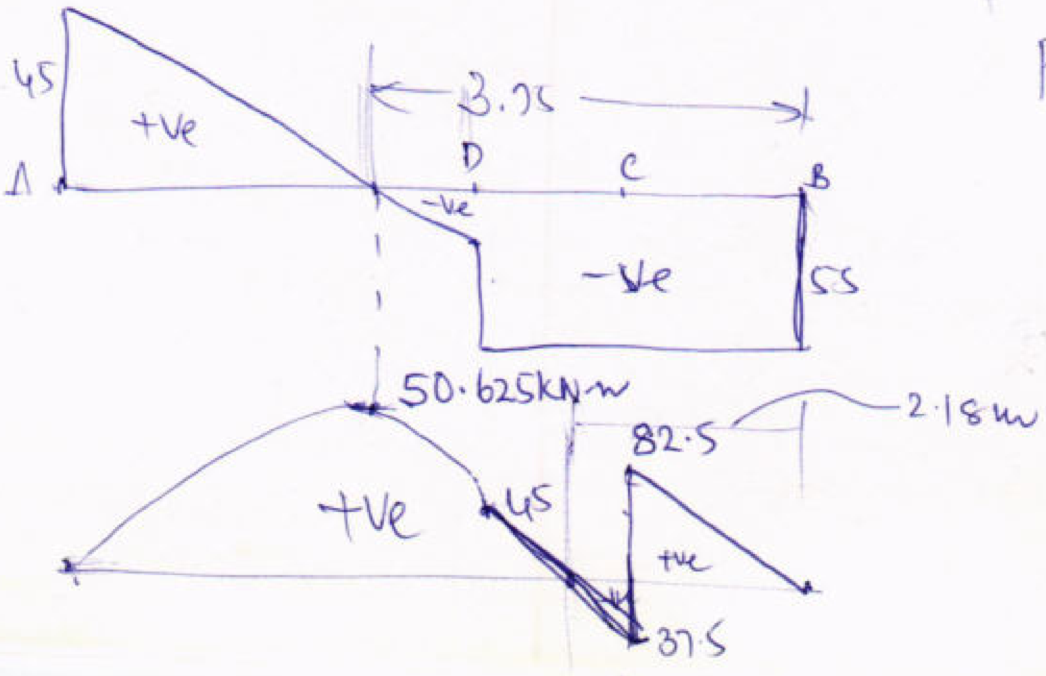
BM is maximum when SF changes its sign from +ve to −ve SF = 0

SF = 0 = 20x − 60 − 55 + 40 = 0
20x = 75
$\boxed{x = 3.75}$

Max BM = 55x

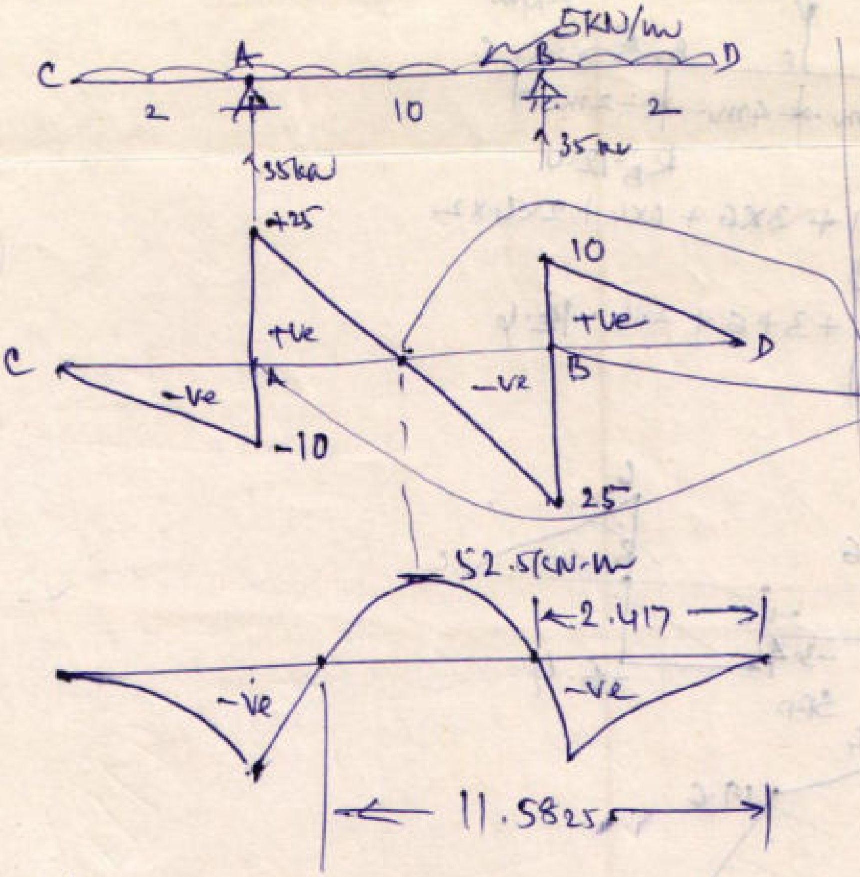
Max BM = 50.625 kN-m

Point of Contraflexure
55x − 120 = 0
x = 2.18 m



Point of Contraflexure

Draw SF and BMD for loads shown in fig. Indicate where the inflexion and contraflexure points are located. Also locate the Max BM with its magnitude. 12 marks



Calculation of reactions

$\Sigma M_A = 0$

$R_B \times 10 = 5 \times 14 \times 5$

$R_B = 35$ kN

$R_A = 5 \times 14 - 35 = 35$ kN

Points of Inflexion

is one where the shear force changes its sign fr +ve to -ve

Calculation of SD

B&D

$SF = +5 \times x$

$x = 0 \quad SF = 0$

$x = 2 \quad SF = +10$

A&B

$SF = +5 \cdot x - 35$ @ $x = 7$, $SF = 0$

$x = 2 \quad SF = -25$ kN

$x = 12 \quad SF = +25$ kN

C&A

$SF = +5(x) - 35 - 35$

when $x = 12 \quad SF = -10$

$x = 14 \quad SF = 0$

Calculation BMD

B&D

$BM = -5 \cdot x \cdot \frac{x}{2}$

$x = 0 \quad M = 0$

$x = 2 \quad M = -10$ kN-m

A&B

$BM = -5 \cdot x \cdot \frac{x}{2} + 35(x-2)$ BM @ 7, $M = -5 \cdot 7 \cdot \frac{7}{2} + 35(5)$

$M = 52.5$ kN-m

$x = 2 \rightarrow M = -10$ kN-m

$x = 12 \rightarrow M = -10$ kN-m

C&A

$BM = -5 \cdot x \cdot \frac{x}{2} + 35(x-2) + 35(x-12)$ @ $x = 12$, $BM = -10$ kN-m

$x = 12 \quad BM = -10$ kN-m

$x = 14, \quad M = 0$

Points of Contraflexure

$-\frac{5x^2}{2} + 35(x-2) = 0$

$-\frac{5x^2}{2} + 35x - 70 = 0$

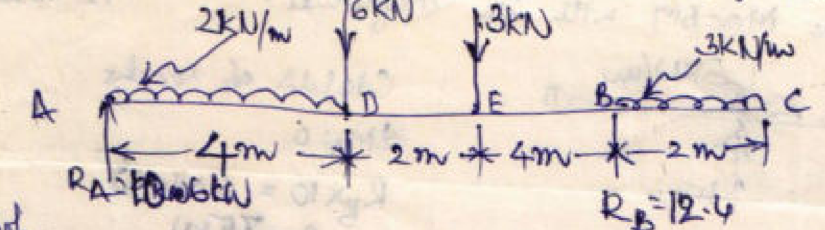
$+2.5x^2 - 35x + 70 = 0$

$x = \frac{-(-35) \pm \sqrt{(-35)^2 - 4(2.5)(70)}}{2(2.5)}$

$x = 11.5825$ m

$x = 2.4174$

Draw SFD and BMD for beam shown in fig indicating principal values 16 Marks June/Dec 06



Soln

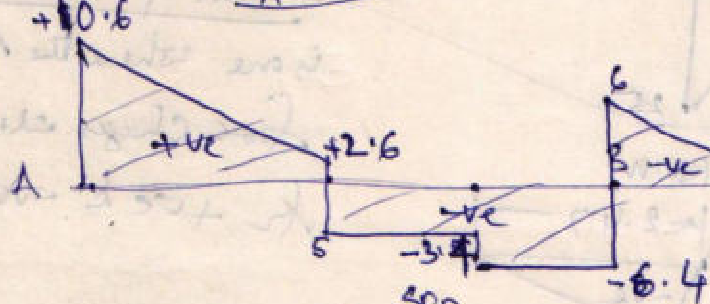
$R_B \times 10 = 3\times2\times11 + 3\times6 + 6\times4 + 2\times4\times2$

$\boxed{R_B = 12.4KN}$

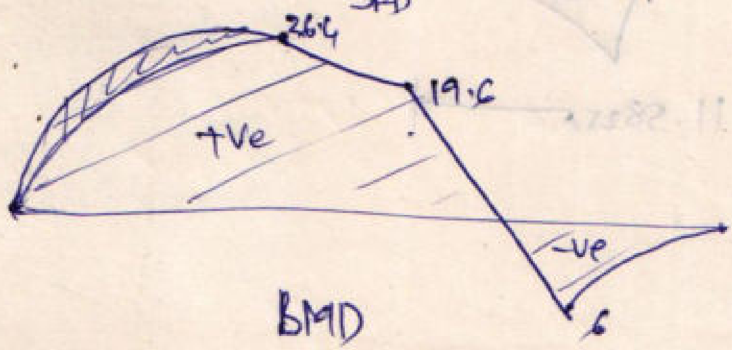
$R_A = 3\times2 + 3 + 6 + 2\times4 - 12.6$

$\boxed{R_A = 10.6KN}$

(14)



SFD



BMD

Calculation of SF

B & C $SF = +3\times x$

$x=0$ $SF = 0$

$x=2$ $SF = +6KN$

E and B $SF = +3\times2 - 12.4$

$SF = -6.4KN$

D and E $SF = +3\times2 - 12.4 + 3$

$SF = -3.4KN$

A and D

$SF = +3\times2 - 12.4 + 3 + 6 + 2(x-8)$

When $x=8$ $SF = +2.6KN$

$x=12$ $= +10.6 KN$

Calculation of BM

B & C

$M = -3 \cdot \frac{x^2}{2}$

$x=0$ $M = 0$

$x=2$ $M = -6$ KN-m

B & E

$M = -(3\times2)(x-1) + 12.4(x-2)$

$x=2$ $M = -6$ KN-m

$x=6$ $M = +19.6$ KN-m

D & E

$M = -(3\times2)(x-1) + 12.4(x-2) - 3(x-6)$

$x=6$ $M = 19.6$ KN-m

$x=8$ $M = +26.4$ KN-m

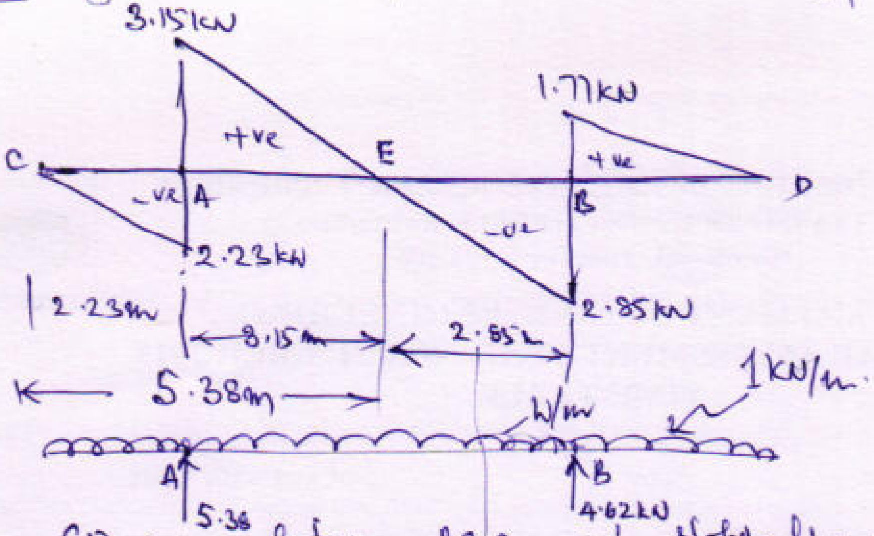
A and D

$M = -(3\times2)(x-1) + 12.4(x-2) - 3(x-6) - 6(x-8) - 2\frac{(x-8)^2}{2}$

$x=8$ $M = +26.4$ KN-m

$x=12$ $M = 0$

Problem: Fig shows the SFD for a SSB. obtain the loading pattern and draw BMD.



$$5.38 - 2.23 = 3.15$$
$$6.00 - 3.15 = 2.85$$

Soln

SF is +ve between B & D. and sloping line is from 0 to 1.77kN. from D to B

$$\text{Load} = \frac{1.77\,kN}{1.77\,m} = 1\,kN/m$$

$$SF = +W.x$$
$$W = \frac{1.77}{1.77} = 1\,kN/m$$

From B to A SF changes from -ve to +ve

At B $SF = 1.77 + 2.85 = 4.62\,kN \uparrow$

SF = 1
-2.85

$$SF = -2.85 = +1 \times 1.77 - R_B$$
$$R_B = 1.77 + 2.85$$
$$= 4.62$$

SF at ②-②: $1 \times 1.77 - 4.62 + W \times 2.85 = 0$

$$2.85\,W = 2.85$$
$$W = \frac{2.85}{2.85} = 1\,kN/m$$

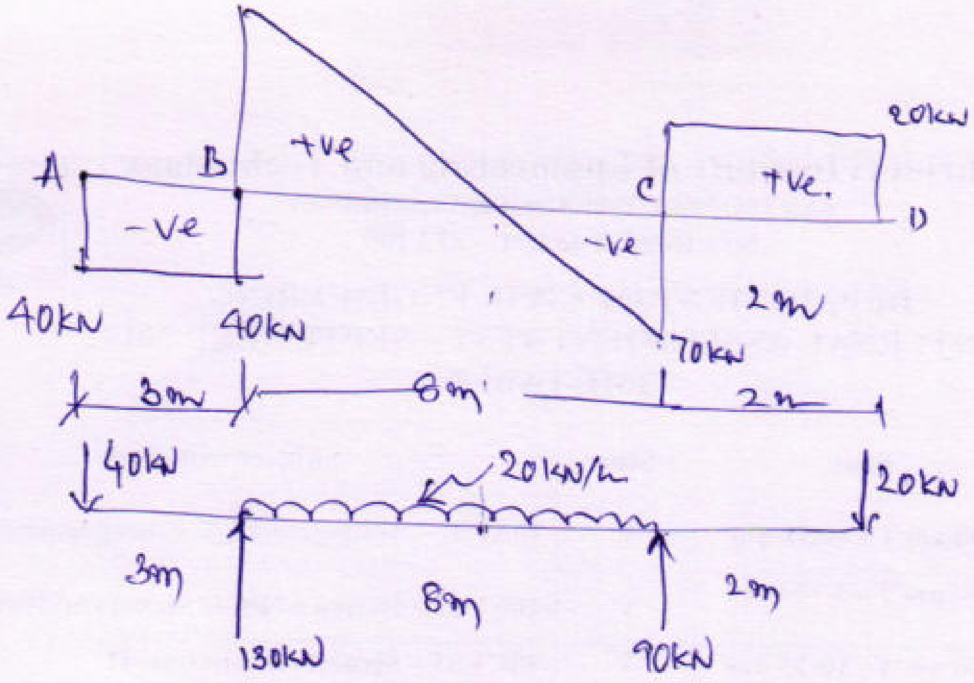
For C to A SF slopes from 0 to $-2.23\,kN$

$$\text{Load} = \frac{2.23}{2.23} = 1\,kN$$

At A, it raises to $+3.15\,kN$

$$SF = 3.15 = -1 \times 2.23 + R_A$$
$$\boxed{R_A = 5.38\,kN}$$

4 8.



Soln: Between C & D SF is Constant.

At D = 20kN ↓

At D, SF = +20 = +20 kN

At C, it Changes to − 70kN

$SF = -70 = +20 - R_C$ $\boxed{R_C = 90kN}$ ↑

Between B & C, SF Changes from −70 to +90 kN over 8m

$SF = +90 = +20 - 90 - W \cdot 8$

$8W = 20 - 90 - 90$

$W = \frac{160}{8} = 20kN/m$

Between A & B, SF is Const.

At A = −40 kN ↓

90
130
220

160
20
40
220

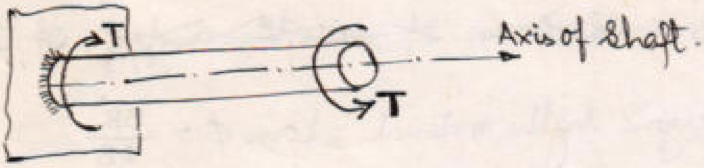
$+20 - R_C = 70$

$R_C = 70$

-ve

TORSION OF SHAFTS

A member is said to be in torsion when it is subjected to moment about its axis. The effect of torsional moment on the member is to twist it and hence a torsional moment is also called as twisting moment or torque.



Shafts transmitting power from engine to the rear axle of automobile, from motor to machine tools and from turbine to generator/motor are the common examples of members in torsion.
Torsion appears at the corners of two-way slabs.

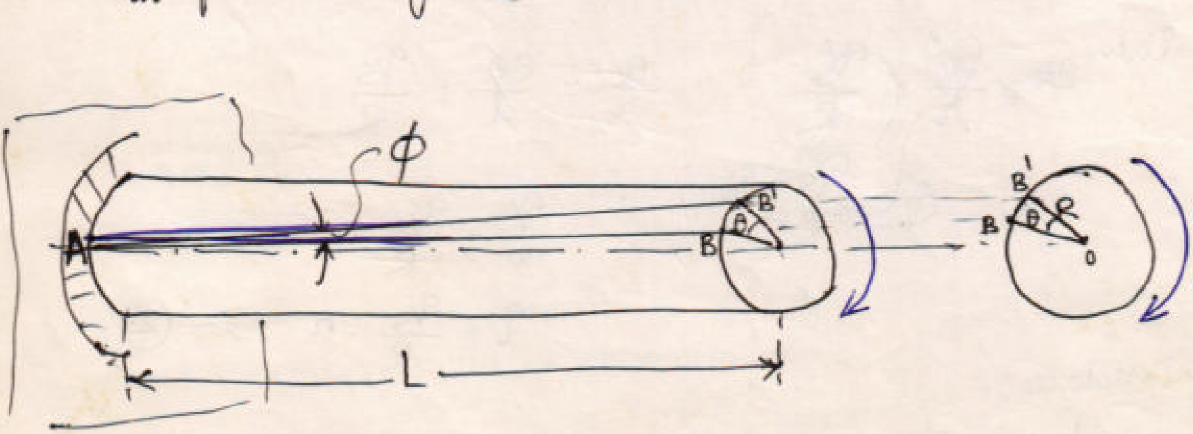
Pure torsion:

A member is said to be in pure torsion when its cross sections are subjected to only torsional moments and not accompanied by axial forces or bending moment.

Assumptions in the theory pure ~~bending~~ Torsion:

1) The material is homogeneous and isotropic
2) The stresses are within the elastic limit i.e., shear stress is proportional to shear strain.
3) Cross sections which are plane before applying twisting moment remain plane even after the application of twisting moment i.e., no warping takes place.
4) Radial lines remain radial even after applying torsional moment.
5) The twist along the shaft is uniform.

Derivation of torsional equations:



Consider a shaft of length 'L', radius R fixed at one end subjected to torque T at the other end as shown in fig.

Let 'O' be the Centre of Circular section and B a point on the surface. AB be the line on the shaft parallel to the axis of the shaft. Due to torque 'T' applied at the end, the point B moves to B'. ~~If φ is the angle of~~ If $\angle BAB'$ is ϕ,

Then Shear strain undergone by the material $= \tan\phi = \frac{BB'}{AB}$

Since ϕ is very small $\tan\phi \simeq \phi$ and thus $\phi = \frac{BB'}{L}$

$$\therefore BB' = L\phi.$$

Let θ be the angle of twist in length 'L', then $B'B = R\theta$

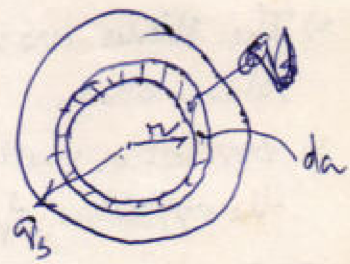
$$R\theta = L\phi$$

$$\phi = \frac{R\theta}{L}$$

(Shear strain)

If q_s is the shear stress and C is modulus of rigidity, then

$$\phi = \frac{q_s}{C}$$



$$\therefore \frac{q_s}{C} = \frac{R\theta}{L}$$

$$\frac{q_s}{R} = \frac{C\theta}{L} \quad \text{——— (1)}$$

If ~~a point~~ an elemental area 'da' is Considered at a distance 'r' from Centre ~~instead of on the surface~~ ~~then~~ and let q be the shear stress on the elemental area 'da';

then $\frac{q}{r} = \frac{q_s}{R}$, $q = \frac{q_s}{R}\cdot r$ $\qquad \frac{q}{r} = \frac{C\theta}{l} = \frac{q_s}{R}$

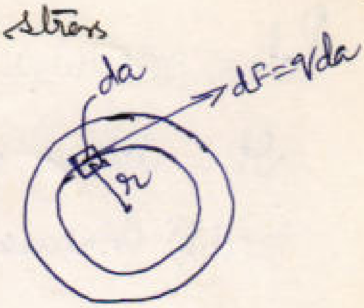
$$\therefore \frac{q}{r} = \frac{q_s}{R}$$

$$q = \frac{q_s}{R}\cdot r \quad \text{—— (2)}$$

Moment of resistance:

For equilibrium, the external torque 'T' must be balanced by internal torque due to internal stresses.

The resisting force on the elemental area 'da' due to shear stress



$$dF = q\,da$$

∴ Resisting torsional moment $= dT = dF \times r$
$$= q\,r\,da$$

From eqn (2) $$dT = \frac{q_s}{R} \cdot r^2\,da.$$

∴ Total resisting torsional moment $T = \frac{q_s}{R}\int r^2\,da$

But $\int r^2\,da$ is called polar moment of inertia of the section. Representing it by notation J,

$$T = \frac{q_s}{R} \cdot J.$$

Then $$\frac{T}{J} = \frac{q_s}{R}.$$

From (2) $$\frac{T}{J} = \frac{q}{R}.$$

From (1) $$\frac{T}{J} = \frac{q}{r} = \frac{C\theta}{l}$$

Where T = Torsional moment N-mm
J = polar moment of inertia mm^4
q = Shear stress at a distance 'r' N/mm^2
r = Distance from the axis
C = Modulus of rigidity N/mm^2
θ = Angle of twist in radians
L = length of shaft in mm.

Power Transmitted:

Consider a shaft subjected to a mean torque T and rotating at N revolutions per minute (rpm)

Work done per second = Mean Torque × Angle turned per second

$$\boxed{P = T_{mean} \times \frac{2\pi N}{60}}$$

If T is in N-m

P = Power watts
T = Torque N-m
N = rpm

Then work done per second (or transmitted) $= P = \frac{2\pi N T}{60}$ N-m/sec

Power is expressed in watts, that equal to 1 N-m/sec

1 watt = 1 N-m/sec = 1000 N-mm/sec

Polar modulus:

It is defined as "the ratio of polar moment of inertia to the extreme radial distance of the fibre from the centre". It is denoted by Z_p.

ie

$$Z_p = \text{Polar modulus} = \frac{\text{Polar moment of Inertia}}{\text{Extreme radial distance of fibre}} = \frac{J}{R}$$

$$Z_p = \frac{J}{R}$$

From Torsion eqn.

$$T = \frac{J}{R} \cdot q_s \quad \text{or} \quad \boxed{T = Z_p q_s}$$

ie

Torque capacity will be maximum if polar modulus is maximum

Hence polar modulus is a measure of strength of shaft in resisting torsion.

a) For a solid circular shaft of dia 'd'

$$Z = \frac{J}{R} = \frac{\frac{\pi d^4}{32}}{d/2} = \frac{\pi d^3}{16}$$

b) For hollow circular shaft with external dia d_1 and internal dia d_2

$$Z = \frac{J}{R} = \frac{\frac{\pi(d_1^4 - d_2^4)}{32}}{d_1/2} = \frac{\pi}{16 d_1}(d_1^4 - d_2^4)$$

Torsional rigidity or Stiffness of the shafts:

From torsion eqn

$$\frac{T}{J} = \frac{C\theta}{l}$$

$\boxed{K = CJ}$

$$T = \frac{CJ\theta}{L} \quad \text{ie} \quad CJ = \frac{T.L}{\theta}$$

When CJ is more → θ will be less.
More CJ → more capability against twisting

Where CJ is called torsional rigidity or stiffness of shaft which is torque required to produce unit angle of twist in unit length.

~~A load of 240 kN~~ A steel shaft transmits 125 kW at 175 rpm. The dia of shaft is 100mm. Determine the torque on the shaft and the maximum shearing stress induced. Also calculate the twist of the shaft in a length of 6m. Take $C = 8.5 \times 10^4$ N/mm². (3)

Feb 2002

Soln: power transmitted $P = 125$ kW

$= 125 \times 10^3$ N-m/sec

$= 125 \times 10^6$ N-mm/sec

$N = 175$ rpm

Now $$P = \frac{2\pi N T}{60}$$

$$T = \frac{60P}{2\pi N} = \frac{60 \times 125 \times 10^6}{2\pi \times 175} = 6820926.132 \text{ N-mm}$$

$d = 100$ mm

$$J = \frac{\pi d^4}{32} = 9.8174 \times 10^6 \text{ mm}^4$$

From Torsion Eqn

$$\frac{T}{J} = \frac{q_s}{R}$$

$$q_s = \frac{T}{J} \cdot R$$

$$= \frac{6820926.132}{9.8174 \times 10^6} \times 50$$

$$= 34.73 \text{ N/mm}^2.$$

also

$$\frac{T}{J} = \frac{C\theta}{l}$$

$$\theta = \frac{T \cdot l}{JC} = \frac{6820926.132}{9.8174 \times 10^6} \times \frac{6 \times 10^3}{8.5 \times 10^4}$$

$= 0.0490$ radians

$= 0.0490 \times \frac{180}{\pi} = 2.8099^\circ$

$= 2^\circ 48' 35.9''$

What size of shaft should be used for the rotor of a 7.5 KW motor operating at 3600 rpm if the shearing stress is not to exceed 60MPa in the shaft?

Aug 2001

$q_s = 60 N/mm^2$

$P = 7.5 \times 10^3$ watts
$= 7.5 \times 10^6$ N-mm/sec

$$P = \frac{2\pi N T}{60}$$

$$T = \frac{60P}{2\pi N} = \frac{60 \times 7.5 \times 10^6}{2 \times \pi \times 3600} = 19894.36 \text{ N-mm}$$

From Torsion eqn.

$$T = \frac{J}{R} \cdot q_s$$

$$= \frac{\frac{\pi d^4}{32}}{d/2} \cdot q_s$$

$$T = \frac{\pi d^3}{16} \cdot q_s$$

$$d = \sqrt[3]{\frac{16T}{\pi q_s}} = \sqrt[3]{\frac{16 \times 19894.36}{\pi \times 60}}$$

$= 11.908$ mm $\simeq \underline{12 \text{ mm}}$

* Derive the torsion eqn for circular members $\frac{T}{J} = \frac{\tau}{r} = \frac{G\theta}{L}$ with usual notations. State clearly the assumptions made.

Aug 2001

What must be the length of 5mm dia aluminium wire so that it can be twisted through one complete revolution without exceeding shearing stress of 42 N/mm². Take $C = 27$ GN/m².

March 2000

$l = ?$

$d = 5mm$

$\theta = 360° = 2\pi$ radians

$q_s = 42 N/mm^2$

$C = 27 \times 10^9 N/m^2$

$= \frac{27 \times 10^9}{1000 \times 1000} = 27 \times 10^3 N/mm^2$

From Torsion eqn

$$\frac{q_s}{R} = \frac{C\theta}{l}$$ 1 marks

$$\therefore l = \frac{C\theta \cdot R}{q} = \frac{27 \times 10^3 \times 2\pi \times 2.5}{42}$$

$= 10097.97$ mm

$= 10.098$ mts

4 marks.

The shearing stress in a solid shaft is not to exceed $50 N/mm^2$ when the torque transmitted is 30000 N-m. Determine the minimum dia of the shaft.

Aug 1999

Soln:
$q_s = 50 N/mm^2$

$T = 30000$ N-m
$= 30000 \times 10^3$ N-mm

From

$$\frac{T}{J} = \frac{q_s}{R}$$

$$T = \frac{\frac{\pi d^4}{32}}{d/2} \cdot q_s \qquad T = \frac{\pi d^3}{16} \cdot q_s$$

$$d = \sqrt[3]{\frac{16T}{\pi \cdot q_s}} = \sqrt[3]{\frac{16 \times 30000 \times 10^3}{\pi \times 50}}$$

$= \cancel{\ \ }$ mm.
145.11 mm

P.T a hollow shaft is stronger than solid shaft of the same material, length and weight.

March 2000

Soln: Let 'd' be the dia of solid shaft.
d_1 be the outer dia of hollow shaft.
d_2 be the inner dia of hollow shaft.

The two shafts have equal weight and length and are of same material.
Hence, equating the ~~weight of solid~~ areas of cross sections,

$$\frac{\pi d^2}{4} = \frac{\pi}{4}(d_1^2 - d_2^2)$$

$$d^2 = (d_1^2 - d_2^2) \quad \therefore \quad d = (d_1^2 - d_2^2)^{1/2}$$

Let T_s be the torque resisting capacity of solid shaft and
T_h be torque resisting capacity of hollow shaft

From torsion eqn

$$\frac{T_s}{J_s} = \frac{q_{solid}}{d/2} \qquad T_s = \frac{J_s \, q_s}{d/2}$$

$$T_s = \frac{\pi d^4}{32} \Big/ \frac{d}{2} \cdot q_s = \frac{\pi d^3}{16} q_s$$

and $T_h = J_h \dfrac{q_s}{\frac{d_1}{2}} = \dfrac{\pi}{32}(d_1^4 - d_2^4)\, q_s \Big/ \dfrac{d_1}{2}$

$$= \frac{\pi}{16 d_1}[d_1^4 - d_2^4]\, q_s$$

Then $\dfrac{T_h}{T_s} = \dfrac{d_1^4 - d_2^4}{d^3 d_1}$ $d = (d_1^2 - d_2^2)^{1/2}$

Substituting the value of d in the above eqn.

$$\frac{T_h}{T_s} = \frac{(d_1^2 + d_2^2)(d_1^2 - d_2^2)}{d_1\,(d_1^2 - d_2^2)^{3/2}}$$

$$= \frac{(d_1^2 + d_2^2)}{d_1(d_1^2 - d_2^2)^{1/2}}$$

$$= \frac{d_1^2\left(1 + \left(\frac{d_2}{d_1}\right)^2\right)}{d_1\left[d_1^2\left(1 - \left(\frac{d_2}{d_1}\right)^2\right)\right]^{1/2}}$$

d_1 = Ext. dia
d_2 = Int. dia.
$\therefore \dfrac{d_2}{d_1} < 1$

$$\frac{T_h}{T_s} = \frac{\cancel{d_1^2}\left(1 + \left(\frac{d_2}{d_1}\right)^2\right)}{\cancel{d_1^2}\left[\sqrt{1 - \left(\frac{d_2}{d_1}\right)^2}\right]} = \frac{1 + \left(\frac{d_2}{d_1}\right)^2}{\sqrt{1 - \left(\frac{d_2}{d_1}\right)^2}} > 1$$

Numerator is greater than 1
Denominator is less than 1.

$\therefore \; T_h > T_s$ (Torque taken by hollow shaft is more than that of solid shaft)

i.e. Hollow shaft is stronger than solid shaft when their weights are same.

<u>To proove that hollow shaft is stiffer: (than solid shaft)</u>

Stiffness of a shaft may be defined as torque required to produce unit rotation in unit length. Let this be denoted by 'K'.

$\frac{T}{J} = \frac{C\theta}{l}$ $T = CJ \times \frac{1}{1}$ $K = T = CJ$ $K = CJ$.

Let k_s be the stiffness of solid shaft and k_h that of hollow shaft.

$$k_s = G\frac{\pi d^4}{32}$$

$$k_h = C\frac{\pi}{32}(d_1^4 - d_2^4) \qquad d^2 = d_1^2 - d_2^2$$

$$\therefore \quad \frac{k_h}{k_s} = \frac{d_1^4 - d_2^4}{d^4}$$

$$= \frac{(d_1^2-d_2^2)(d_1^2+d_2^2)}{(d_1^2-d_2^2)^2} = \frac{(d_1^2+d_2^2)}{(d_1^2-d_2^2)} > 1$$

$$k_h > k_s$$

Hollow shaft is stiffer than solid shaft when their weights are same.

Calculate the maximum intensity of shear stress induced and the angle of twist produced in degree in solid shaft of 120mm dia and 10m long transmitting 120kW at 160 rpm. Take $G = 80kN/mm^2$.

Aug 1999

$C = 80\times10^3\ N/mm^2$

$P = 120\ kW$

$= 120\times10^3\ N\text{-}m/sec$

$= 120\times10^6\ N\text{-}mm/sec$

$N = 160\ rpm$

$1\,watt = 1\ N\text{-}m/sec$

$= 1\times10^3\ N\text{-}mm/sec$

$$P = \frac{2\pi NT}{60}$$

$$T = \frac{60P}{2\pi N} = \frac{60\times120\times10^6}{2\times\pi\times160}$$

$$= 7161969.6\ N\text{-}mm$$

From Torsion eqn

$$\frac{T}{J} = \frac{q_s}{R}$$

$R = 60mm$

$J = \frac{\pi d^4}{32} = \frac{\pi\times120^4}{32}$

$= 20357500\,mm^4$

$$q_s = \frac{T.R}{J} = \frac{7161969.6\times60\times32}{\pi\times120^4}$$

$$\boxed{q_s = 21.10\ N/mm^2.}$$

also $\frac{T}{J} = \frac{C\theta}{l}$

$$\theta = \frac{T.l}{CJ} = \frac{7161969.6\times10000}{80\times10^3\times20357500}$$

$$= 0.0439762 \text{ radians} \times\frac{180}{\pi}$$

$$= 2.519^\circ$$

$\pi - 180^\circ$

A solid cylindrical shaft is to transmit 300kW at 100 rpm. If the shear stress is not to increase 80 N/mm², find the diameter. Also what percentage saving in weight would be obtained if this shaft is replaced by a hollow one whose internal diameter equals 0.6 times of external dia, the length, material and maximum stress being the same. March 2000

Soln:

$$P = \frac{2\pi NT}{60}$$

$P = 300 kW$

$= 300 \times 10^3 W$

$= 300 \times 10^6$ N-mm/sec

$$T = \frac{60P}{2\pi N} = \frac{60 \times 300 \times 10^6}{2 \times \pi \times 100}$$

$= 2864720$ N-mm

From Torsion Eqn

$J = \frac{\pi d^4}{32}$

$= \frac{\pi \times}{}$

$$\frac{T}{J} = \frac{q_s}{R}$$

$$T = \frac{\pi d^4}{32} \cdot \frac{2}{d} \cdot q_s$$

$$d^3 = \frac{16T}{\pi q_s} \qquad d = \sqrt[3]{\frac{16 \times 2864720}{\pi \times 80}}$$

$$\boxed{d = 122.17 \text{ mm}}$$

When the shear stress, torque are same, the polar modulus of both hollow ~~[illegible]~~ and solid shafts are same.

$$\frac{J_{Solid}}{R_{Solid}} = \frac{J_{Hollow}}{R_{Hollow}}$$

d_1 = Ext. dia

d_2 = Int. dia

d = Solid

$d_2 = 0.6 d_1$

$$\frac{\pi d^3}{16} = \frac{\pi}{16 d_1}(d_1^4 - d_2^4)$$

$$d^3 = \frac{1}{d_1}(d_1^4 - (0.6 d_1)^4)$$

$$d^3 = \frac{d_1^4 (1 - (0.6)^4)}{d_1}$$

$$d^3 = d_1^3 (0.8704)$$

$$d_1 = \sqrt[3]{\frac{122.17^3}{0.8704}} = 127.95 \text{ mm} \qquad d_2 = 76.77 \text{ mm}$$

Percentage saving in weight = $\dfrac{(A_S - A_H)\cancel{L\rho}}{A_S \cancel{L\rho}} \times 100$

$= \dfrac{\frac{\pi}{4}(122.27^2 - (127.95^2 - 76.77^2))}{\frac{\pi}{4} \times 122.27^2} \times 100$

$\boxed{\text{percentage saving in weight} = 29.91\ \%}$

Design a shaft to transmit 1 MW of power at 300 rpm, the stress in the shaft should not exceed 60 MPa and the angle of twist should not be more than 1° in a length of 10 dia. Assume $\underset{E}{C} = 80$ GPa for the material.

August 2000

$P = 1$ MW
$= 1 \times 10^6$ W
$= 1 \times 10^9$ N-mm/sec

$N = 300$ rpm

$\dfrac{2\pi NT}{60} = P$

$T = \dfrac{60P}{2\pi N}$

$= \dfrac{60 \times 1 \times 10^9}{2 \times \pi \times 300}$

$= 31830960$ N-mm.

Diameter based on shear stress consideration (Strength criteria)

$q_s = 60$ N/mm²

$\dfrac{T}{J} = \dfrac{q_s}{R}$

$T = \dfrac{\pi d^3}{16} \cdot q_s$

$d = \sqrt[3]{\dfrac{16T}{\pi q_s}} = \sqrt[3]{\dfrac{16 \times 31830960}{\pi \times 60}} = 139.28$ mm ✓

Diameter based on angle of twist consideration (Stiffness criteria)

$C = 80 \times 10^3$ N/mm²

$\dfrac{T}{J} = \dfrac{C\theta}{l}$

$\theta = 1^\circ = \dfrac{\pi}{180}$ radians,

$l = 10\,d$

$\boxed{d = 139.28 \text{ mm}}$

$\dfrac{T}{C\theta} = \dfrac{\frac{\pi d^4}{32}}{10d}$

$\dfrac{T}{C\theta} = \dfrac{\pi d^3}{320}$

$d = \sqrt[3]{\dfrac{320 \times 31830960 \times 180}{\pi \times 80 \times 10^3 \times \pi}} = 132.42$ mm

* ~~Higher~~ value of diameter to be adopted.

A hollow steel shaft of 300mm extnl dia and 200mm internal dia has to be replaced by a solid brass shaft. The polar modulus will be same for both the cases. Find the dia of brass shaft and also work out the ratio of their torsional rigidity. Take C_S as $2.4 C_B$.

March 2001

Soln:

Polar modulus of steel = Polar modulus of brass

$$\frac{\pi (300^4 - 200^4)}{32 \times 150} = \frac{\pi d_b^3}{16}$$

$$\frac{\pi d_b^3}{16} = 4254222.3$$

$$d_b = \sqrt[3]{\frac{16 \times 4254222.3}{\pi}}$$

$$\boxed{d_b = 278.78\,mm}$$

$$J_{Hollow} = \frac{\pi (300^4 - 200^4)}{32} = 6.381 \times 10^8\, mm^4$$

$$J_{Solid} = \frac{\pi \times 278.78^4}{32} = 5.929 \times 10^8\, mm^4$$

Ratio of torsional rigidity:

$$\frac{(CJ)_{Steel}}{(CJ)_{Brass}} = \frac{2.4 C_B \times 6.381 \times 10^8}{C_B \times 5.929 \times 10^8} = 2.58$$

A hollow shaft with diameter ratio 3/5 is required to transmit 482 kW at 125 rpm with a uniform twisting moment. The shearing stress in the shaft must not exceed 65 N/mm² and twist in a length of 2 m must not exceed 1°. Taking $C = 8 \times 10^4$ N/mm², determine the minimum external dia of the shaft satisfying above two conditions.

Soln:

$P = 482$ kW
$= 482 \times 10^3$ W
$= 482 \times 10^6$ N-mm/sec

$$\frac{2\pi N T}{60} = P$$
$$T = \frac{60P}{2\pi N} = \frac{60 \times 482 \times 10^6}{2\pi \times 125}$$
$$\boxed{T = 36821811 \text{ N-mm}}$$

If d_1 = External dia
d_2 = internal dia
$d_2 = 0.6 d_1$

From shear stress considerations: (Strength criteria)

$$\frac{T}{J} = \frac{q_s}{R}$$
$$T = \frac{\pi}{16 d_1}(d_1^4 - (0.6 d_1)^4) \times q_s$$
$$T = \frac{\pi}{16}\frac{d_1^{4}(1-0.6^4)}{d_1} q_s$$
$$d_1^3 = \frac{16T}{\pi q_s \times 0.8704} = \sqrt[3]{\frac{16 \times 36821811}{\pi \times 65 \times 0.8704}}$$
$$= 149.10 \text{ mm}.$$

From angle of twist criteria

$C = 8 \times 10^4$ N/mm²
$\theta = 1° = \frac{\pi}{180}$ radians
$l = 2000$ mm

$$\frac{T}{J} = \frac{C\theta}{l}$$
$$J = \frac{Tl}{C\theta}$$
$$\frac{\pi d_1^4 (1-0.6^4)}{32} = \frac{Tl}{C\theta}$$
$$d_1^4 = \frac{32\, TL}{C\theta \pi \times 0.8704}$$
$$= \frac{32 \times 36821811 \times 2000 \times 180}{8 \times 10^4 \times \pi \times \pi \times 0.8704}$$
$$d_1 = 157.62 \text{ mm}.$$

* Higher value of the above two to be chosen i.e. 149.10 mm.

A Shaft 100mm dia and 2m long is subjected to a torque of 8kN-m. Find the maximum shear stress and the angle of twist. If the central 1m length of the shaft is turned down to 60mm dia and same torque is applied, what will be the change in shear stress and the angle of twist? C=80GPa.

March 2001

Soln:

By torsion eqn.

$d = 100mm$

$l = 2m = 2000mm$

$T = 8kN\text{-}m.$

$J = \frac{\pi \times 100^4}{32}$

$= 9.817 \times 10^6 \, mm^4$

$\frac{T}{J} = \frac{C\theta}{l}$

$q_s = \frac{T}{J} \cdot R$

$= \frac{8 \times 10^6 \times 32 \times 50}{\pi \times 100^4}$

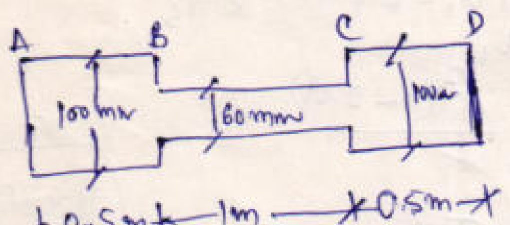
$= 40.74$ MPa.

$\theta = \frac{T}{J} \frac{l}{C} = \frac{8 \times 10^6 \times 32}{\pi \times 100^4} \times \frac{2000}{80 \times 10^3}$

$= 0.020$ radians

$= 1.167°$

ii)



(Not to be done as it is not included in Syllabus).

$J_{at\,centre} = \frac{\pi \times 60^4}{32}$

$= 1272345 \, mm^4$

$q_{s\,at\,centre} = \frac{T}{J} \cdot R = \frac{8 \times 10^6 \times 30}{1272345} = 188.628$ MPa.

$\underline{\text{Rotation}}$ θ_{total} = Rotation of AB + Rotation of BC + Rotation of CD

$\theta_{tl} = \Sigma \left(\frac{Tl}{CJ}\right) = \frac{8 \times 10^6}{80 \times 10^3} \left[\frac{1 \times 10^3}{9.817 \times 10^6} + \frac{2 \times 0.5 \times 10^3}{1272345}\right]$

$= 0.0887$ radians. $\times \frac{180}{\pi}$

$= 5.086°$

A hollow Shaft has an external dia of 360mm and a bore of 220mm. When transmitting a power, it is observed that the angle of twist is 0.5° in a length of 3.5 mts. If $C = 8\times10^4$ N/mm² and $T_{max} = 1.4\,T_{mean}$,

Calculate 1) The power transmitted by the shaft when speed is 5 RPS

2) The max shear stress induced in the shaft.

Aug 99

Soln.

$$J = I_p = \frac{\pi}{32}[360^4 - 220^4] = 1.42\times10^9 \text{ mm}^4$$

From torsion eqn

$$\frac{T_{max}}{J} = \frac{C\theta}{L}$$

$$T_{max} = C\theta \cdot \frac{J}{L}$$

$\theta = 0.5\times\frac{\pi}{180}$

$$= \frac{8\times10^4 \times 0.5\times\pi/180 \times 1.42\times10^9}{3500}$$

$$= 283.24\times10^6 \text{ N-mm}$$

$N =$ rpm

$= 60\times5 = 300$ rpm

$$\text{Power } P = \frac{2\pi N T_{mean}}{60}$$

$$T_{mean} = \frac{T_{max}}{1.4}$$

$$= \frac{283.24\times10^6}{1.4}$$

$$= 202.3\times10^6 \text{ N-mm}$$

$$= 202.3\times10^3 \text{ N-m}$$

$$= \frac{2\times\pi\times300\times202.\times10^3}{60}$$

$$= 6355890 \text{ W}$$

$$= 6355.89 \text{ kW}$$

The max shear stress $q = \frac{T_{max}}{J}\cdot R$

$$= \frac{283.24\times10^6}{1.42\times10^9}\times180$$

$$= 35.90 \text{ N/mm}^2$$

Determine the ratio of power transmitted by a hollow shaft and a solid shaft when both have same weight, length, material and speed. The dia of Solid shaft is 150mm and external dia of hollow shaft is 250mm.

Aug 1999

Soln: Since the weight of both shafts are same, their areas of cross section must be same.

$$A_S = A_H$$

$$\frac{\pi \times 150^2}{4} = \frac{\pi}{4}\left[250^2 - d_2^2\right]$$

$$d_2 = 200\,mm.$$

Power transmitted by Solid shaft $\quad P_s = \frac{2\pi N T_s}{60}$

Hollow shaft $\quad P_H = \frac{2\pi N T_H}{60}$

$$\frac{P_S}{P_H} = \frac{T_S}{T_H} = \frac{\frac{J_s}{R_s} \cdot q_s}{\frac{J_h}{R_h} \cdot q_H} = \frac{J_s\, \cancel{q_s}}{J_h \cdot \cancel{q_H}} \cdot \frac{R_h}{R_s}$$

But $q_s = q_H$

$$= \frac{\frac{\pi}{32}(150)^4 \times 125}{\frac{\pi}{32}\left[250^4 - 200^4\right] \times 75}$$

$J_s = \pi \times$

$$\boxed{\frac{P_S}{P_H} = 0.5402}$$

or $\boxed{\frac{P_H}{P_S} = 1.8511}$

1.55×10^{10}

A solid shaft rotating at 500 rpm transmits 30 kW. Maximum torque is 20% more than mean torque. Allowable shear stress 65 MPa and modulus of rigidity 81 GPa, angle of twist in the shaft should not exceed 1° in 1 mt length. Determine the suitable diameter. 10 marks

Jan 2009

Soln

$$P = 30\,kW = 30 \times 10^6 \text{ N-mm/sec}$$

$$N = 500 \text{ rpm}$$

$$T_{max} = 1.2\, T_{mean}$$

$$q_s = 65 = 65 \text{ N/mm}^2$$

$$C = G = 81\,GPa = 81 \times 10^9 \text{ N/m}^2 = \frac{81 \times 10^9}{10^6} = 81 \times 10^3 \text{ N/mm}^2$$

$$\theta = 1^\circ = 1 \times \frac{\pi}{180} \text{ radians}$$

π - 180°
180 - π
1° - ?

$$L = 1\,m = 1000\,mm$$

$$P = \frac{2\pi N T}{60} \qquad T = \frac{60P}{2\pi N} = \frac{60 \times 30 \times 10^6}{2 \times \pi \times 500}$$

$$= 572.95 \times 10^3 \text{ N-mm}$$

$$J = \frac{\pi}{32} d^4 \qquad T_{max} = 1.2 \times T_{mean}$$

$$= 687.54 \times 10^3 \text{ N-mm}$$

From Consideration of Shear stress:

$$\frac{T}{J} = \frac{q_s}{d/2}$$

$$T = \frac{\pi d^4}{32} \cdot \frac{2}{d} \cdot q_s$$

$$T = \frac{\pi d^3}{16} q_s$$

$$d = \sqrt[3]{\frac{16\,T}{\pi\, q_s}} = \sqrt{\frac{16 \times 687.54 \times 10^3}{\pi \times 65}}$$

$$\boxed{d = 37.76\,mm}$$

From Consideration of angle of twist

$$\frac{T_{max}}{J} = \frac{C\theta}{l}$$

$$J = \frac{Tl}{C\theta}$$

$$\frac{\pi d^4}{32} = \frac{Tl}{C\theta}$$

$$d = \sqrt[4]{\frac{6.8754 \times 10^5 \times 1000 \times 180 \times 32}{81 \times 10^3 \times \cancel{180} \times \pi}} = 47.17\,mm$$

$= \cancel{14.91}$ mm

$\boxed{\cancel{d = 14.91\,mm}}$

Find the diameter of the shaft required to transmit 60 kW at 150 rpm if the maxm torque is 25% more than the mean torque for maxm permissible shear stress of 60 MN/m². Find also the angle of twist for a length of 4 m. Take G = 80 GPa.

Jan 2008 10 marks

$P = 60\,kW = 60 \times 10^6$ N-mm/sec

$N = 150$ rpm

$T_{max} = 1.25\, T_{mean}$

$q_s = 60\,MN/m^2 = 60\, N/mm^2$

$G = 80\,GPa = 80 \times 10^3\, N/mm^2$

$P = \frac{2\pi N T}{60}$

$T = \frac{60P}{2\pi N}$

$T = \frac{60 \times 60 \times 10^6}{2 \times \pi \times 150}$

$T_{mean} = 3.819 \times 10^6$

$T_{max} = 1.25 \times 3.819 \times 10^6$

$\boxed{T_{max} = 4.774 \times 10^6 \text{ N-mm}}$

From Shear Stress Consideration

$$\frac{T}{J} = \frac{q_s}{R}$$

$$T = \frac{\pi d^4}{32} \cdot \frac{q_s}{d} \cdot 2$$

$$T = \frac{\pi d^3}{16} \cdot q_s$$

$$d = \sqrt[3]{\frac{16T}{\pi q_s}}$$

$$d = \sqrt[3]{\frac{16 \times 4.774 \times 10^6}{\pi \times 60}} = 74.00\text{ mm}$$

$l = 4\,m = 4000$ mm

$$\frac{q_s}{R} = \frac{G\theta}{l}$$

$$\theta = \frac{q_s \cdot l}{R \cdot G} = \frac{60 \times 4000}{37 \times 80 \times 10^3}$$

$$= 0.08108 \text{ radians}$$

$\pi - 180°$

$1 - \frac{180}{\pi} \times 1$

$$\theta = \frac{0.08108 \times 180}{\pi}$$

$$\boxed{\theta = 4.64°}$$

A hollow shaft is to transmit 300 kW at 80 rpm. If the shear stress is not to exceed 60 N/mm² and internal diameter is 0.6 times the external diameter, find the external and internal diameters assuming that maximum torque is 1.4 times the mean torque.

July 2009 (Old scheme) 8 marks

$P = 300\,kW = 300 \times 10^6$ N-mm/sec

$N = 80$ rpm

$T_{max} = 1.4\,T_{mean}$.

$q_s = 60\,N/mm^2$

d_2 = External diameter

$d_1 = 0.6 d_2$

$$P = \frac{2\pi N T}{60}$$

$$T = \frac{60 P}{2\pi N}$$

$$= \frac{60 \times 300 \times 10^6}{2\pi \times 80}$$

$$T_{mean} = 35.80 \times 10^6 \text{ N-mm}$$

$T_{max} = 1.4\,T_{mean}$

$T_{max} = 50.13$ N-mm

From Shear stress Considerations

$$\frac{T}{J} = \frac{q_s}{R}$$

$$J = \frac{T \cdot R}{q_s}$$

$$\frac{\pi(d_2^4 - (0.6 d_2)^4)}{32} = \frac{50.13 \times 10^6 \times \frac{d_2}{2}}{60}$$

$$2 \times \pi \frac{d_2^4}{3} \frac{(1 - 0.1296)}{d_2} = 50.13 \times 10^6 \times 0.53$$

$$d_2 = \sqrt[3]{\frac{\cancel{8} \times 50.13 \times 10^6 \times 0.533}{\pi \times 0.8704 \times 2}}$$

$$\boxed{d_2 = 170.61 \text{ mm}}$$

$$\boxed{d_1 = \cancel{183}.10 \text{ mm}}$$ 101.87 mm

A solid shaft transmits 250 kW at 100 rpm. If the shear stress is not to exceed 75 MPa, what should be the diameter of the shaft? If this shaft is to be replaced by a hollow one, whose dia ratio is 0.6, determine the size and percentage saving in weight, the maximum shearing stress being same.

Soln:

$P = 250\,kW = 250 \times 10^6$ N-mm/sec

$N = 100$ rpm

$$P = \frac{2\pi N T}{60}$$

$$T = \frac{60 \times 250 \times 10^6}{2\pi \times 100} = 23.87 \times 10^6 \text{ N-mm}$$

$$\frac{T}{J} = \frac{q_s}{R} \Rightarrow \frac{J}{R} = \frac{T}{q_s}$$

$$\frac{\pi d^3}{16} = \cancel{250} \cdot \frac{T}{q_s}$$

$$d = \sqrt[3]{\frac{16 \times 23.87 \times 10^6}{75 \times \pi}} = 117.468\,mm$$

Now,

$$\frac{T}{J_{solid}} = \frac{q_s}{R_{solid}} \quad ; \quad \frac{T}{J_{hollow}} = \frac{q_s}{R_{Hollow}} \qquad \frac{T}{q_s} = \frac{J_{solid}}{R_{solid}} = \frac{J_{hollow}}{R_{hollow}}$$

$$\frac{\pi d^3}{16} = \frac{\pi}{16 d_1}(d_1^4 - d_2^4) = \frac{\pi d_1^4}{16 d_1}(1 - 0.6^4)$$

$$d^3 = d_1^3 (0.8704)$$

$$d_1 = \sqrt[3]{\frac{117.468^3}{0.8704}}$$

$$\boxed{d_1 = 123.03\,mm} \quad \boxed{d_2 = 73.81\,mm}$$

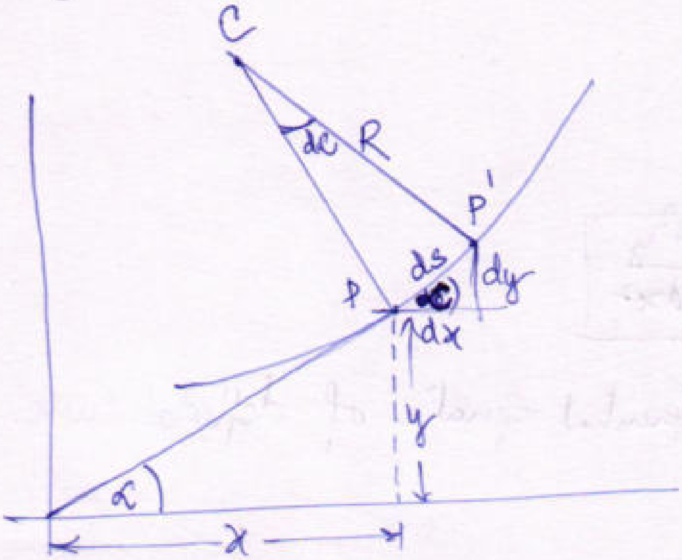
$$\text{Percentage Saving in weight} = \frac{(A_S - A_H)L \cdot \rho}{A_S L \cdot \rho} \times 100$$

$$= \frac{\frac{\pi}{4}(117.468)^2 - \frac{\pi}{4}(123.03^2 - 73.81^2)}{\frac{\pi}{4} \times 117.468^2} \times 100$$

$$\boxed{\text{Saving in weight} = 29.78\,\%}$$

Differential Equation for elastic curve

Consider a member AB of span 'L' subjected to a uniform bending moment M so that the member is bent into a circular shape. Let 'R' be the radius of curvature of bent member as shown in the figure.



Consider an elemental length $PP' = ds$ of the curve. As the distance 'ds' is very small,

$$ds^2 = dx^2 + dy^2$$

From fig,

$$ds = R\,d\alpha = \sqrt{(dx^2+dy^2)} \quad \text{—①}$$

$$\frac{1}{R} = \frac{d\alpha}{\sqrt{(dx^2+dy^2)}}$$

$$R\,d\alpha = \sqrt{(dx^2+dy^2)}$$

Also

$$\tan\alpha = \frac{dy}{dx}$$

Differentiating,

$$\sec^2\alpha \cdot d\alpha = \frac{d^2y}{dx^2} \cdot dx$$

$$(1+\tan^2\alpha)\,d\alpha = \frac{d^2y}{dx^2} \cdot dx$$

$$\left(1+\left(\frac{dy}{dx}\right)^2\right)d\alpha = \frac{d^2y}{dx^2} \cdot dx$$

$$d\alpha = \frac{(d^2y/dx^2)\cdot dx}{\left[1+\left(\frac{dy}{dx}\right)^2\right]} \quad \text{—②}$$

Substituting ② in ①

$$\frac{R\cdot\left(\frac{d^2y}{dx^2}\right)\cdot dx}{\left[1+\left(\frac{dy}{dx}\right)^2\right]} = \sqrt{(dx^2+dy^2)}$$

$$= \sqrt{dx^2\left(1+\frac{dy^2}{dx^2}\right)}$$

$$\frac{R\cdot\left(\frac{d^2y}{dx^2}\right)\cdot dx}{\left[1+\left(\frac{dy}{dx}\right)^2\right]} = dx\sqrt{\left(1+\left(\frac{dy}{dx}\right)^2\right)}$$

Then

$$\frac{1}{R} = \frac{\frac{d^2y}{dx^2}}{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}$$

Since dy and dx are very small quantities, their squares are much smaller. Therefore, the term $\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}$ can be taken as 1.

Thus,

$$\frac{1}{R} = \frac{d^2y}{dx^2}$$

From bending equation

$$\frac{M}{I} = \frac{f}{y} = \frac{E}{R}$$

$$M = \frac{1}{R} EI$$

$$\therefore \boxed{M = EI \frac{d^2y}{dx^2}}$$

This equation is called differential equation of deflected curve.

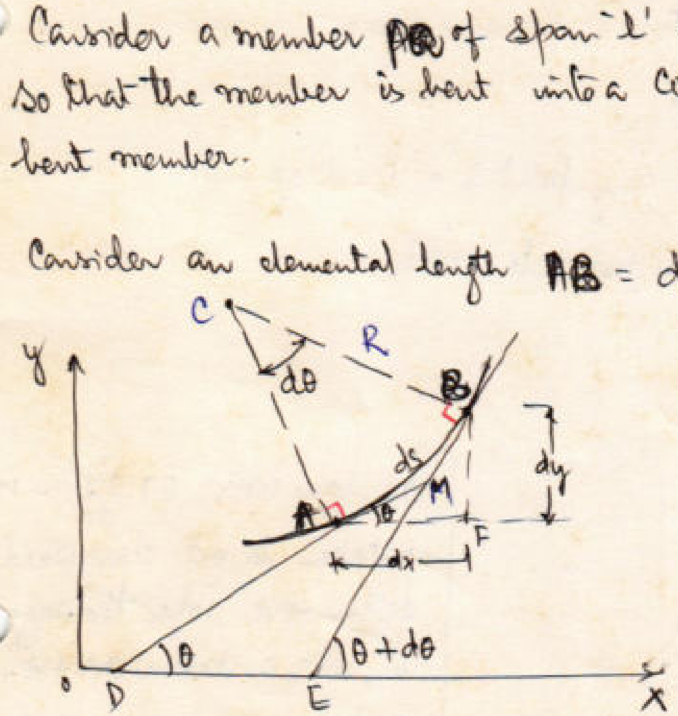
Deflection of Beams

Materials used for beams are elastic and hence under the action of loads on the beam, the axes deflect. A beam has to be designed not only based on Strength requirements but also from the Consideration of deflections which should be within specified limits. In machineries, the deflections in Components Cause Severe problems in functioning and in buildings/Structures excessive deformation gives rise to psychological fear/anxiety/ DisComfort and leads to breaking of Ceiling, flooring or roofing materials.

Differential equation for deflection:

Consider a member PQ of span 'l' Subjected to a uniform bending moment M so that the member is bent into a Circular shape. Let 'R' be the radius of the bent member.

Consider an elemental length AB = dS of a Curve. AC and BC are the normals at A and B respectively.



Let tangents drawn at A and B make angles θ and θ+dθ with x axis at D and E respectively.

Let M be the intersection point of these two tangents.

$\therefore \angle DME = d\theta$.

and $\angle DME + \angle AMB = 180^\circ$

Then $\angle AMB + \angle ACB = 360^\circ - 90^\circ - 90^\circ = 180^\circ$

Hence $\angle AMB + \angle ACB = \angle AMB + \angle DME$.

$\therefore \angle ACB = \angle DME = d\theta$.

$\therefore dS = R\,d\theta$ —— ①

Since ds is elemental length, ABF Can be treated as a triangle.

$\frac{dS}{dx} = \sec\theta$

$\frac{dy}{dx} = \tan\theta$

From Eq ①

Then $\frac{1}{R} = \frac{d\theta}{dS}$

~~Differentiating w.r.t x~~,

Differentiating w.r.t x

$\frac{d^2y}{dx^2} = \sec^2\theta \frac{d\theta}{dx}$.

$$= \cancel{\sec\theta \cdot \frac{d\theta}{\theta} \cdot \frac{ds}{dx}}$$

From eqn 1 $\frac{1}{R} = \frac{d\theta}{ds}$

$$= \sec^2\theta \frac{d\theta}{ds} \cdot \frac{ds}{dx}$$

$$= \sec^2\theta \frac{1}{R} \cdot \sec\theta$$

$$\frac{d^2y}{dx^2} = \sec^3\theta \frac{1}{R}$$

$$\frac{1}{R} = \frac{d^2y/dx^2}{\sec^3\theta}$$

but $\sec^2\theta = 1+\tan^2\theta$

$(\sec^2\theta)^{3/2}$

$$= \frac{\frac{d^2y}{dx^2}}{(1+\tan^2\theta)^{3/2}}$$

$$\cancel{= \frac{d^2y}{dx^2}}$$

For a practical member, $\tan\theta$ at any point is a small quantity

Hence $\tan^2\theta = \left(\frac{dy}{dx}\right)^2$ is neglected compared to unit.

$$\therefore \quad \frac{1}{R} = \frac{d^2y}{dx^2}.$$

From bending eqn.

$$\frac{M}{I} = \frac{E}{R}.$$

$$\frac{1}{R} = \frac{M}{EI}$$

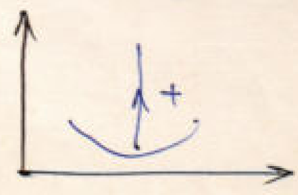
Hence

$$\frac{M}{EI} = \frac{d^2y}{dx^2}$$

and $\boxed{M = EI.\frac{d^2y}{dx^2}}$

In some books, $EI\frac{d^2y}{dx^2} = -M$ is taken to get downward defln +ve when the sagging moment is taken as +ve. and hogging moment is considered −ve

This equation is called differential eqn of deflection curve.



Note: The following sign conventions are used.

i) the y axis is upward.

ii) Curvature is concave towards positive y axis

iii) This type of curvature occurs due to sagging moment. Taken as +ve moment.

EI is called Flexural rigidity.

If the curve is concave towards +ve y axis, then $\frac{d^2y}{dx^2} > 0$ Hence $\frac{1}{R} > 0$. Hence taken as positive

Conversely if the curve is concave towards -ve y axis, then $\frac{d^2y}{dx^2} < 0$, then $\frac{1}{R} < 0$, Hence it is taken as negative.

Other Useful Equations:

Deflection $= y$

Slope $= \theta = \frac{dy}{dx}$

Moment $= M = EI \frac{d^2y}{dx^2}$

Shear force $: F = \frac{dM}{dx} = EI \frac{d^3y}{dx^3}$

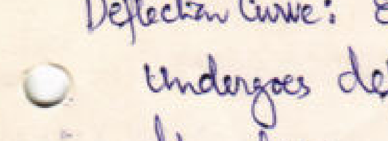
loading intensity $= q = \frac{dF}{dx} = EI \frac{d^4y}{dx^4}$

Some definitions.

Deflection: Distance through which a point on the longitudinal axis displaces in a direction transverse to longitudinal axis is known as deflection of the beam at that point. (y)

Deflection Curve: Elastic Curve: A beam subjected to transverse loads undergoes deflection. The deflected shape of longitudinal axis of the beam is known as deflection curve. Deflection curve is also known as Elastic curve.

Slope: The slope is the angle between the tangent to the deflection curve at that point and longitudinal axis or an axis parallel to longitudinal axis ($\frac{dy}{dx}$)



Derive the differential equation for the elastic line. March 2001

Derive the equation $EI \frac{d^2y}{dx^2} = -M$ with usual notations Sept 2000, Mar 2000

Double integration method:

In this method, the first moment M, at any distance 'x' from one of the supports (usually left hand support) is written with the sagging moment as positive

$$EI\frac{d^2y}{dx^2} = M$$

$$EI\frac{dy}{dx} = \int_0^x M\,dx + C_1$$

$$EI\,y = \int_0^x\int_0^x M\,dx + C_1x + C_2$$

The constants C_1 and C_2 are found by making use of boundary conditions. Useful boundary conditions are listed below:

(a) At simply supported / roller ends:

deflection $y = 0$

(b) At fixed ends:

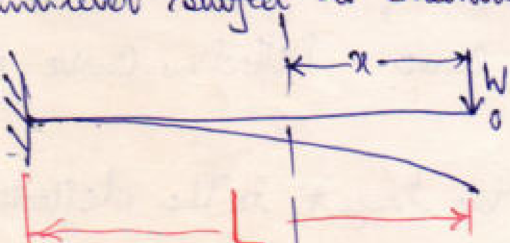
deflection $y = 0$

Slope $\frac{dy}{dx} = 0$

(c) At point of symmetry $\frac{dy}{dx} = 0$

General Cases:

Cantilever subject to concentrated load at free end:



Consider a cantilever of span 'L' subjected to load 'W' at free end. Consider a section at a distance 'x' from free end.
Taking hogging moment −ve

$$M_x = -Wx.$$

$$EI\frac{d^2y}{dx^2} = -Wx$$

$$EI\frac{dy}{dx} = -\frac{Wx^2}{2} + C_1$$

At $x = L$ $\frac{dy}{dx} = 0$

$$0 = -\frac{WL^2}{2} + C_1$$

$$C_1 = WL^2/2$$

$$EI\frac{dy}{dx} = -\frac{Wx^2}{2} + \frac{WL^2}{2}$$

$$EI\,y = -\frac{Wx^3}{6} + \frac{WL^2}{2}x + C_2$$

The boundary condition is

at $x = L$ $y = 0$

$$\therefore\quad 0 = -\frac{WL^3}{6} + \frac{WL^3}{2} + C_2$$

$$C_2 = -WL^3\left(\frac{1}{2} - \frac{1}{6}\right) = \frac{-WL^3}{3}$$

$\therefore$ $EI\,y = \frac{-Wx^3}{6} + \frac{WL^2}{2}.x - \frac{WL^3}{3}$

At free end i.e. $x = 0$,

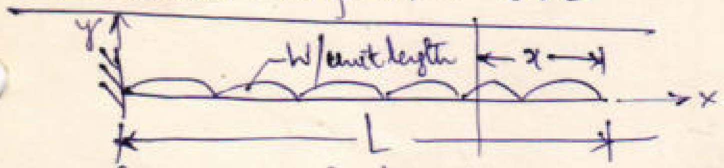
$$\text{Slope} = \frac{dy}{dx} = \frac{1}{EI}\left[\frac{WL^2}{2}\right] = \frac{WL^2}{2EI}$$

and $$y_{max} = \frac{1}{EI}\left[-\frac{WL^3}{3}\right] = -\frac{WL^3}{3EI}$$

i.e. Deflection is downward $\frac{WL^3}{3EI}$

A Cantilever Subjected to UDL

Feb 2002



Consider a Cantilever of span 'l' subjected to load 'w' over the whole span.

Consider a section at a distance 'x' from free end,

Taking hogging moment -ve,

$$M_x = -W.\frac{x^2}{2}$$

$$EI\frac{d^2y}{dx^2} = -\frac{Wx^2}{2}$$

$$EI\frac{dy}{dx} = -\frac{Wx^3}{6} + C_1$$

at $x = L$ $\quad \frac{dy}{dx} = 0$

$$0 = -\frac{Wl^3}{6} + C_1$$

$$C_1 = \frac{Wl^3}{6}$$

Then $$EI\frac{dy}{dx} = -\frac{Wx^3}{6} + \frac{Wl^3}{6}$$

Integrating, $$EI\,y = -\frac{Wx^4}{24} + \frac{Wl^3}{6}.x + C_2$$

at $x = L$, $\quad y = 0$

$$\therefore \quad 0 = -\frac{Wl^4}{24} + \frac{Wl^4}{6} + C_2$$

$$C_2 = -\frac{Wl^4}{8}$$

$$\therefore \quad EI\,y = -\frac{Wx^4}{24} + \frac{Wl^3}{6}.x - \frac{Wl^4}{8}$$

At free end where $x = 0$,

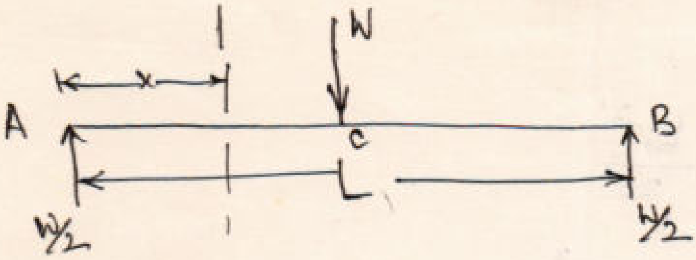
$$\boxed{\frac{dy}{dx} = \frac{Wl^3}{6EI}}$$

and y at $x = 0$

$$\left[y_{max} = -\frac{Wl^4}{8EI} \quad \text{downward}\right]$$

$$\boxed{y_{max} = \frac{Wl^4}{8EI}}$$

Simply supported beam with load at central span:



Consider a simply supported beam AB of span 'L' carrying central concentrated load 'W' at C, the centre of its span.

Reactions $R_A = R_B = W/2$

$$M_x = R_A x = \frac{Wx}{2}$$

$$EI\frac{d^2y}{dx^2} = \frac{Wx}{2}$$

$$EI\frac{dy}{dx} = \frac{Wx^2}{4} + C_1$$

When $x = l/2$, $\frac{dy}{dx} = 0$ $\therefore C_1 = \frac{-WL^2}{16}$

$$EI\frac{dy}{dx} = \frac{Wx^2}{4} - \frac{WL^2}{16} \quad \text{——— (1)}$$

Integrating again, we get, $EIy = \frac{Wx^3}{12} - \frac{WL^2}{16}\cdot x + C_2$

When $x=0$, $y=0$, then $C_2 = 0$

$$EIy = \frac{Wx^3}{12} - \frac{WL^2}{16}\cdot x \quad \text{——— (2)}$$

Deflection at mid span ie y_{max} at $x = L/2$ is

$$y_{max} = \frac{1}{EI}\left[\frac{WL^3}{96} - \frac{WL^3}{32}\right]$$

$$= \frac{1}{EI}\left[\frac{WL^3 - 3WL^3}{96}\right] = -\frac{WL^3}{48EI} \text{ downward.}$$

Slope at support $x=0$

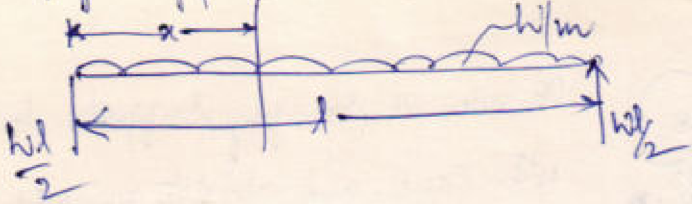
$$EI\frac{dy}{dx} = -\frac{WL^2}{16}$$

$$\frac{dy}{dx} = \frac{-1}{EI}\frac{WL^2}{16} = \frac{-WL^2}{16EI}$$

$$\boxed{y_{max} = \frac{WL^3}{48EI}}$$

$$\boxed{\text{Slope } \theta = \frac{WL^2}{16EI}}$$

Simply supported beam with UDL over whole span: Aug 1999



Consider a simply supported beam with UDL w/m over the whole span

$R_A = R_B = wl/2$

$$M_x = \frac{wl}{2}.x - w\frac{x^2}{2}$$

$$EI\frac{d^2y}{dx^2} = \frac{wl}{2}.x - \frac{wx^2}{2}$$

$$EI\frac{dy}{dx} = \frac{wlx^2}{4} - \frac{wx^3}{6} + C_1$$

at $x = l/2$, $\frac{dy}{dx} = 0$

$$0 = \frac{wl}{4}\left(\frac{l^2}{4}\right) - \frac{wl^3}{48} + C_1$$

$$C_1 = -wl^3\left[\frac{1}{16} - \frac{1}{48}\right] = -wl^3\left[\frac{3-1}{48}\right] = \frac{-wl^3}{24}$$

$$EI\frac{dy}{dx} = \frac{wlx^2}{4} - \frac{wx^3}{6} - \frac{wl^3}{24}$$

$$EIy = \frac{wl\,x^3}{12} - \frac{wx^4}{24} - \frac{wl^3.x}{24} + C_2$$

when $x = 0$ $y = 0$

$$C_2 = 0$$

Hence $$EIy = \frac{wl}{12}x^3 - \frac{wx^4}{24} - \frac{wl^3.x}{24}$$

Max deflection y_c which occurs at centre 'c' $x = l/2$

$$y_{max} = \frac{1}{EI}\left[\frac{wl}{12}.\frac{l^3}{8} - \frac{w}{24}.\frac{l^4}{16} - \frac{wl^4}{48}\right]$$

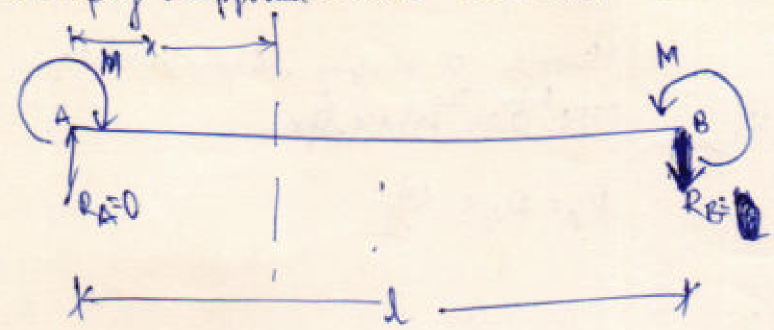
$$= \frac{1}{EI}\left[\frac{wl^4}{96} - \frac{wl^4}{384} - \frac{wl^4}{48}\right]$$

$$= \frac{wl^4}{EI}\left[\frac{4-1-8}{384}\right] = \frac{-5}{384}\frac{wl^4}{EI}$$

$$\boxed{y_{max} = \frac{5}{384}\frac{wl^4}{EI}}$$

Slope at $x=0$, $EI\frac{dy}{dx} = \frac{-wl^3}{24}$. $\frac{dy}{dx} = \frac{-wl^3}{24EI}$

Simply supported beam with end moments:



Consider a simply supported loaded with equal and opposite moments M as shown in fig.

Here $R_A = R_B = 0$

$R_B \times l = M - M$

$R_B = 0$

$R_A + R_B = 0$

$R_A = -R_B$

$= +0$

$M_x = M \qquad EI\frac{d^2y}{dx^2} = M$

$$EI\frac{dy}{dx} = M.x + C_1$$

when $x = l/2 \quad \frac{dy}{dx} = 0$

$$-\frac{Ml}{2} = C_1$$

$$\boxed{EI\frac{dy}{dx} = M.x - \frac{Ml}{2}}$$ — Equation for slope

$$EI\,y = M.\frac{x^2}{2} - \frac{ML}{2}.x + C_2$$

when $x = 0 \quad y = 0$

$$C_2 = 0$$

then $$\boxed{EI\,y = \frac{Mx^2}{2} - \frac{ML}{2}.x}$$ — Equation for deflection

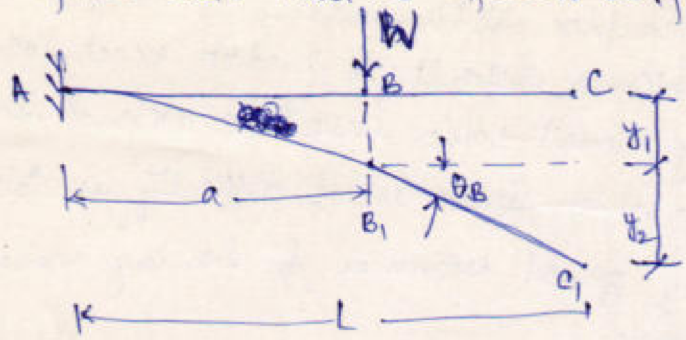
when $x = 0 \quad \theta_A = \frac{-ML}{2EI} \qquad$ ^{ly} at $x = L \quad \theta_B = \frac{ML}{2EI}$

$x = l/2 \quad y = y_{max} = \frac{Ml^2}{8} - \frac{ML^2}{4}$

$$y = \frac{1}{EI}\frac{ML^2}{4}\left[\frac{1}{2} - 1\right] = -\frac{ML^2}{8EI}$$

A Cantilever beam of span L is subjected to a point load W at a distance 'a' from fixed end. Find the deflection at free end.

Homework. Bhavitalli



Soln: Let AC be the Cantilever subjected to load 'W' at B as shown in fig. Let AB_1C_1 be the deflected shape.

Now

$$\text{Deflection at } B = y_1 = \frac{Wa^3}{3EI}$$

$$\text{and Slope at } B = \theta_B = \frac{Wa^2}{2EI}.$$

Since the portion BC is not subjected to any moment it remains straight and its slope is θ_B.

$$\therefore \text{ Deflection at } C = \text{Deflection at } B + (L-a) \text{ Slope at } B$$

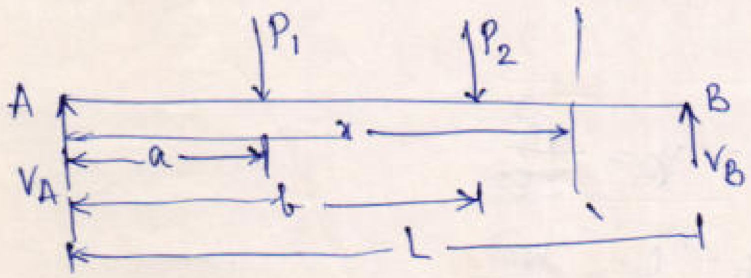
$$= y_1 + (L-a)\theta_B$$

$$= \frac{Wa^3}{3EI} + (L-a)\frac{Wa^2}{2EI}$$

Macaulay's Method:

Estimation of deflection will be more complex if the loading is complex and not symmetrical. In such cases, it becomes difficult to predict exact location of maximum deflections. The bending moment values have to be defined separately for different segments of the beam ~~thedyp~~ which causes lot of difficulty in calculations. Hence the speciality of the method lies in writing general expression for bending moment and retaining original forms in integration.

The method is illustrated with the example of a simply supported beam as shown in fig.



Let V_A be the reaction at A. The expression for bending moment in general

General. $$M_x = V_A \cdot x - P_1(x-a) - P_2(x-b) \qquad — ①$$

$$EI\frac{d^2y}{dx^2} = V_A x - P_1(x-a) - P_2(x-b)$$

$$EI\frac{dy}{dx} = C_1 + V_A\frac{x^2}{2} - P_1\frac{(x-a)^2}{2} - P_2\frac{(x-b)^2}{2} \qquad — ②$$

Here a) Normally constant of integration is written first, since it is applicable to all points.

b) $\int(x-a)dx$ is written $\frac{(x-a)^2}{2}$ [instead of $\frac{x^2}{2} - ax$. Both integrations are correct, the only difference is const of integration is different which is still arbitrary.

$$EIy = C_2 + C_1x + V_A\frac{x^3}{6} - \frac{P_1(x-a)^3}{6} - P_2\frac{(x-b)^3}{6} \qquad — ③$$

C_1 and C_2 are found from boundary conditions.

~~12.350, 17.50~~. [It is very important to note that if the expression in brackets $(x-a)$ or $(x-b)$ becomes negative, the terms containing this factor is to be omitted.

at $x=0$, $y=0$, then $C_2=0$.

at $x=L$ $y=0$ $\quad 0 = C_1L + \frac{V_AL^3}{6} - \frac{P_1(L-a)^3}{6} - \frac{P_2(L-b)^3}{6}$

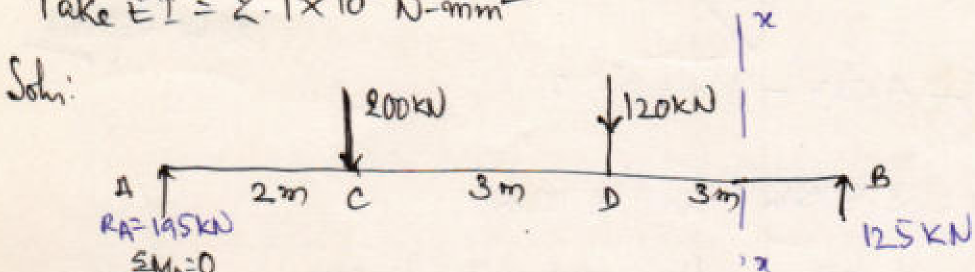
Hence C_1 can be found. once C_1 and C_2 are known, slope and deflection at any point can be found using eqns ② and ③.

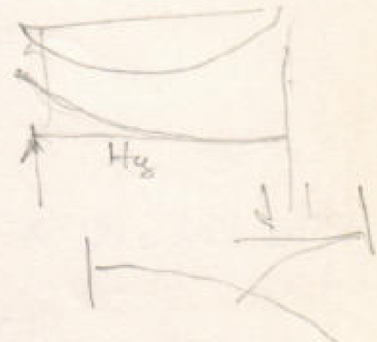
(C)

Determine the max deflection and the deflections under the loads for the simply supported beam ACDB with AC = 2m, CD = 3m and DB = 3m carrying point loads of 200KN and 120KN acting at C and D respectively. Take $EI = 2.1\times10^{15}$ N-mm²

Aug 99

Soln:





$\Sigma M_A = 0$

$R_B \times 8 = 120\times5 + 200\times2$

$R_B = 125\text{KN}$

$\Sigma V = 0$

$R_A + R_B = 320$

$R_A = 320 - 125 = 195\text{KN}.$

Bending moment at section x-x $= M = EI\frac{d^2y}{dx^2} = 195\cdot x - 200(x-2) - 120(x-5)$

Then

$$EI\frac{dy}{dx} = +195\cdot\frac{x^2}{2} - 200\frac{(x-2)^2}{2} - 120\frac{(x-5)^2}{2} + C_1$$

$$EIy = 195\frac{x^3}{6} - 200\frac{(x-2)^3}{6} - 120\frac{(x-5)^3}{6} + C_1x + C_2$$

when $x=0$, $y=0$

$$0 = C_2$$

$x=8$ $y=0$

$$0 = 16640 - 7200 - 540 + 8C_1$$

$$C_1 = -1112\cdot5$$

KN/m² · m⁴ · m

KN - m³

General equation for deflection is

$$EIy = \frac{195x^3}{6} - \frac{200(x-2)^3}{6} \frac{120(x-5)^3}{6} - 1112\cdot5x$$

at $x=2m$ $\quad EIy = \frac{195\times2^3}{6} - 1112\cdot5\times2 = -1965\text{KN-m}^3$

$$y = \frac{-1965\times10^{12}}{2.1\times10^{15}} = -0.935\text{ mm}$$

at $x=5$ $\quad EIy = \frac{195\times5^3}{6} - \frac{200(3)^3}{6} - 1112\cdot5\times5 = -2400\text{ KN-m}^3 = -2400\times10^3\times(10^3)^3 = -2400\times10^{12}$

$$Y = \frac{-2400\times10^{12}}{2.1\times10^{15}} = 1.142\text{ mm}$$

Maximum deflection would occur at a point where slope is zero. i.e. $\frac{dy}{dx} = 0$ - The max deflection occurs in portion CD.

$$97.5x^2 - 100(x-2)^2 - \cancel{60(x-5)^2} - 1112.5 = 0$$

Hence this term to be neglected

$$-2.5x^2 + 400x - 1512.5 = 0$$

$$x = 3.875\,m \quad \text{or } 156.12\,m$$

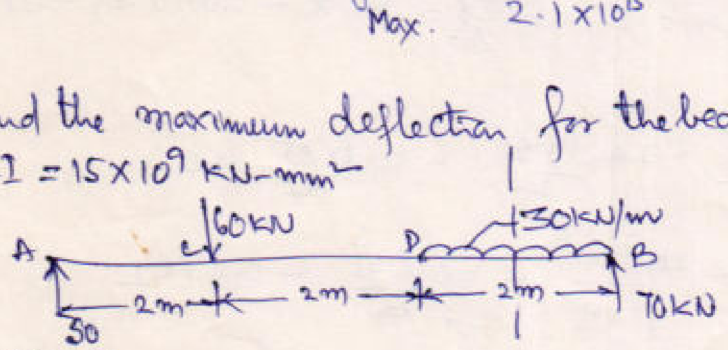
The Value of $x = 3.875\,m$ is OK. Hence assumption is OK.

$$EIy = \frac{195(3.875)^3}{6} - \frac{200(1.875)^3}{6} - 1112.5 \times 3.875$$

$$y_{Max.} = \frac{-2639.63 \times 10^{12}}{2.1 \times 10^{15}} = -1.256\,mm$$

Find the maximum deflection for the beam loaded as shown in fig.
$EI = 15 \times 10^9$ KN-mm^2

Aug/Sept 99



$\Sigma M_A = 0$

$R_B \times 6 = 60 \times 2 + 30 \times 2 \times 5$

$R_B = 70\,KN$

$R_A + R_B = 60 + 60$

$R_A = 120 - 70 = 50\,KN.$

$$M = 50.x - 60(x-2) - 30(x-4)\frac{(x-4)}{2}$$

$$EI\frac{d^2y}{dx^2} = 50x - 60(x-2) - 15(x-4)^2$$

$$EI\frac{dy}{dx} = c_1 + 50\frac{x^2}{2} - \frac{60(x-2)^2}{2} - \frac{15(x-4)^3}{3}$$

and $EIy = c_1x + c_2 + 25\frac{x^3}{3} - 10(x-2)^3 - \frac{15}{12}(x-4)^4$

At $x = 0$, $y = 0$. $\quad c_2 = 0$

At $x = 6$ $y = 0$ $\quad 0 = 6c_1 + \frac{25 \times 6^3}{3} - 10(4)^3 - \frac{5}{4}(2)^4$

$$c_1 = -\frac{1140}{6} = -190$$

General Eqn

$$EIy = -190x + \frac{25x^3}{3} - 10(x-2)^3 - \frac{15}{12}(x-4)^4$$

~~Maximum deflection would occur at a point where slope is zero.~~

~~i.e. $\frac{dy}{dx} = 0$. The max deflection occurs in portion CD. (Assumption with CD)~~

$$\cancel{0 = 97.5x^2 - 100(x-2)^2 - 60(x-5)^2 - 1112.5}$$

$$\cancel{97.5x^2 - 100x^2 - 400 + 400x - 1112.5 = 0}$$

$$\cancel{-2.5x^2 + 400x - 1512.5 = 0}$$

$$\cancel{x = \frac{-400 \pm \sqrt{(400)^2 - 4(-2.5)(-1512.5)}}{2(-2.5)}}$$

$$\cancel{= \frac{-400 \pm \sqrt{144875}}{-5} = 3.875\,m \text{ or } 156.12\,m}$$

~~The value of $x = 3.875$ m. Hence assumption is OK.~~

$$\cancel{EIy = \frac{195(3.875)^3}{6} - \frac{200(1.875)^3}{6} - 1112.5 \times 3.875}$$

$$\cancel{y = \frac{-2639.63 \times 10^{12}}{2.1 \times 10^{15}} = -1.256\,mm}$$

Assuming the deflection to be maximum in the portion CD and substs the value of C_1 in the expression for slope,

$\frac{dy}{dx} = 0$ at max deflectn

$$0 = -190 + 25x^2 - 30(x-2)^2 - \cancel{5(x-4)^3} \quad \text{Hence this term to be neglected.}$$

$$= -190 + 25x^2 - 30(x^2 + 4 - 4x)$$

$$0 = -190 + 25x^2 - 30x^2 - 120 + 120x$$

$$0 = -5x^2 + 120x - 310$$

$$0 = x^2 - 24x + 62$$

$$\cancel{= x^2 - 12x - 2x + 24}$$

$$\cancel{= 2x(x-12) - 2(x-12)}$$

Hence assumption is correct. $x = 2.94$ mts

$$x = \frac{+24 \pm \sqrt{(-24)^2 - 4(1)(62)}}{2(1)}$$

$$\cancel{x = \ldots \text{ or } 2.940}$$

$$x = 2.94\,m \text{ or } 21.05.$$

~~Assume to be max in portion AB,~~

$$\cancel{0 = -190 + 25x^2}$$

$$\cancel{x^2 = \frac{-190}{25}}$$

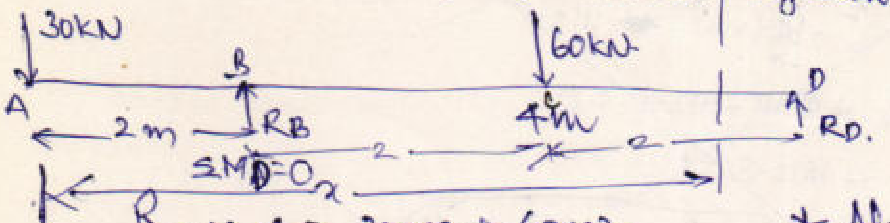
$$EIy_{max} = -190(2.94) + 25\frac{(2.94)^3}{3} - 10(0.94)^3$$

$$y_{max} = \frac{-355.13\,kN\text{-}m^3}{EI} = \frac{-355.13 \times (1000)^3\ kN\text{-}mm^3}{15 \times 10^9}$$

$$\boxed{y_{max} = -23.67\ mm}$$

6m long beam ABCD Consists of overhang AB = 2m and BD = 4m Carries Concentrated loads at free end 'A' of magnitude 30 kN and at C, the Centre of BD 60 kN. Determine the maximum deflection and State where it occurs.

March 2000.



$R_B \times 4 = 30 \times 6 + 60 \times 2$

* Always the section to be Considered at right end

$R_B = 75\text{ kN}$

$R_B + R_D = 30 + 60$

$R_D = 90 - 75 = 15\text{ kN}$

$$M_x = -30 \cdot x + 75(x-2) - 60(x-4)$$

$$EI\frac{d^2y}{dx^2} = -30 \cdot x + 75(x-2) - 60(x-4)$$

$$EI\frac{dy}{dx} = -30 \cdot \frac{x^2}{2} + 75\frac{(x-2)^2}{2} - 60\frac{(x-4)^2}{2} + C_1$$

$$EI\,y = -30 \cdot \frac{x^3}{6} + 75\frac{(x-2)^3}{6} - 60\frac{(x-4)^3}{6} + C_1x + C_2$$

At $x = 2$, $y = 0$

$$0 = -30 \times \frac{8}{6} + C_1 \times 2 + C_2$$

$$2C_1 + C_2 = 40 \quad \text{--- (1)}$$

At $x = 6$, $y = 0$

$$0 = -30 \times 36 + 75\frac{(4)^3}{6} - 60\frac{(2)^3}{6} + 6C_1 + C_2$$

$$6C_1 + C_2 = 360 \quad \text{--- (2)}$$

Solving (1) and (2)

$4C_1 = 320$

$\boxed{C_1 = 80}$ $\quad \boxed{C_2 = -120}$

$$\therefore EI\,y = -\frac{30x^3}{6} + \frac{75(x-2)^3}{6} - \frac{60(x-4)^3}{6} + 80x - 120$$ — General eqn for deflection

y_{max} may be at two locations. 1) Deflection may be maximum at Slope is zero.
2) Deflection may be maximum at A.

Location of max deflection:

At maximum deflection, slope is 0 between C and D.

$$\frac{dy}{dx}=0 \qquad -15x^2+37.5(x-2)^2-30(x-4)^2+80=0$$

$$-15x^2+37.5(x^2+4-4x)-30(x^2+16-8x)+80=0$$

$$-15x^2+37.5x^2+150-150x-30x^2-480+240x+80=0$$

$$-7.5x^2+90x-250=0$$

$$x^2-12x+33.33=0$$

$$\therefore\ x=\frac{12\pm\sqrt{(-12)^2-4(1)(33.33)}}{2(1)}$$

$$=\frac{12\pm 3.27}{2}=7.63\,m \text{ or } 4.37\,m.$$

At $x=4.37m$

$$EI\,y_{max}=-5(4.37)^3+12.5(2.37)^3-10(0.37)^3+80\times 4.37-120$$

$$y_{max}=\frac{-21.77}{EI}$$

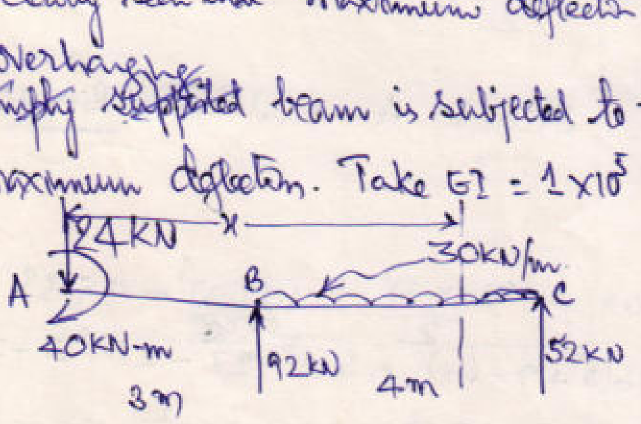
At $x=0$

$$EI\,y_{max}=-120 \qquad y_{max}=\frac{-120}{EI}$$

It is clearly seen that maximum deflection occurs at A, the free end of beam.

A ~~Simply supported~~ overhanging beam is subjected to the loads as shown in fig. Determine the maximum deflection. Take $EI = 1\times10^5$ N-m² for the beam.

March 2001.



$EI = 1\times10^5\times10^9$ N-mm²

To find reactions R_B and R_C

$$\Sigma M_C=0$$

$$R_B\times 4=30\times4\times2+24\times7-40$$

$$R_B=92\,kN.$$

$$R_C+R_B=24+30\times4=144\,kN$$

$$R_C=144-92=52\,kN.$$

$$M_x=-24x+40+92(x-3)-30\frac{(x-3)^2}{2}$$

$$EI\frac{d^2y}{dx^2}=-24x+40+92(x-3)-30\frac{(x-3)^2}{2}$$

$$EI\frac{dy}{dx} = -24\frac{x^2}{2} + 40x + \frac{92(x-3)^2}{2} - \frac{30(x-3)^3}{6} + C_1$$

$$EI\,y = -\frac{24x^3}{6} + \frac{40x^2}{2} + \frac{92}{6}(x-3)^3 - \frac{30(x-3)^4}{24} + C_1x + C_2$$

When $x = 3$ $y = 0$

$$0 = -4(3)^3 + 20(3)^2 + 15.33 - 0 + C_1 3 + C_2$$

$$3C_1 + C_2 = 108 - 180 = -72 \quad \text{—— ①}$$

When $x = 7$ $y = 0$

$$0 = -4(7)^3 + 20(7)^2 + 15.33(4)^3 - 1.25(4)^4 + 7C_1 + C_2$$

$$7C_1 + C_2 = -269.12 \quad \text{———— ②}$$

Subtract ① from ②

$$4C_1 = -197.33$$

$$C_1 = -49.33 \qquad C_2 = 75.99.$$

Location of max deflection : It can be A and between B and C

At max deflection $\frac{dy}{dx} = 0$

$$0 = -12x^2 + 40x - \frac{92(x-3)^2}{2} - 5(x-3)^3 - 49.33$$

$$= -12x^2 + 40x - 46(x^2+9-6x) - 5(x^3 - 27 - 2(x)^2 3 + 2 \times 3^2) - 49.33$$

$$= -12x^2 + 40x - 46x^2 - 414 + 276x - 5x^3 + 135 + 30x^2 - 90x - 49.33$$

$$0 = -5x^3 - 28x^2 + 226 - 326.33$$

$$0 = -12x^2 + 40x - 46(x-3)^2 - 5(x-3)^3 - 49.33$$

:

Maximum deflection at A, $x = 0$

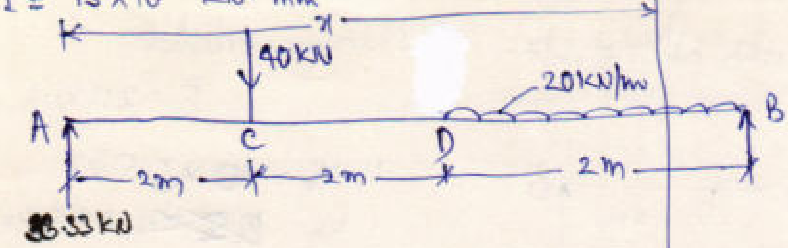
$$EI\,y_{max} = 0 + 0 + 0 + 75.99$$

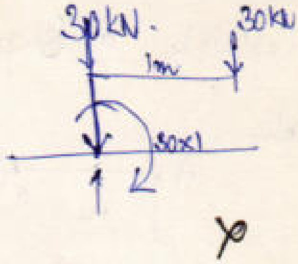
$$y_{max} = \frac{75.99 \times 10^{12}\ \text{N-mm}^3}{1 \times 10^{14}\ \text{N-mm}^2} = 0.76\,\text{mm}$$

$75.99 \times 10^3 \times 10^9$
$= 75.99 \times 10^{12}$ N-mm^3

Find the maximum deflection and max slope for the beam loaded as shown in fig

$EI = 15 \times 10^9$ kN-mm^2.





$V_A \times 6 = 40 \times 4 + 20 \times 2 \times 1$

$V_A = 33.33$ kN.

$M_x = 33.33x - 40(x-2) - \frac{20(x-4)^2}{2}$ kN-m

$EI\frac{d^2y}{dx^2} = 33.33x - 40(x-2) - 10(x-4)^2$ kN-m

$EI\frac{dy}{dx} = 33.33\frac{x^2}{2} - 40\frac{(x-2)^2}{2} - 10\frac{(x-4)^3}{3} + C_1$ kN-m^2

$EIy = 33.33\frac{x^3}{6} - \frac{40}{6}(x-2)^3 - \frac{10}{12}(x-4)^4 + C_1x + C_2$

At $x=0$, $y=0$ $C_2 = 0$

$x=6$ $y=0$ $C_1 = -126.667$

Assuming deflection to be max in portion CD, $\frac{dy}{dx} = 0$

$0 = 33.33\frac{x^2}{2} - 20(x-2)^2 - 126.667$.

$= 33.33\frac{x^2}{2} - 20x^2 + 80x - 80 - 126.667$

$x^2 - 24x + 62 = 0$

$x = 2.945$ m. Hence assumption is correct.

$EIy_{max} = 33.33\frac{(2.945)^3}{6} - \frac{40}{6}(0.945)^3 - \cancel{\frac{10}{12}(2.945-4)^4} - 126.667 \times 2.945$

$= -236.759$ kN-m^3

$= -236.759 \times 10^9$ kN-mm^3

$y_{max} = \frac{-236.759 \times 10^9}{15 \times 10^9}$

$= -15.784$ mm.

Maximum slope occurs at ends. $\theta_A = -126.667$ kN-m^2 $\left(\frac{dy}{dx}\right)_A = \frac{-126.667 \times 10^6}{15 \times 10^9}$

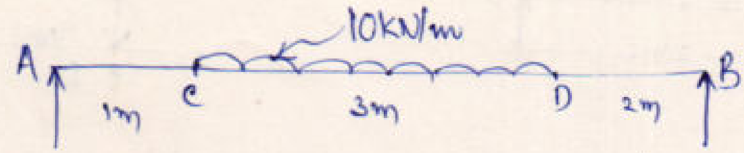
$\theta_B = 133.33$

$= -8.44 \times 10^{-3}$ radians

A beam AB is 6m long has a moment of inertia of 450×10^6 mm⁴. It is supported at A and B Carries a uniformly distributed load of 10kN/m from C to D as shown in fig. Calculate a) Slope at A b) Deflection at mid span c) Maximum deflection

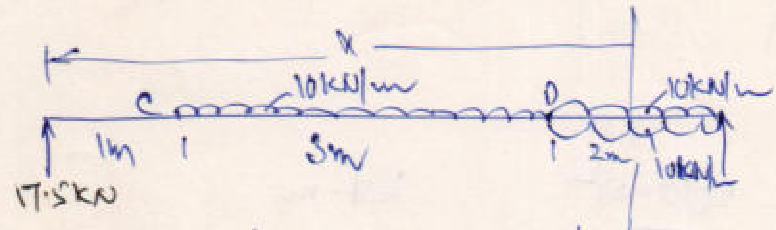
$E = 200\ kN/mm^2$

Soln



$V_A \times 6 = 10 \times 3.5 \times 3$

$V_A = 17.5\ kN$



* To maintain the generality of the expression, as required in Macaulay's method downward udl is extended upto right support and equal udl is applied in upward direction.

$$\therefore M_x = 17.5 \cdot x - \frac{10(x-1)^2}{2} + \frac{10(x-4)^2}{2} \quad kN\text{-}m$$

$$EI \frac{d^2y}{dx^2} = 17.5x - 5(x-1)^2 + 5(x-4)^2 \quad kN\text{-}m$$

$$EI \frac{dy}{dx} = \frac{17.5x^2}{2} - \frac{5(x-1)^3}{3} + \frac{5(x-4)^3}{3} + C_1 \quad kN\text{-}m^2$$

$$EI\, y = \frac{17.5x^3}{6} - \frac{5(x-1)^4}{12} + \frac{5}{12}(x-4)^4 + C_1x + C_2 \quad kN\text{-}m^3$$

At $x=0$, $y=0$, $C_2=0$

$x=6$ $y=0$, $C_1 = -62.70$

(i) Slope at A ie at $x=0$.

$$EI\left(\frac{dy}{dx}\right)_A = -62.708\ kN\text{-}m^2$$

$$\frac{dy}{dx} = \frac{-62.708 \times 10^6 \times 10^3}{200 \times 10^3\, 450 \times 10^6} = 6.967 \times 10^{-4} \text{ radians}$$

(ii) Deflection at mid span $x = 3m$

$$EI(y)_{Centre} = \frac{17.5(3)^3}{6} - \frac{5(2)^4}{12} + \cancel{\frac{5}{12}} - 62.70$$

$$EI\, y_{Centre} = -116.04\ kN\text{-}m^3 = -116.04 \times 10^9\ kN\text{-}mm^3$$

$$y_{Centre} = \frac{-116.04 \times 10^9}{200 \times 450 \times 10^6} = -1.289\ mm.$$

iii) To find maximum deflection:

$\frac{dy}{dx} = 0$ at y_{max}

Assume it to be in portion CD. (2m and 4)

$$0 = 17.5\frac{x^2}{2} - 5\frac{(x-1)^3}{3} - 62.708$$

By trial and error, $x = 2.92$

Then y_{max} occurs at 2.92m

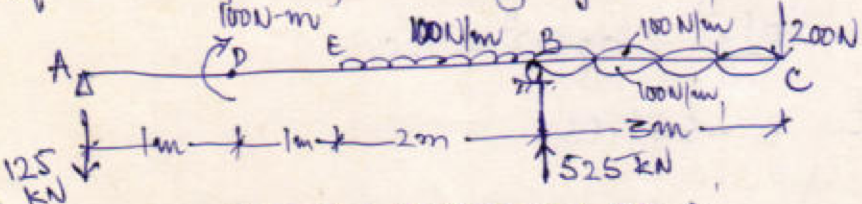
$$EI\, y_{max} = 17.5\frac{(2.9)^3}{6} - \frac{5}{12}(\cancel{x})^4 - 62.708 \times 2.92$$

$$= -116.153 \text{ kN-m}^3$$

$$y_{max} = \frac{-116.153 \times 10^3 \times 10^9}{200 \times 10^3 \times 450 \times 10^6} = -1.290 \text{ mm. Downward.}$$

Determine the equation of the elastic curve of the prismatic beam supported and loaded as shown in fig. Determine also the slope of the deflection curve at the left end and the deflection at the right end of the beam in terms of the uniform flexural rigidity EI.

Aug 2001



$R_B \times 4 = 200 \times 7 + 100 \times 2 \times 3 + 100$ } $R_A + R_B = 200 + 100 \times 2 = 400$

$R_B = \frac{2100}{4} = 525$ kN $\qquad R_A = 400 - 525 = -125$ kN

* To maintain the generality of the expression, as required in Macaulay's method, downward udl is extended upto ~~right support~~ free end and equal udl is applied in upward direction.

$$\therefore\ M_x = -125x + 100 - 100\frac{(x-2)^2}{2} + 100\frac{(x-4)^2}{2} + 525(x-4)$$

$$EI\frac{d^2y}{dx^2} = -125x - \frac{100}{2}(x-2)^2 + 50(x-4)^2 + 525(x-4) + 100(x-1)^0$$

$$EI\frac{dy}{dx} = -125\frac{x^2}{2} - 50\frac{(x-2)^3}{3} + \frac{50(x-4)^3}{3} + 525\frac{(x-4)^2}{2} + 100(x-1) + C_1$$

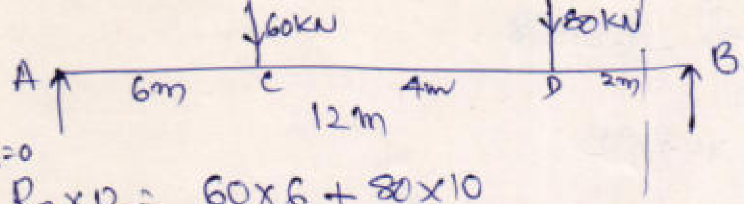
$$EI\,y = -125\frac{x^3}{6} - \frac{50}{12}(x-2)^4 + \frac{50}{12}(x-4)^4 + \frac{525}{6}(x-4)^3 + 100\frac{(x-1)^2}{2} + C_1x + C_2$$

Continued in Page 11 A *

A beam of uniform section is 12m long and is simply supported at the ends. It carries concentrated loads of 80KN and 60KN at distances of 2m and 6m from the right end respectively. Calculate the deflections under each load and also maxm deflecn

$E = 2\times10^5\ N/mm^2$ and $I = 1800\times10^6\ mm^4$.

Feb 2002



$\Sigma M_A = 0$

$R_B \times 12 = 60\times6 + 80\times10$

$R_B = 96.67\ KN$

$\Sigma V = 0 \quad R_A + R_B = 140$

$R_A = 43.33\ KN$

$M_x = +43.33\times x - 60(x-6) - 80(x-10)$

$EI\frac{d^2y}{dx^2} = 43.33x - 60(x-6) - 80(x-10) \quad KN-m$

$EI\frac{dy}{dx} = 43.33\frac{x^2}{2} - \frac{60(x-6)^2}{2} - \frac{80(x-10)^2}{2} + C_1 \quad KN-m^2$

$EIy = 43.33\frac{x^3}{6} - \frac{30(x-6)^3}{3} - \frac{40}{3}(x-10)^3 + C_1x + C_2 \quad KN-m^3$

When $x=0 \quad y=0$

$C_2 = 0$

When $x = 12, y = 0$

$0 = 43.33\frac{(12)^3}{6} - 30\frac{(6)^3}{3} - \frac{40}{3}(2)^3 + 12C_1$

$C_1 = \frac{-10212.37}{12} = -851.03$

General eqn for deflecn

$EIy = 43.33\frac{x^3}{6} - 10(x-6)^3 - \frac{40}{3}(x-10)^3 - 851x$

When $x = 6 \quad y_C$

$EIy_C = 43.33\times6^2 - 851\times6$

$= \frac{-3546.12\times10^{12}}{2\times1800\times10^{11}} = -9.850\ mm$

When $x = 10, y = y_D$

$EIy_D = 43.33\times\frac{10^3}{6} - 10(4)^3 - 851\times10$

$y_D = -\frac{1928.33\times10^{12}}{2\times1800\times10^{11}} = -5.356\ mm.$

Continued from page 10

* When $x=0$, $y=0$

$C_2 = 0$.

When $x=4$, $y=0$

$$0 = -125 \times \frac{4^3}{6} - \frac{50}{12}(2)^4 + \frac{100}{2}(3)^2 + 4C_1$$

$$C_1 = \frac{949.99}{4} = 237.5$$

General eqn for deflection:

$$EI\,y = -125\frac{x^3}{6} - \frac{50}{12}(x-2)^4 + \frac{50}{12}(x-4)^4 + \frac{525}{6}(x-4)^3 + \frac{100}{2}(x-1)^2 + 237.5x \quad \text{—} (1)$$

General eqn for ~~deflection~~ slope:

$$EI\frac{dy}{dx} = -125\frac{x^2}{2} - \frac{50}{3}(x-2)^3 + \frac{50}{3}(x-4)^3 + \frac{525}{2}(x-4)^2 + 100(x-1) + 237.5$$

θ_A ~~:~~ θ_A is at $x=0$

$$\boxed{\theta_A = \frac{237.5}{EI}}$$

Deflection at C i.e. at $x = 7$m

$$EI\,y_C = \frac{-125(7)^3}{6} - \frac{50}{12}(5)^4 + \frac{50}{12}(3)^4 + \frac{525}{6}(3)^3 + \frac{100}{2}(6)^2 + 237.5\times 7$$

$$\boxed{y_c = \frac{3587.5}{EI}}$$

TO find maxm deflection

>6 and <10

Assuming max deflection occurs between C and D, i.e. $\frac{dy}{dx} = 0$. between C and D.

$$0 = \frac{43.33}{2}x^2 - \frac{60}{2}(x-6)^2 - \cancel{\frac{80(x-10)^2}{2}} + \cancel{237.5} - 851.83$$

$$= 21.67x^2 - 30(x^2+36-12x) \cancel{+237.5} - 851$$

$$= 21.67x^2 - 30x^2 - 1080 + 360x \cancel{+237.5} - 851$$

$$= -8.33x^2 + 360x \cancel{-842.5} - 1931$$

$$8.33x^2 - 360x + \cancel{842.5}\,1931 = 0$$

$$x^2 - 43.21x + \cancel{101.14}\,231.81 = 0$$

$x = \cancel{6.99}$ m or 40.72 ~~both are irrelevant.~~

$$x = \frac{+43.21 \pm \sqrt{(43.21)^2 - 4(1)(231.81)}}{2(1)}$$

$$= \frac{43.21 \pm 30.65}{2}$$

$$= 6.276 \text{ and } 36.93$$

~~Again~~ between A and C max deflection

~~$0 = 21.67x^2 + 237.5$~~

?

Hence

$$EI\,y_{max} = \cancel{\ldots}\ \frac{43.33}{6}(6.276)^3 - 10(0.276)^3 - 851(6.276)$$

$$EI\,y_{max} = -3555.88 \text{ kN-m}^3$$

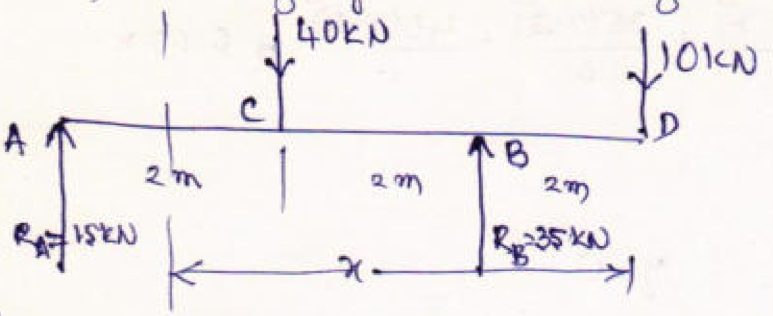
$$\therefore y_{max} = -\frac{3555.88\times 10^{12}}{2\times 1800\times 10^{11}} = 9.877 \text{ mm}$$

A Steel beam having a uniform Cross section is 14 m span and is simply supported at its ends. It Carries a Concentrated load of 120 KN and 80 KN at two points 3m and 4.5m from the left end and right end respectively. If moment of inertia of the section is 160×10^7 mm^4 and $E = 210$ GPa, Calculate the deflection of the beam at load points.

Aug 2000

Determine the deflection under the loads in the beam shown in fig 6. Take flexural rigidity as EI throughout.

Jan 09



Soln:

$\Sigma M_A = 0 \qquad -R_B \times 4 + 10 \times 6 + 40 \times 2 = 0$

$\boxed{R_B = \frac{140}{4} = 35kN}$

$\Sigma V = 0$

$R_A + R_B = 50 \qquad \boxed{R_A = 50 - 35 = 15KN}$

$EI\left(\frac{d^2y}{dx^2}\right) = M_x = +35 \times (x-2) - 10 \cdot x - 40(x-4)$

$EI\frac{dy}{dx} = -10\frac{x^2}{2} + 35\frac{(x-2)^2}{2} - 40\frac{(x-4)^2}{2} + C_1$

$\cancel{EI\left(\frac{dy}{dx}\right)} = EI(y) = -\frac{10x^3}{6} + \frac{35(x-2)^3}{6} - \frac{40(x-4)^3}{6} + C_1x + C_2$

When $x = 2$, $y = 0$.

$-\frac{10(2)^3}{6} + C_1(2) + C_2 = 0$

$2C_1 + C_2 = 13.333$ ——— (1)

$x = 6$, $y = 0$

$0 = -\frac{10(6)^3}{6} + \frac{35(4)^3}{6} - \frac{40(2)^3}{6} + 6C_1 + C_2$

$6C_1 + C_2 = 40$ ——— (2)

Solving (1) and (2)

$C_2 = 13.333 - 2C_1$

$C_2 = 40 - 6C_1$

$13.33 - 2C_1 = 40 - 6C_1$

$4C_1 = 40 - 13.33$

$\boxed{C_1 = 6.667}$

$\boxed{C_2 = 0}$

Slope eqn : $EI\left(\frac{dy}{dx}\right) = -10\frac{x^2}{2} + \frac{35(x-2)^2}{2} - 40\frac{(x-4)^2}{2} + 6.67$

Deflⁿ eqn : $EI(y) = -10\frac{x^3}{6} + \frac{35(x-2)^3}{6} - \frac{40(x-4)^3}{6} + 6.67x$

Deflectn at D → $x = 0$

$$EI(y) = 0 \qquad y = 0$$

at C → $x = 4$

$$EI(y) = \frac{-10(4)^3}{6} + \frac{35(2)^3}{6} + 6.67\times4$$

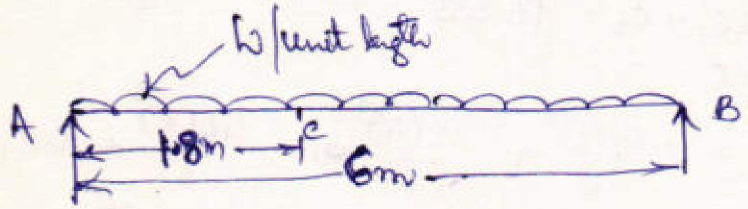
$$\boxed{y = -\frac{33.32}{EI}\text{ mm}}$$

A Steel girder of 6m length acting as a beam carries a UDL of W kN/m run throughout its length as shown in fig. If $I = 30\times10^{-6}\ m^4$ and depth 270 mm, Calculate.

(i) Magnitude of 'W' so that the maxm stress developed in the beam section does not exceed 72 MN/m²

(ii) The slope and deflection in the beam at a distance of 1.8m from one end. Take $E = 200\ GN/m^2$

June July 2009



$I = 30\times10^{-6}\times10^{12}$

$\boxed{I = 30\times10^{6}\ mm^4}$

$y = \frac{d}{2} = \frac{270}{2} = 135\ m$

$f = 72\ MN/m^2 = \frac{72\times10^6}{1000^2} = 72\ N/mm^2$

$M = \frac{Wl^2}{8} = \frac{W\cdot6^2\times10^6}{8} = \frac{36W\times10^6}{8}$

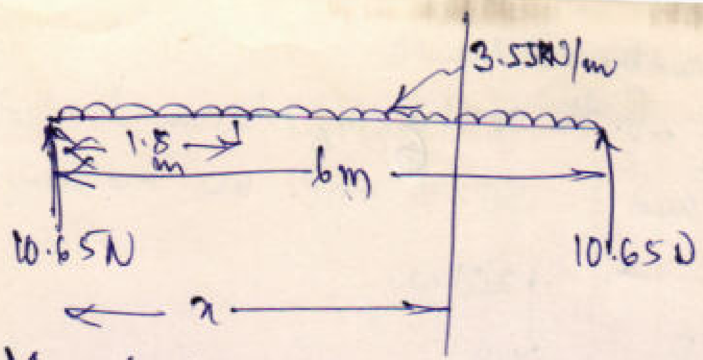
$E = \frac{200\times10^9}{10^6} = 200\times10^3\ N/mm^2$

$\frac{M}{I} = \frac{f}{y}$

$\frac{36W}{8}\times10^6 = \frac{72\times30\times10^6}{135}$

$W = \frac{72\times30\times10^6\times8}{36\times10^6\times135}$

$W = 3.55\ kN/m$



$M_x = +(10.65 \times x) - 3.55 \cdot \frac{x^2}{2}$

$N/m^2 \cdot m^4 \cdot m$

$N-m^3$

$EI \frac{d^2y}{dx^2} = 10.65 \cdot x - 3.55 \frac{x^2}{2}$

$EI \left(\frac{dy}{dx}\right) = 10.65 \frac{x^2}{2} - 3.55 \frac{x^3}{6} + C_1$

At $x = l/2 = 6$ $\frac{dy}{dx} = 0$

$0 = 10.65 \times \frac{3^2}{2} - 3.55 \times \frac{3^3}{6} + C_1$

$C_1 = -31.95$

$EI \left(\frac{dy}{dx}\right) = 10.65 \frac{x^2}{2} - 3.55 \frac{x^3}{6} - 31.95$

$EI(y) = 10.65 \frac{x^3}{6} - 3.55 \frac{x^4}{24} - 31.95 \cdot x + C_2$

At $x = 0$ $y = 0$ $C_2 = 0$

$EI(y) = 10.65 \cdot \frac{x^3}{6} - 3.55 \frac{x^4}{24} - 31.95 \cdot x$

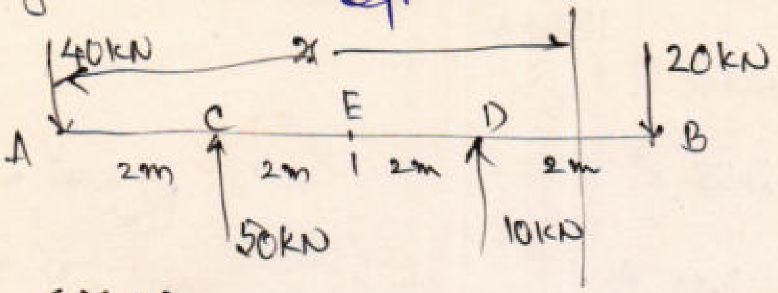
When $x = 1.8$

$EI(y) = 10.65 \cdot \frac{1.8^3}{6} - 3.55 \cdot \frac{1.8^4}{24} - 31.95 \times 1.8$

$EI(y) = 1.775 \times 1.8^3 - 0.1479 \times 1.8^4 - 31.95 \times 1.8 = -48.71 kN-m^3$

$= \frac{-48.71 \times 10^9 \times 10^3}{200 \times 10^3 \times 30 \times 10^6} = 8.11 \times 10^{3-3}$ mm $= \underline{8.11 mm}$

Calculate the deflections at E and the ends A and B for the overhanging beam loaded as shown in fig. Take $E = 200\,GPa$ and $I = 50\times10^{6}\ mm^4$. Use Macaulay's method also.



$\Sigma M_C = 0$

$-R_D \times 4 + 20\times 6 - 40\times 2 = 0$

$R_D = \frac{40kN}{4} = 10kN$

$R_C = 50kN.$

$M_x = -40.x + 50(x-2) + 10(x-6)$

$EI\frac{dy}{dx} = -\frac{40x^2}{2} + \frac{50(x-2)^2}{2} + \frac{10(x-6)^2}{2} + C_1$

$EI\,y = -\frac{40x^3}{6} + \frac{50(x-2)^3}{6} + \frac{10(x-6)^3}{6} + C_1x + C_2$

At $x=2,\ y=0$

$0 = \frac{-40(2)^3}{6} + C_1(2) + C_2$

$\boxed{2C_1 + C_2 = 53.33}$ — (1)

At $x=6,\ y=0$

$0 = -40.\frac{6^3}{6} + \frac{50(4)^3}{6} + 6C_1 + C_2$

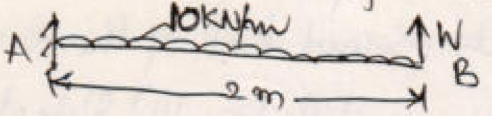
$\boxed{6C_1 + C_2 = +906.66}$ — (2)

Solving (1) and (2) $-4C_1 = 959.99$

$C_1 = -239.99 \approx -240$ $\boxed{C_1 = -240}$

A prismatic Cantilever beam Carries UDL of intensity 10KN/m over the entire span as shown in fig. What is the value of load 'W' in KN for zero deflection at the free end B (12)

Aug 2001



Soln:-

The deflection at B due to UDL : $y_B = \frac{-WL^4}{8EI}$

$\frac{kN/m \times m^4}{kN/m^2 \cdot m^4} = \frac{kN \times m^4}{kN m^3} = \frac{kN\, m^4}{kN\, m^3} = m$ $\quad = \frac{10 \times 2^4}{8 \times EI} = \frac{160}{8EI} = \frac{-20}{EI}$

(downward)

If the deflection at B is to be zero, then deflection due to point load 'W' acting upwards must be equal to $\frac{20}{EI}$.

The deflection at B due to point load acting upwards : $\frac{Wl^3}{3EI}$

$$= \frac{W \times 2^3}{3EI}$$

Then

$$\frac{8W}{3EI} = \frac{20}{EI}$$

$$W = \frac{3 \times 20}{8} \cdot \frac{EI}{EI}$$

$$= \frac{60}{8} = 7.5\,KN.$$

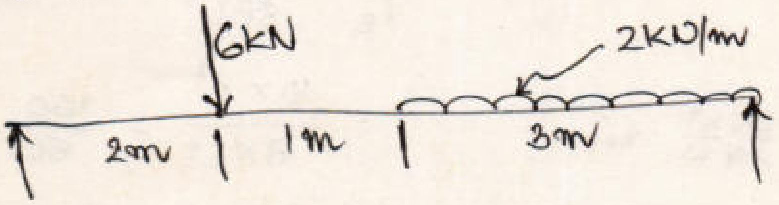
$$\boxed{W = 7.5\,KN}$$

A beam AB of span 6m span is simply supported at the ends and is loaded with a point load of 6KN at 2m from left support and uniformly distributed load of 2KN/m for the second half of the beam. Find i) Deflection at mid span ii) Maximum deflection iii) Slope at left support

Take $E = 20\,GPa$ and $I = 2 \times 10^7\ mm^4$.

June 2010

14 marks.



Columns & Struts

A Column is any structural member subjected to Compression. Columns are relatively long in proportion to their cross sectional area. The term Column is commonly referred to a vertical member whereas strut is used to denote an inclined member subjected to Compression. Compression members are also called by following names:

1) post. ⟶ Wooden Column
2) Stanchion ⟶ Steel Column
3) Pillar ⟶ Massive dimensioned Column [pillars of ancient temples]
4) pier ⟶ Intermediate supports of a bridge

Effective length:- The length of a Column invariably means the actual length between supports where as effective length is not necessarily actual length. It depends on nature of fixity at the ends. It is defined as that length of a Column that is subjected to buckling. It is defined as the distance between points of Contraflexure. The effective length of a Column hinged at both ends is equal to its actual length measured from Centre to Centre of the hinges. l_e = Effective length.

Short Column:

A Column is defined as Short Column if the ratio of effective length to least radius of gyration does not exceed 50

Slenderness ratio $= \frac{l_e}{k} \not> 50$.

Long Column:

A Column is defined as long Column if the ratio of effective length to least radius of gyration exceeds 50.

A Short Column fails by Crushing.
A long Column fails by buckling

In crushing, the material fails. In buckling geometrical section fails.
In Crushing, the full strength of material is used. In buckling, only a portion of material strength is used.
In crushing, the stresses will reach the ultimate stress value. In buckling, the stresses will be within proportionality limit.

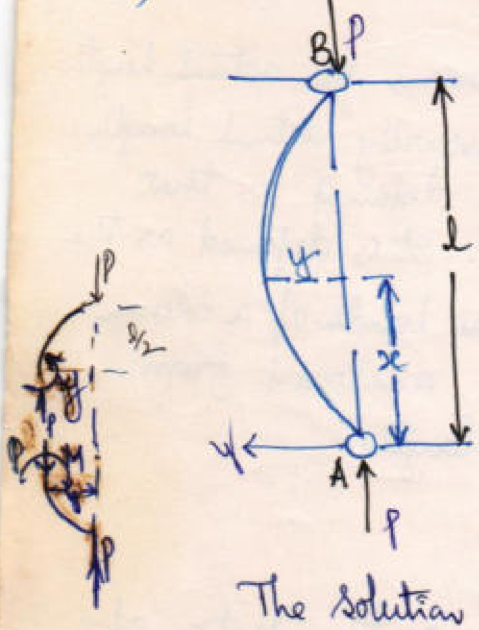
Euler's theory on Columns:

The following assumptions are made:

1) The material of the column is homogeneous and isotropic
2) The section of the column is uniform through out.
3) The column is initially straight and loaded axially.
4) The column fails by buckling alone
5) The self weight of the column is negligible.

The expressions for buckling load with various end conditions are as follows:

a) Both ends hinged (pinned) (March 2000) Expressions.



Consider the long column AB of length 'l'. It's both ends A and B are hinged. Due to axial compressive load 'P', let the deflection at distance 'x' from A be 'y'.

The bending moment at the section is given by

$M = -Py$ [-ve sign as x increases curvature decreases]

$EI\frac{d^2y}{dx^2} + Py = 0$ — 1 or $\frac{d^2y}{dx^2} + \frac{P}{EI}y = 0$

The solution to this differential equation is | Homogeneous linear differential equation of second order

$$y = C_1 \cos\left(x\sqrt{\frac{P}{EI}}\right) + C_2 \sin\left(x\sqrt{\frac{P}{EI}}\right)$$

where C_1 and C_2 are the constants of integration. They can be found using the boundary conditions.

Now at $x=0$, $y=0$ $\therefore C_1 = 0$

at $x=l$, $y=0$. ie $0 = 0 + C_2 \sin\left(l\sqrt{P/EI}\right)$

[Here $C_2 \neq 0$, because in that case $y=0$, which means there is no lateral deflection. Hence $\sin\left(l\sqrt{\frac{P}{EI}}\right) = 0$

ie. $l\sqrt{\frac{P}{EI}} = 0, \pi, 2\pi, 3\pi, \ldots$

Here the first value 0 can not be taken because it means $P=0$.

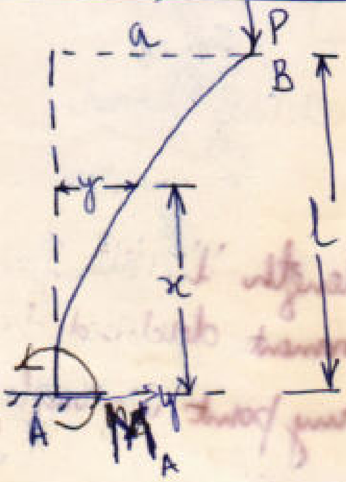
Considering the least practical value, $\left(l\sqrt{\frac{P}{EI}}\right) = \pi$

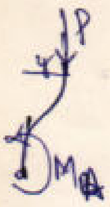
$$l^2 \frac{P}{EI} = \pi^2$$

$$\boxed{P = \frac{\pi^2 EI}{l^2}}$$

This load is called critical or buckling load. Pcr or Euler's crippling load

<u>One end fixed and other end free</u> Aug 99





In the fig, end A is fixed and the end B is free.
Let deflection at distance 'x' from A be y.
The bending moment at the section is given by

$$M = P(a-y) \quad \text{Where 'a' is max}^m \text{ displacement}$$

$$EI\frac{d^2y}{dx^2} + Py = Pa.$$

$$\frac{d^2y}{dx^2} + \frac{P}{EI}y = \frac{P}{EI}a.$$

The soln for this differential equation is

$$y = C_1 \cos\left(x\sqrt{\tfrac{P}{EI}}\right) + C_2 \sin\left(x\sqrt{\tfrac{P}{EI}}\right) + a$$

~~At x=0,~~ $\cancel{\frac{dy}{dx}=0}$

$$\frac{dy}{dx} = -C_1\sqrt{\tfrac{P}{EI}}\,\sin\left(x\sqrt{\tfrac{P}{EI}}\right) + C_2\sqrt{\tfrac{P}{EI}}\,\cos\left(x\sqrt{\tfrac{P}{EI}}\right)$$

Then, At $x=0$, $\frac{dy}{dx}=0$

$$0 = C_2\sqrt{\tfrac{P}{EI}}$$. Since $\frac{P}{EI}$ is not zero, $C_2 = 0$.

$$\therefore \quad y = +C_1 \cos\left(x\sqrt{\tfrac{P}{EI}}\right) + a$$

At $x=0$, $y=0$,

$$0 = C_1 + a$$

$$C_1 = -a$$

$$y = a - a\cos\left(x\sqrt{\tfrac{P}{EI}}\right)$$

At $x = l$, $y = a$

$$a = a - a\cos\left(l\sqrt{\tfrac{P}{EI}}\right)$$

$$a\cos\left(l\sqrt{\tfrac{P}{EI}}\right) = 0$$

Since 'a' cannot be zero. $\cos\left(l\sqrt{\tfrac{P}{EI}}\right) = 0$

i.e. $l\sqrt{\frac{P}{EI}} = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$

The least significant value is $\pi/2$. Then $l\sqrt{\frac{P}{EI}} = \frac{\pi}{2}$

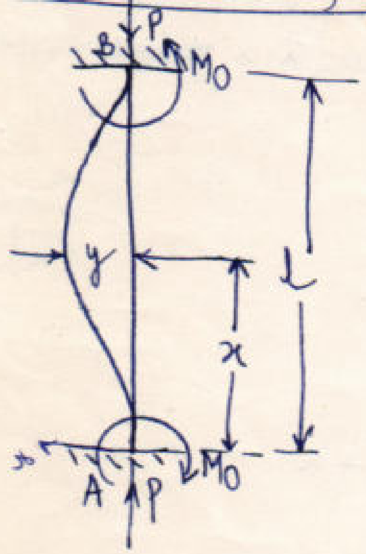
$$l^2 \frac{P}{EI} = \frac{\pi^2}{4}$$

$$\boxed{P = \frac{\pi^2 EI}{4l^2}}$$

$2x=$ $2dx$ $\cos 2x\,dx$ $2\frac{\cos u}{du}$ $2\sin u$ $2\sin 2x$

This is Euler Crippling load, Critical load, buckling load.

Both ends are fixed :



Consider a Column AB of length 'l' with fixed ends as shown in fig. Let end moment developed be M_0. Then bending moment at any point is given by

$M = M_0 - Py$ i.e. $EI\frac{d^2y}{dx^2} = M_0 - Py$

$$\frac{d^2y}{dx^2} + \frac{Py}{EI} = \frac{M_0}{EI}$$

The soln of the above differential eqn is

$$y = C_1\cos\left(x\sqrt{\tfrac{P}{EI}}\right) + C_2\sin\left(x\sqrt{\tfrac{P}{EI}}\right) + \frac{M_0}{P}$$

$$\therefore \frac{dy}{dx} = -C_1\sqrt{\tfrac{P}{EI}}\sin\left(x\sqrt{\tfrac{P}{EI}}\right) + C_2\sqrt{\tfrac{P}{EI}}\cos\left(x\sqrt{\tfrac{P}{EI}}\right)$$

From Condition $\frac{dy}{dx} = 0$ at $x=0$, We get

$$C_2\sqrt{\tfrac{P}{EI}} = 0$$

Since P Can not be zero, $C_2 = 0$.

At $x=0$, $y=0$

$$0 = C_1 + \frac{M_0}{P}$$

$$C_1 = -\frac{M_0}{P}$$

$$\therefore y = -\frac{M_0}{P}\cos\left(x\sqrt{\tfrac{P}{EI}}\right) + \frac{M_0}{P}.$$

From the Condition $x=l$, $y=0$,

$$0 = -\frac{M_0}{P}\cos\left(l\sqrt{\tfrac{P}{EI}}\right) + \frac{M_0}{P}$$

or $\cos\left(l\sqrt{P/EI}\right) = 1$

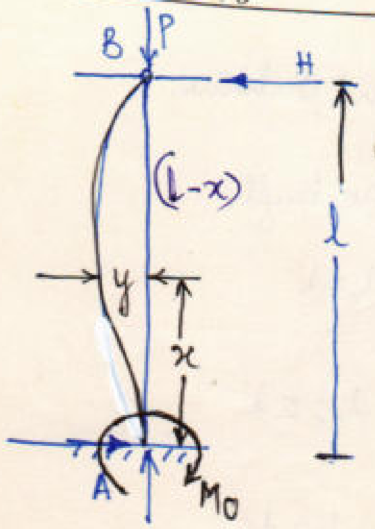
$\therefore l\sqrt{\frac{P}{EI}} = 0, 2\pi, 4\pi.$

Taking the least significant value,

$$l\sqrt{\tfrac{P}{EI}} = 2\pi$$

$$\boxed{P = \frac{4\pi^2 EI}{l^2}}$$

one end is fixed and the other end hinged : ③

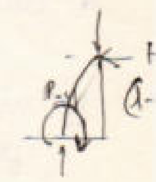


Consider a Column as shown in fig.

Let the fixed end moment at end A be M_0.

For equilibrium, horizontal force H develops such that $H = \frac{M_0}{l}$

Moment at any distance 'x' from A is given by



$$EI\frac{d^2y}{dx^2} = H(l-x) - Py$$

i.e. $$\frac{d^2y}{dx^2} + \frac{P}{EI}y = \frac{H(l-x)}{EI}$$

The soln for this differential eqn is

$$y = C_1\cos\left(x\sqrt{\frac{P}{EI}}\right) + C_2\sin\left(x\sqrt{\frac{P}{EI}}\right) + \frac{H(l-x)}{P}$$

$$\frac{dy}{dx} = -C_1\sqrt{\frac{P}{EI}}\sin\left(x\sqrt{\frac{P}{EI}}\right) + C_2\sqrt{\frac{P}{EI}}\cos\left(x\sqrt{P/EI}\right) - \frac{H}{P}$$

At $x=0$, $\frac{dy}{dx} = 0$

$$0 = C_2\sqrt{\frac{P}{EI}} - \frac{H}{P}$$

$$C_2 = \frac{H}{P}\sqrt{\frac{EI}{P}}$$

At $x=0$, $y = 0$

$$0 = C_1 + \frac{Hl}{P}$$

$$C_1 = -\frac{Hl}{P}$$

At $x = l$ $y = 0$.

$$0 = -\frac{Hl}{P}\cos\left(l\sqrt{P/EI}\right) + \frac{H}{P}\sqrt{\frac{EI}{P}}\sin\left(l\sqrt{P/EI}\right)$$

$$\tan\left(l\sqrt{\frac{P}{EI}}\right) = \frac{Hl}{P} \Big/ \frac{H}{P}\sqrt{\frac{EI}{P}}$$

$$\tan\left(l\sqrt{\frac{P}{EI}}\right) = l\sqrt{\frac{P}{EI}}$$

$\tan\phi = \phi$ Here ϕ: 4.4935 radians.

$l\sqrt{\frac{P}{EI}} = 4.4935$

$$P = 20.1915\frac{EI}{l^2} = 2.0458\frac{\pi^2 EI}{l^2} = \frac{2\pi^2 EI}{l^2}$$

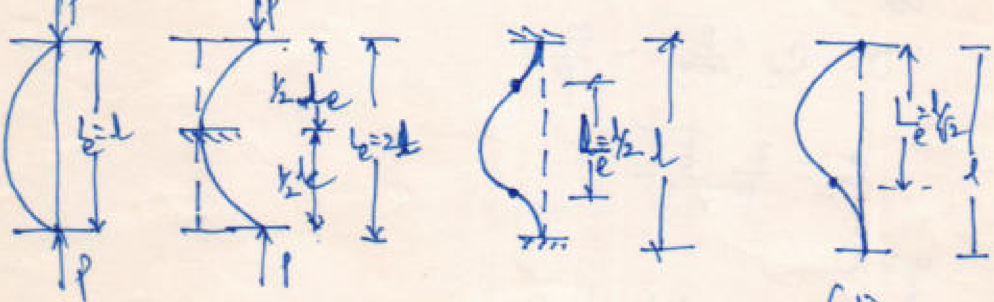
$$\boxed{P = \frac{2\pi^2 EI}{l^2}}$$

Effective length:

Table gives the expression for effective length for various end conditions.

End Conditions	Crippling load	Effective length.
a Both ends hinged	$\frac{\pi^2 EI}{l_e^2}$	$l_e = l$
b. one end fixed other end free	$\frac{\pi^2 EI}{l_e^2}$	$l_e = 2l$
c Both ends fixed	$\frac{\pi^2 EI}{l_e^2}$	$l_e = l/2$
d. one end fixed other end hinged	$\frac{\pi^2 EI}{l_e^2}$	$l_e = \frac{l}{\sqrt{2}} = 0.707l$

The effective length is the distance between inflexion points on the elastic curve or hinges.



(a) (b) (c) (d)

Limitation of Euler's theory:

Euler's theory assumes the failure is due to buckling only.

Euler's load for crippling is $P_{cr} = \frac{\pi^2 EI}{l^2}$

$$= \frac{\pi^2 E A k^2}{l^2}$$

$$\frac{P_{cr}}{A} = \frac{\pi^2 E}{(l/k)^2}$$

ie. $$f_{cr} = \frac{\pi^2 E}{(l/k)^2}$$ where f_{cr} = Critical stress.

The term $\frac{L_e}{K}$ is called slenderness ratio. As slenderness ratio increases, critical stress decreases. As $\frac{L}{K}$ approaches zero, the critical stress tends to infinity. But this can not happen. Before this stage, the material gets crushed.

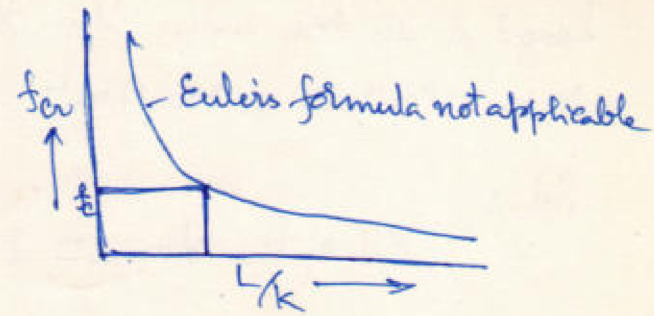
Hence Euler's theory is valid upto a limiting value of crippling stress which is the crushing stress.

i.e. $f_{cr} = f_c$

Then

$$\frac{\pi^2 E}{(L/k)^2} = f_c$$

$$\left(\frac{L}{k}\right)^2_{lim} = \frac{\pi^2 E}{f_c}$$



This slenderness ratio is the limiting value. Below this value of (L/k) Euler's formula does not give load carrying capacity.

Rankine's formula:-

Euler's formula is applicable for long and axially loaded columns. Many empirical formulae have been proposed to predict the critical loads on other types such as short, intermediate columns. Among them, Rankine's formula is widely accepted and gives fairly good results for columns of all slenderness ratios.

According to Rankine, the actual critical load P_{cr} on the column is given by

$$\frac{1}{P_{cr}} = \frac{1}{P_c} + \frac{1}{P_E}$$

Where P_{cr} = Actual crippling load
P_c = Crushing load
P_E = Euler's buckling load

$$\therefore \frac{1}{P_{cr}} = \frac{P_c + P_E}{P_c P_E}$$

$$P_{cr} = \frac{P_c P_E}{P_c + P_E} = \frac{P_c}{1 + \frac{P_c}{P_E}}$$

Now $P_c = f_c A$ and $P_E = \frac{\pi^2 EI}{l^2}$

$$P_{cr} = \frac{f_c A}{1 + f_c A \times \left(\frac{l^2}{\pi^2 EI}\right)}$$

$$= \frac{f_c A}{1 + f_c A\left(\frac{l^2}{\pi^2 E A k^2}\right)}$$

$$= \frac{f_c A}{1 + \left(\frac{f_c A}{\pi^2 E A}(l/k)^2\right)}$$

$$P_{cr} = \frac{f_c A}{1 + \frac{f_c}{\pi^2 E}(l/k)^2}$$

$$\boxed{P_{cr} = \frac{f_c A}{1 + a\left(\frac{l}{k}\right)^2}}$$

Where $a = \frac{f_c}{\pi^2 E}$ is called Rankine's Constant.

The eqn is called Rankine's Equation.

Find the Euler's Crippling load for a hollow Cylindrical steel Column 40 mm external dia and 4 mm thick. Consider the length of Column 2.3 m and hinged at its both ends. Also determine the Crippling load by Rankine's formula using Constants as 335 N/mm² and 1/7500. Take E = 205 KN/mm².

Aug 99

Soln:

MI of the Column = $I = \frac{\pi}{64}(40^4 - 32^4)$

$= 7.42 \times 10^4 \ mm^4$

Euler's buckling load $= P_{cr} = \frac{\pi^2 EI}{l_e^2} = \frac{\pi^2 \times 205 \times 10^3 \times 7.42 \times 10^4}{(2.3 \times 10^3)^2}$

$= 28379.31 \ N = 28.4 \ KN$

Rankine's buckling load:

$A = \frac{\pi}{4}\left[40^2 - 32^2\right] = 452.4 \ mm^2$

Radius of gyration $= K = \sqrt{\frac{I}{A}} = \sqrt{\frac{7.42 \times 10^4}{452.4}} = 12.8 mm.$

$\frac{f_c A}{1 + a\left[\frac{L}{K}\right]^2}$ $\quad P_{Rankine} = \frac{335 \times 452.4}{1 + \frac{1}{7500}\left[\frac{2.3 \times 10^3}{12.8}\right]^2}$

$= 28568.07 N = 28.6 \ KN.$

The external and internal diameters of a hollow Cast iron Column are 6 cms and 4 cms respectively. If the length of this Column is 3 m and both of ends are fixed, determine the Crippling load using Rankine's formula. Take value of $P_c = 550 \ N/mm^2$ and $a = \frac{1}{1600}$ in Rankine's formula.

Aug/Sept 99

Soln:

MI of the Column $= I = \frac{\pi (60^4 - 40^4)}{64} = 510508.79 \ mm^4$

Euler's buckling load $= P_{cr} = \frac{4\pi^2 EI}{l^2} = \frac{4 \times \pi^2 \times E \times 510508.79}{(3000)^2} = 2.24E$ ~~KN~~

Rankine's formula:

Area $= \frac{\pi}{4}(6^2 - 4^2) = 15.70 \ cm^2 = 1570.79 \ mm^2$

$k = \sqrt{\frac{I}{A}} = \sqrt{\frac{510508.79}{1570.79}} = 18.02 \ mm.$

$P_{Rankine} = \frac{550 \times 1570.79}{1 + \frac{1}{1600}\left[\frac{1.5 \times 10^3}{18.02}\right]^2}$

$l_e = \frac{1}{2} l = \frac{1}{2} \times 3000 = 1500$

$= 162069.7 N$ ~~= 19455.08 N~~

$= 162.069 \ KN.$

A 2m long column of hollow circular section has outer dia 180mm and is subjected to a load of 212.5 kN and designed for the yield stress of 340 N/mm². The column is fixed at one end and free at other end. Determine the inner diameter. Take E = 210 GPa and FOS = 4. Mar 2001

Soln:

$l = 2m = 2000mm.$ $l_e = 2l$

d_1 = External dia = 180mm

d_2 = Int. dia = d_2

Working load = 212.5 kN.

Critical load = 4 × 212.5 kN

$P_{cr} = 850 kN.$

$$I = \frac{\pi(180^4 - d_1^4)}{64}$$

$$A = \frac{\pi}{4}(180^2 - d_1^2)$$

$$k^2 = \frac{I}{A} = \frac{\frac{\pi}{64}(180^4 - d_1^4)}{\frac{\pi}{4}(180^2 - d_1^2)} = \frac{1}{16}(180^2 + d_1^2)$$

~~Calculation~~.

$$a = \frac{f_c}{\pi^2 E} = \frac{340}{\pi^2 \times 210 \times 10^3} = 1.64 \times 10^{-4}$$

Using Rankine's formula,

$$P_{cr} = \frac{f_c A}{1 + a\left(\frac{l_e}{k}\right)^2} = \frac{340 \times \frac{\pi}{4}(180^2 - d_1^2)}{1 + (1.64\times10^{-4})\frac{(2\times2000)^2 \times 16}{(180^2 + d_1^2)}} = 850\times10^3$$

$$850\times10^3 = \frac{340\times\frac{\pi}{4}(180^2 - d_1^2)(180^2 + d_1^2)}{(180^2 + d_1^2) + (1.64\times10^{-4})(4000^2)\times16}$$

$$850\times10^3(180^2 + d_1^2) + (1.64\times10^{-4})(4000^2)\times16 \times 850\times10^3 = 340\times\frac{\pi}{4}\times(180^4 - d_1^4)$$

$$850\times10^3\times180^2 + 850\times10^3 d_1^2 + 3.56\times10^{10} - 2.80\times10^{11} = -267 d_1^4$$

$$\cancel{850267 d_1^2 + 5.845\times10^{10} = 0}$$

$$\cancel{d_1^2 = \frac{-5.845\times10^{10}}{850267}}$$

$$267 d_1^4 + 850\times10^3 d_1^2 - 2.168\times10^{11} = 0$$

$$d_1^2 = \frac{-850\times10^3 \pm \sqrt{(850\times10^3) - 4(267)(-2.168\times10^{11})}}{2\times267}$$

$$d_1^2 = 26903.56 = 164 \text{ mm}$$

A hollow cylindrical Column is fixed at both ends. The length of Column is 4.2m and Carries an axial load of 230 kN. Design the column by Rankine's formula. Adopt a FOS of 5. The internal dia may be taken as 0.82 times the external dia. Take $F_c = 550\,N/mm^2$ and $a = \frac{1}{1600}$ Feb-2002

Soln:

$l = 4.2m$

$l_e = \frac{L}{2} = \frac{4200}{2} = 2100\,mm.$

Critical load. P_{cr} = FOS x Working load

$= 5 \times 230$

$= 1150\,kN = 1150 \times 10^3\,N$

d_1 = Ext. dia $\quad d_2 = 0.82 d_1$

d_2 = Int. dia

$A = \frac{\pi}{4}(d_1^2 - d_2^2)$

$= \frac{\pi}{4}(d_1^2 - (0.82)^2 d_1^2)$

$= \frac{\pi}{4} d_1 (1 - 0.82^2) = 0.2573 d_1^2$

$I = \frac{\pi}{64}(d_1^4 - d_2^4)$

$= \frac{\pi}{64}(d_1^4 - (0.82)^4 d_1^4)$

$= \frac{\pi}{64} d_1^4 (1 - 0.82^4)$

$= 0.0269\, d_1^4$

$$k^2 = \frac{I}{A} = \frac{0.0269 d_1^4}{0.2573 d_1^2} = 0.1045 d_1^2$$

$$P_{cr} = \frac{f_c A}{1 + a\left(\frac{l_e}{k}\right)^2} = \frac{550 \times 0.2573 d_1^2}{1 + \frac{1}{1600}\frac{(2100)^2}{0.1045 d_1^2}} = 1150 \times 10^3$$

$$1150 \times 10^3 = \frac{550 \times 0.2573 d_1^2}{\frac{167.2 d_1^2 + 441 \times 10^4}{167.2 d_1^2}}$$

$$1150 \times 10^3 (167.2 d_1^2 + 441 \times 10^4) = 550 \times 0.2573 d_1^2 \times 167.2 d_1^2$$

$$10^8 \times 1.923 d_1^2 + 5.07 \times 10^{12} = 23661.308 d_1^4$$

$$23661.30 d_1^4 - 10^8 \times 1.923 d_1^2 - 5.07 \times 10^{12} = 0$$

Solving for d_1^2

$$d_1^2 = \frac{+1.923 \times 10^8 \pm \sqrt{(-1.923 \times 10^8)^2 - 4(23661.30)(-5.07 \times 10^{12})}}{2(23661.30)}$$

$$d_1^2 = 19255.23$$

Ext. dia $\boxed{d_1 = 138.76\,mm}$

Internal dia $= 0.82 \times 138.76 = 113.78\,mm$

16. Find the shortest length 'L' for a pin ended steel column having a cross section of 70mm × 110mm for which Euler's formula applies. Take $E = 2.1\times10^5$ N/mm² and critical proportional limit is 250 N/mm². Feb 2002.

Soln: Critical length = Actual length.

$P_{cr} = f_{cr} \times A$

$= 250\times70\times110$

$= 1925000$ N.

$P_{cr} = \dfrac{\pi^2 E I}{l^2}$

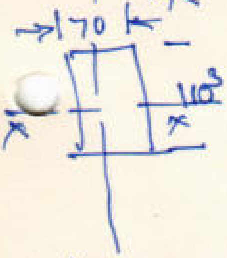
$1925000 = \dfrac{\pi^2\times2.1\times10^5\times3144166.6}{l^2}$

$\cancel{1925000}\, l^2 = \cancel{33852707}$

$l = \cancel{5818.30}$ mm 1839.91

$l = \cancel{5.818}$ mts. 1.84 mts

$I_{xx} = \dfrac{70\times110^3}{12} = 7764166.6$ mm⁴



$I_{yy} = \dfrac{110\times70^3}{12}$

$= 3144166.6$ mm⁴

* Imp. Note: I_{min} is to be chosen

A 28mm diameter column fixed at its ends is acted upon by an axial load. Determine the length of the column. Take compressive stress as 320 MPa and E = 200 GPa. March 2001

'l' can be found out using Euler's and Rankine's formula.

Soln: $f_{cr} = 320$ MPa $A = \dfrac{\pi\times28^2}{4} = 615.75$ mm² $I = \dfrac{\pi\times28^4}{64}$

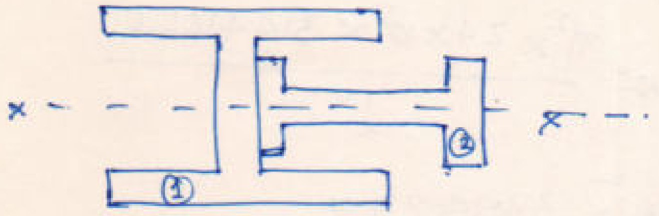
$P_{cr} = 320\times615.75 = 197040.67$ N. $= 30171.85$ mm⁴

$$P_{cr} = \frac{4\pi^2 EI}{l^2} = \frac{4\times\pi^2\times200\times10^3\times30171.85}{l^2} = 197040.67$$

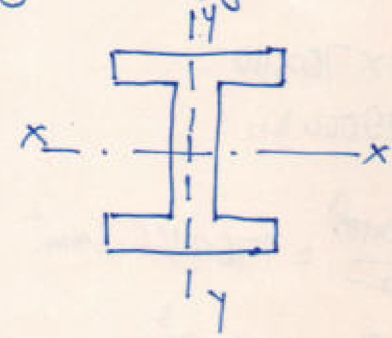
$$l^2 = 1209024.5 \quad l = 1099.55 \text{ mm.}$$

A Column is fabricated using two identical I-joists and its cross section is shown in fig 7(a). Its bottom end is fixed in position and direction whereas its top end is fixed in position but not in direction. The actual length of the Column is 6m. What maximum axial compressive load can be allowed on the Column if the load factor is 3.5 as per Euler's equation. The second moment of area of each of the I sections used are given in fig. $E = 2\times10^5\ N/mm^2$.

Aug 2001



Cross-sections of Columns.



$I_{xx} = 0.6\times10^8\ mm^4$

$I_{yy} = 0.1\times10^8\ mm^4$

Soln:-

End Conditions: One end (Bottom) : Fixed end.

other end (Top) : Hinged end.

$$P_{cr} = \frac{2\pi^2 EI}{l^2}$$

I = Least I is to be choosen.

$$I_{Combined} = I_{xx} \text{ of } 1 + I_{yy} \text{ of } 2. = 0.6\times10^8 + 0.1\times10^8$$
$$= 0.7\times10^8\ mm^4$$

$$P_{cr} = \frac{2\times\pi^2\times2\times10^5\times0.7\times10^8}{(6000)^2} = 7676.337\times10^3\ N.$$

$$\text{Safe load} = \frac{7676.337\ kN}{3.5} = 2193.23\ kN.$$

Determine the ratio of buckling strength of two columns of circular section one hollow and other solid, when both are made up of the same material, have the same length, cross sectional area and end conditions. The internal dia of hollow Column is half of its external diameter. March 2000

Soln:

$E_H = E_S$

$l_H = l_S$

$A_H = A_S$

d = dia of Solid Shaft

d_1 = Ext. dia d_2 = Int dia $d_2 = 0.5 d_1$

From Euler's formula

$$(P_{cr})_H = \frac{\pi^2 EI}{l^2}$$

$$I_{Solid} = \frac{\pi d^4}{64}$$

$$I_{Hollow} = \frac{\pi}{64}(d_1^4 - d_2^4)$$
$$= \frac{\pi\times d_1^4(1-0.5^4)}{64}$$
$$= \frac{\pi\times0.9375 d_1^4}{64}$$

Now $A_S = A_H$

$$\frac{\pi}{4} d^2 = \frac{\pi}{4}(d_1^2 - (0.5d_1)^2)$$

$$= \frac{\pi}{4} d_1^2 (1 - 0.5^2) = \frac{\pi \times 0.75 d_1^2}{4}$$

$$d^2 = 0.75 d_1^2$$

$$\boxed{d = 0.866 d_1}$$

$$\frac{(P_{cr})_{Hollow}}{(P_{cr})_{Solid}} = \frac{\dfrac{\cancel{\pi^2 \times E} \times (\cancel{\frac{\pi}{64}} 0.9375 d_1^4)}{\cancel{l^2}}}{\dfrac{\cancel{\pi^2 \times E} \times (\cancel{\frac{\pi}{64}} d^4)}{\cancel{l^2}}}$$

$$= \frac{0.9375 d_1^4}{(0.866 d_1)^4} = \frac{0.9375}{0.562} \frac{d_1^4}{d_1^4}$$

$$= 1.666$$

$$\boxed{\frac{(P_{cr})_{Hollow}}{(P_{cr})_{Solid}} = 1.666}$$

A solid round bar 4m long and 6cms in dia is used as a Column with both ends hinged. Determine the crippling load if $E = 2 \times 10^5$ N/mm²

$$I = \frac{\pi}{64} \times 60^4 = 636172.5 \text{mm}^4$$

$$P_{cr} = \frac{\pi^2 EI}{l^2} = \frac{\pi^2 \times 2 \times 10^5 \times 636172.5}{(4000)^2}$$

$$= 78484.6 \text{ N.}$$

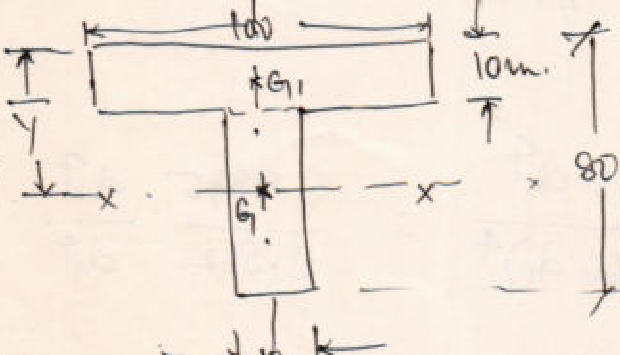
What is meant by the term critical load for a column.

Critical load:- It is that load at which an ideal column undergoes a small lateral deflection with a small lateral load and the deflection does not disappear when the small lateral load is removed. If the critical load is crossed, the column buckles and fails.

Determine the buckling load for a strut of Tee section, the flange width being 100mm, overall depth 80mm and both flange and stem (web) 10mm thick. The strut is 3mts long and is hinged at both ends. Take $E = 200\ GN/m^2$.

$$E = \frac{200 \times 10^9}{1000 \times 1000} = 200 \times 10^3\ N/mm^2$$

Note: To get P_{cr}, least MI is to be found. Hence I_x and I_y is found and least value of them is considered



To find $\bar{Y}$ from top

$$\bar{Y} = \frac{100 \times 10 \times 5 + 10 \times 70 \times \overset{(35+10)}{45}}{(100 \times 10) + (70 \times 10)}$$

$= 21.47mm$

$$I_y = \frac{10^3 \times 10}{12} + \frac{10^3 \times 70}{12}$$

$= 839166.67 mm^4$

$$I_x = \frac{100 \times 10^3}{12} + (100 \times 10)(21.47 - 5)^2 + \frac{70^3 \times 10}{12} + 10 \times 70 (45 - 21.47)^2$$

$= 952990.2 mm^4$

Least $MI = I_y = 839166.67\ mm^4$

To calculate P_{cr}:

Since both ends are hinged

$$P_{cr} = \frac{\pi^2 EI}{l^2} = \frac{\pi^2 \times 200 \times 1000 \times 839166.67}{(3000)^2}$$

$= 184049.85 N$

$= 184.05 kN$

A hollow column of CI whose outside diameter is 200mm, has thickness of 20mm. It is 4.5m long and is fixed at both ends. Calculate the safe load by Rankine's formula using a factor of safety of 4. Calculate slenderness ratio and the ratio of Euler's and Rankine's critical loads. Take $\sigma_c = 550\,N/mm^2$, $\alpha = \frac{1}{1600}$ (Rankine's constant) and $E = 8\times10^4\,N/mm^2$

July 2008 12 marks

Soln:

External dia = 200mm

Internal dia = 160mm.

$$I = \frac{\pi(200^4 - 160^4)}{64} = 46.36\times10^6\ mm^4$$

Both ends are fixed:

$$P_E = \frac{4\pi^2 EI}{l^2} = \frac{4\times\pi^2\times8\times10^4\times46.36\times10^6}{4.5^2\times1000^2}$$

$$\boxed{P_E = 7.23\times10^6\ N}$$

To calculate Rankine's critical load:

$$A = \frac{\pi}{4}(200^2 - 160^2) = 11309.734\ mm^2$$

$$k^2 = \frac{I}{A}$$

$$k = \sqrt{\frac{46.36\times10^6}{11309.734}} = 64.024\ mm$$

$$\text{Slenderness ratio} = \frac{l_e}{k} = \frac{2250}{64.024} = 35.14$$

$l_e = \frac{1}{2}l$ for both ends fixed

$$P_R = \frac{f_c A}{1+\alpha\left(\frac{l_e}{k}\right)^2}$$

Rankine's critical load

$$= \frac{550\times11309.73}{1+\frac{1}{1600}\left(\frac{2250}{64.024}\right)^2}$$

$$\boxed{P_R = 3.51\times10^6\ N}$$

$$\text{Ratio} = \frac{P_E}{P_{CR}} = \frac{7.23\times10^6}{3.51\times10^6} = 2.05$$

$$\boxed{\text{Safe load} = \frac{3.51\times10^6}{4} = 0.8775\times10^6\ N}$$

Find the shortest lgth 'L' for a pin ended steel column having a cross sectional area of 60 mm × 100 mm for which Euler's formula applies. Take E = 200 GPa and proportionality limit of the material = 250 MPa.

Euler's critical load $P_{cr} = \dfrac{\pi^2 EI}{l^2}$

Effective lgth = Actual lgth

$P_{cr} = 250 \times 60 \times 100$

$= 1.5 \times 10^6$ N

I_{min} is to be chosen

$I_{min} = \dfrac{100 \times 60^3}{12} = 1.8 \times 10^6$

$1.5 \times 10^6 = \dfrac{\pi^2 \times 200 \times 10^3 \times 1.8 \times 10^6}{l^2}$

$l^2 = 2.3687 \times 10^6$

$\boxed{l = 1539.05 \text{ mm}}$

$\boxed{l = 1.539 \text{ mts}}$

A hollow cylindrical column with both ends hinged is 6m long and has an outer dia of 120mm and inner dia of 80mm. Compare the crippling load by Euler's and Rankine's formula. What is the length of the column if both the crippling loads are equal? $E = 8\times10^4\ N/mm^2$ $\sigma_y = 550\ N/mm^2$ $\alpha = \frac{1}{1600}$ Jan2016 (10m)

Soln:

$$P_E = \frac{\pi^2 EI}{l^2}$$

$$P_E = \frac{\pi^2 \times 8\times10^4 \times 8.168\times10^6}{6000^2}$$

$$I = \frac{\pi}{64}(120^4 - 80^4) = 8.168\times10^6\ mm^4$$

$$\boxed{P_E = 179.14\times10^3\ N}$$

$$A = \frac{\pi}{4}(120^2 - 80^2) = 6283.18\ mm^4$$

$$P_R = \frac{\sigma_y \cdot A}{1+\alpha\left(\frac{l_e}{k}\right)^2} = \frac{550\times6283.18}{1+\frac{1}{1600}\left(\frac{6000}{36.05}\right)^2}$$

$$k = \sqrt{\frac{I}{A}} = 36.05$$

$$\boxed{P_R = 188.70\times10^3\ N}$$

$$\boxed{\frac{P_E}{P_R} = 0.9493}$$

If the Euler's load & Rankine's loads are equal

$$\frac{\pi^2 EI}{l^2} = \frac{\sigma_y \cdot A}{1+\frac{1}{1600}\left(\frac{l_e}{k}\right)^2}$$

$$\frac{1+\left(\frac{l_e}{k}\right)^2\frac{1}{1600}}{l^2} = \frac{\sigma_y \cdot A}{\pi^2 EI}$$

$$1 + \frac{l_e^2}{1600\times36.05^2} = 5.358\times10^{-7}\, l^2$$

Since both the ends are hinged $l_e = l$

$$2.07\times10^6 + l_e^2 = 1.114\, l_e^2$$

$$0.114\, l^2 = 2.07\times10^6$$

$$\boxed{l = 4261.20\ mm}$$ 4.21m

Compare the crippling loads given by Euler's & Rankine's formula for a Column of circular section 2.3 m long and of 30 mm diameter. The Column is hinged at both ends. Take $\sigma_{yield} = 335\ N/mm^2$ and $\alpha = \frac{1}{7500}$ and $E = 2\times10^5$ MPa. For what ratio of $\left(\frac{l}{k}\right)$, the Euler's formula cease to apply for this Column?

July 16 (12m)

Soln

$$P_E = \frac{\pi^2 EI}{l^2} = \frac{\pi^2 \times 2\times10^5 \times 39.76\times10^3}{(2300)^2}$$

$I = \frac{\pi d^4}{64}$

$= 39.76\times10^3\ mm^4$

$$\boxed{P_E = 14.83\times10^3\ N}$$

$$P_R = \frac{f_c \cdot A}{1+\alpha\left(\frac{l_e}{k}\right)^2}$$

$l_e = l = 2300$

$f_c = \sigma_y$

$k = \sqrt{\frac{I}{A}}$

$K = 7.50$ mm

$$= \frac{335 \times 706.85}{1+\frac{1}{7500}\left(\frac{2300}{7.5}\right)^2}$$

$A = \frac{\pi d^2}{4}$

$= 706.85\ mm^2$

$$\boxed{P_R = 17.48\times10^3\ N}$$

$$\boxed{\frac{P_E}{P_R} = 0.8479}$$

then

$$\frac{\pi^2 EI}{l^2} = P_E = \sigma_y \cdot A = 335\times A$$

$$\frac{\pi^2 E \cdot Ak^2}{l^2} = 335\times A$$

$$\frac{\pi^2 E}{\left(\frac{l_e}{k}\right)^2} = 335$$

$$\left(\frac{l_e}{k}\right)^2 = \frac{\pi^2 E}{335}$$

$$\frac{l_e}{k} = \sqrt{5892.30}$$

$$\boxed{\frac{l_e}{k} = 76.76}$$

For a ratio of $\frac{l_e}{k} = 76.76$, Euler's formula gets ceased.

A hollow cylindrical cast iron column is 4 m long with both ends fixed. (firmly built-in). Design the column to carry an axial load of 250 kN use Rankine's formula and factor of safety = 5. The internal diameter may be taken as 0.80 times the external diameter. Take $f_c = 550\,N/mm^2$ and $\alpha = \frac{1}{1600}$

June-July 2018 (08m)

Soln:

$l = 4000\,mm$

d_1 = Ext. dia

d_2 = Internal dia $= 0.8d_1$

Area of cross section $= \frac{\pi}{4}\left(d_1^2 - (0.8d_1)^2\right)$

$= \frac{\pi}{4} \cdot d_1^2 (1 - 0.8^2)$

$$\boxed{A = 0.282\, d_1^2}$$

Crippling load = Safe load × Factor of safety

$= 250 \times 5 = 1250\,kN = 1250 \times 10^3\,N$

$I = \frac{\pi}{64}\left(d_1^4 - (0.8d_1)^4\right) = \frac{\pi}{64} d_1^4 (1 - 0.8^4)$

$$\boxed{I = 0.0289\, d_1^4}$$

$k = \sqrt{\frac{I}{A}} = \sqrt{\frac{0.0289\, d_1^4}{0.282\, d_1^2}} = 0.3193\, d_1$

$k^2 = 0.102\, d_1^2$

$P_R = \frac{f_c \cdot A}{1 + \alpha\left(\frac{l_e}{k}\right)^2}$

$l_e = \frac{l}{2} = \frac{4000}{2} = 2000$ fixed end.

$1250 \times 10^3 = \frac{550 \times 0.282\, d_1^2}{1 + \frac{1}{1600} \cdot \left(\frac{2000}{0.3193 d_1}\right)^2}$

$= \frac{550 \times 0.282\, d_1^2}{1 + \frac{24.52 \times 10^3}{d_1^2}}$

$\frac{550 \times 0.282\, d_1^2}{1 + \frac{24.52 \times 10^3}{d_1^2}}$

$\frac{550 \times 0.282\, d_1^2}{\frac{d_1^2 + 24.52 \times 10^3}{d_1^2}}$

$1250 \times 10^3 = \frac{550 \times 0.282\, d_1^2 \cdot d_1^2}{d_1^2 + 24.52 \times 10^3}$

$1250 \times 10^3 (d_1^2 + 24.52 \times 10^3) - 550 \times 0.282\, d_1^4 = 0$

$-155.1\, d_1^4 + 1250 \times 10^3 d_1^2 + 3.065 \times 10^{10} = 0$

$d_1^4 - 8.059 \times 10^3 d_1^2 - 197.61 \times 10^6 = 0$

$\frac{-(-8.059 \times 10^3) \pm \sqrt{(8.059 \times 10^3)^2 + 4(1) \times 197.6 \times 10^6}}{2(1)}$

$\frac{+(8.059 \times 10^3) \pm 29.24 \times 10^3}{2}$

$d_1^2 = 18649.50$

$$\boxed{d_1 = 136.56\,mm}$$

$$\boxed{d_2 = 0.8 d_1 = 109.25\,mm}$$

CBCS SCHEME

USN 1 S V 1 9 C V 4 1 7

18CV32

Third Semester B.E. Degree Examination, Aug./Sept.2020
Strength of Materials

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. 2. Any revealing of identification, appeal to evaluator and /or equations written eg. 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Sketch a typical stress-strain curve for a ductile material and explain briefly the salient features of the curve. **(05 Marks)**

b. Derive an expression for the deformation of a rectangular tapering bar of uniform thickness. **(05 Marks)**

c. Determine the value of P that will not except a maximum deformation of 2mm or a stress of 120 MPa in steel, 80 MPa in Aluminium and 115 MPa in bronze (Fig.Q1(c)). Given the following data:

$A_b = 600\ mm^2,\ E_b = 0.84\times10^5\ N/mm^2$
$A_a = 800\ mm^2,\ E_a = 0.7\times10^5\ N/mm^2$
$A_s = 400\ mm^2,\ E_s = 2.1\times10^5\ N/mm^2$

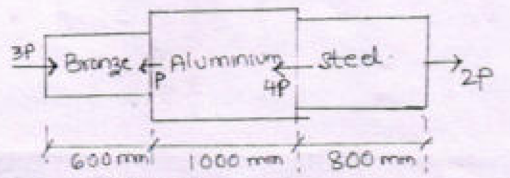


Fig.Q1(c) **(10 Marks)**

OR

2 a. Derive the relationship between Young's modulus and bulk modulus. **(05 Marks)**

b. A load of 270 kN is acting on a RCC column of size 200mm × 200mm. The column is reinforced with 10 bars of 12mm diameter each. Determine the stress in steel and concrete. $E_s = 16.5\ E_c$. **(05 Marks)**

c. A bar of brass 25mm diameter is enclosed in a steel tube of 50mm external diameter and 25mm internal diameter. The bar and tube are both initially 1m long and rigidly fastened at both the ends. Find the stresses in the two materials when the temperature rises from 10°C to 90°C.

If the composite bar is then subjected to an axial tensile load of 60 kN, find the resulting stresses given that : $E_s = 200\times10^3$ MPa, $E_b = 100\times10^3$ MPa, $\alpha_s = 11.6\times10^{-6}/°C$, $\alpha_b = 18.7\times10^{-6}/°C$. **(10 Marks)**

Module-2

3 a. Explain the maximum shear stress theory. **(05 Marks)**

b. Explain the procedure for determining stresses in a general two dimensional stress system using Mohr's circle. **(05 Marks)**

c. At a point in a strained material, the state of stresses is as shown in Fig.Q3(c), Determine the principal stresses, maximum shear stress and sketch the orientation of the principal planes.

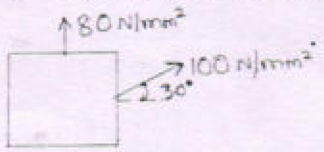


Fig.Q3(c) **(10 Marks)**

OR

4 a. In a thin cylinder, show that he hoop stress is twice the longitudinal stress. **(08 Marks)**

b. The maximum stress permitted in a thick cylinder of internal diameter 100mm and external diameter 150mm is 16 N/mm^2. If the internal pressure is 12 N/mm^2, what external pressure can be applied? Plot curves showing the variation of Hoop stress and radial stress through the material. **(12 Marks)**

Module-3

5 a. Define the terms:
(i) Bending Moment (ii) Point of Inflexion. **(04 Marks)**

b. Draw SFD and BMD for the cantilever beam shown in Fig.Q5(b).

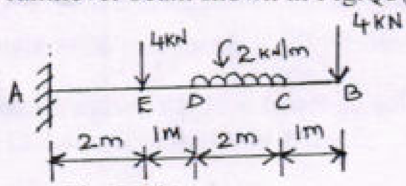


Fig.Q5(b) **(06 Marks)**

c. Draw SFD and BMD for a simply supported beam carrying two point loads of 12 kN at 1/3rd span from either supports in addition to a UDL of 10 kN/m throughout span of beam is 6m. **(10 Marks)**

OR

6 a. Establish the relationship between shear force, bending moment and load intensity. **(06 Marks)**

b. Draw SFD and BMD for the beam shown in Fig.Q6(b). Locate maximum shear force maximum bending moment and poiñt of contraflexure.

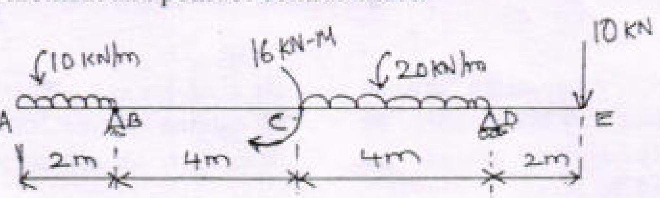


Fig.Q6(b) **(14 Marks)**

Module-4

7 a. Derive the simple bending equation in the form $\frac{M}{I} = \frac{f}{y} = \frac{E}{R}$ with usual notations. **(08 Marks)**

b. A beam of I section consists of 180mm × 15mm flanges and a web of 280mm × 15mm. It is subjected to a bending moment of 120 kN-m and a shear force of 60 kN. Sketch the bending stress distribution and shear stress distribution along the depth of the section. **(12 Marks)**

OR

8 a. Derive the torsion equation for a circular shaft subjected to pure torsion. **(10 Marks)**

b. A solid shaft of 60mm diameter is to be replaced by a hollow shaft of same length. The outer diameter of hollow shaft is same as that of solid shaft. If the angle of twist per unit torsional moment is the same in both cases, determine the inner diameter of hollow shaft. Take the modulus of rigidity of hollow shaft to be three times that of solid shaft. **(10 Marks)**

Module-5

9 a. Derive an expression for the slope and deflection of a simply supported beam carrying a central concentrated load. **(08 Marks)**

b. A simply supported beam of constant cross section is 10m long. It is loaded with two point loads of 100 kN and 80 kN at points 2m and 6m from the left end respectively. Calculate the deflection under each load the maximum deflection. Take E = 200 GPa and $I = 18\times10^{8}$ mm^4. **(12 Marks)**

OR

10 a. Distinguish between long and short columns. **(04 Marks)**

b. What are the limitations of Euler's column theory? **(04 Marks)**

c. A hollow cast iron column whose outside diameter is 200mm has a thickness of 20mm. It is 4.5m long and fixed at both ends. Calculate (i) Slenderness ratio (ii) Ratio of Euler's and Rankine's critical loads. Take E = 100 GPa, $\alpha = \frac{1}{1600}$ and $\sigma_c = 550$ N/mm^2. **(12 Marks)**

* * * * *

CBCS SCHEME

USN | | | | | | | | | |

15CV/CT32

Third Semester B.E. Degree Examination, June/July 2019
Strength of Materials

Time: 3 hrs. Max. Marks: 80

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Define : (i) Modulus of Rigidity (ii) Poisson's ratio **(04 Marks)**

b. Prove that the total extension of a uniformly tapering rod of diameter D_1 and D_2, when the rod is subjected to an axial load 'P' is given by $dl = \frac{4PL}{\pi E D_1 D_2}$. **(06 Marks)**

c. An axial pull of 40,000 N is acting on a bar consisting of three sections of length 300mm, 250mm and 200mm and of diameters 20mm, 40mm and 50mm respectively. If the Young's modulus = 2×10^5 N/mm^2, determine (i) Stress in each section (ii) total extension of the bar. **(06 Marks)**

OR

2 a. Explain elasticity and elastic limit. **(04 Marks)**

b. A steel bar 300mm long, 50mm wide and 40mm thick is subjected to a pull of 300 kN in the direction of its length. Determine the change in volume. Take E = 2×10^5 N/mm^2 and Poisson's ratio = 0.25. **(06 Marks)**

c. A reinforced short concrete column 250mm × 250mm in section is reinforced with 8 steel bars. The total area of steel bars is 2500 mm^2. The column carries a load of 390 kN. If the modulus of elasticity for steel is 15 times that of concrete. Find the stresses in concrete and steel. **(06 Marks)**

Module-2

3 a. Differentiate between thin cylinder and a thick cylinder. Find an expression for the radial pressure and hoop stress at any point in case of a thick cylinder. **(10 Marks)**

b. A rectangular bar of cross section area of 11,000 mm^2 is subjected to a tensile load 'P' as shown in Fig.Q3(b). The permissible normal and shear stresses on the oblique plane BC are given as 7 N/mm^2 and 3.5 N/mm^2 respectively. Determine the safe value of 'P'. **(06 Marks)**

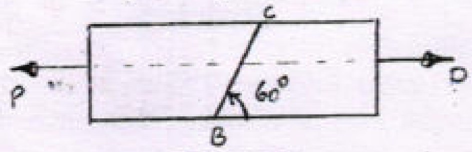


Fig.Q3(b)

OR

4 a. Determine the maximum and minimum hoop stress across the section of a pipe 400mm internal diameter and 100mm thick, when the pipe contains a fluid at a pressure of 8 N/mm^2. Also sketch the radial pressure distribution and hoop stress distribution across the section. **(08 Marks)**

b. At a point in a strained material the principal tensile stresses across two perpendicular planes are 80 N/mm^2 and 40 N/mm^2. Determine normal stress, shear stress and the resultant stress on a plane inclined at 20° with the major principal plane. Determine also the obliquity. **(08 Marks)**

Module-3

5 a. Define (i) Shear force (ii) Bending moment. **(02 Marks)**

b. Draw the SF and BM diagrams for a cantilever of length 'L' carrying a point load 'W' at the free end. **(04 Marks)**

c. Draw the SF and BM diagrams of a simply supported beam of length 7 mt carrying uniformly distributed loads as shown in Fig.Q5(c). **(10 Marks)**

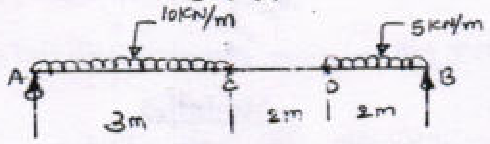


Fig.Q5(c)

OR

6 A horizontal beam 10mt long is carrying a uniformly distributed load of 1 kN/m. The beam is supported on two supports 6 mt apart. Find the position of the supports, so that bending moment on the beam is as small as possible. Also draw the SF and BM diagram. **(16 Marks)**

Module-4

7 a. Define the terms : (i) Neutral axis (ii) Section modulus. **(04 Marks)**

b. A hollow mild steel tube 6m long 40mm internal diameter and 5mm thick is used as a strut with both ends hinged. Find the crippling load and safe load taking factor of safety as 3. Take $E = 2\times10^5$ N/mm^2. **(06 Marks)**

c. The external and internal diameter of a hollow cast iron column are 50mm and 40mm respectively. If the length of this column is 3m and both of its ends are fixed, determine the crippling load using Rankine's formula. Take the values of $\sigma_c = 550$ N/mm^2 and $\alpha = \frac{1}{1600}$ in Rankine's formula. **(06 Marks)**

OR

8 a. Define (i) Buckling load (ii) Slenderness ratio. **(04 Marks)**

b. A timber beam of rectangular section of length 8m is simply supported. The beam carries a U.D.L. of 12 kN/m ran over the entire length and a point load of 10 kN at 3m from the left support. If the depth is two times the width and the stress in the timber is not to exceed 8 N/mm^2, find the suitable dimensions of the section. **(12 Marks)**

Module-5

9 a. List the theories of failures. **(04 Marks)**

b. A hollow shaft of external diameter 120mm transmits 300 kW power at 200 r.p.m. Determine the maximum internal diameter if the maximum stress in the shaft is not to exceed 60 N/mm^2. **(06 Marks)**

c. Determine the diameter of a solid steel shaft which will transmit 90 kW at 160 r.p.m. Also determine the length of the shaft if the twist must not exceed 1° over the entire length. The maximum shear stress is limited to 60 N/mm^2. Take the value of modulus of rigidity = 8×10^4 N/mm^2. **(06 Marks)**

OR

10 a. Derive the relation for a circular shaft when subjected to a torsion as given below:

$$\frac{T}{J} = \frac{\tau}{R} = \frac{C\theta}{L}$$ **(08 Marks)**

b. State and explain theory of maximum principal strain theory. **(08 Marks)**

* * * * *

USN

10CV/EV33

Third Semester B.E. Degree Examination, June/July 2019
Strength of Materials

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions, selecting atleast TWO questions from each part.***

PART - A

1 a. Draw stress – strain curve for mild steel specimen subjected to axial tension and indicate salient points. **(05 Marks)**

b. Determine the elongation caused by an axial load of 54 kN applied to flat bar of 12.5mm thickness, tapering from 100mm to 50mm in a length of 450mm. Take E = 200 GPa. **(07 Marks)**

c. A stepped bar is subjected to forces as shown in fig. Q1(c). Determine the magnitude of force P. Also find the net deformation induced. Take E = 210 GPa. **(08 Marks)**

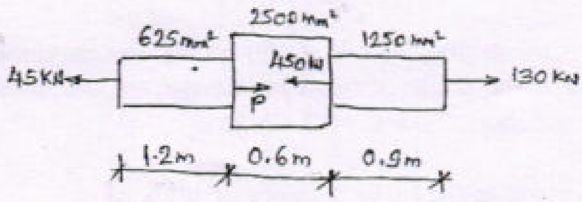


Fig.Q1(c)

2 a. Establish the relationship between E, K and Poisson's ratio. **(08 Marks)**

b. A bar 30mm in diameter was subjected to tensile load of 54kN and the measured extension on a 300mm gauge length was 0.112mm and change in diameter was 0.00366mm. Calculate Poisson's ratio and the values of three modulii. **(12 Marks)**

3 a. What are principal stresses and principal planes? **(06 Marks)**

b. An element has a tensile stress of 600MN/m^2 and a compressive stress of 400 MN/m^2 acting on two mutually perpendicular planes. It has two equal shear stresses of 200MN/m^2 on these planes. Determine i) Magnitude and direction of principal stresses and ii) Magnitude and direction of maximum shear stress. **(14 Marks)**

4 a. Explain the terms i) Shear force ii) Bending moment iii) Point of contra flexure. **(06 Marks)**

b. Draw SFD and BMD for the beam loaded as shown in fig. Q4(b), indicating the salient values. **(06 Marks)**

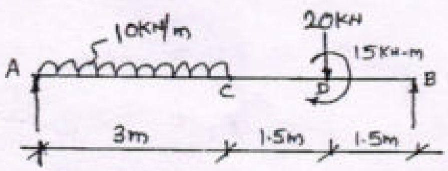


Fig.Q4(b)

c. Draw SFD and BMD for the overhanging bean loaded as shown in fig. Q4(c), indicating the salient values and locate the point of contra flexure, if any. **(08 Marks)**

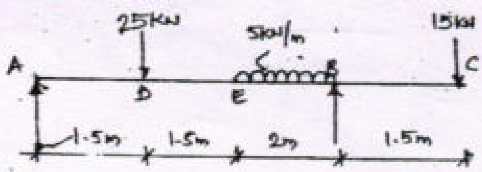


Fig.Q4(c)

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

PART - B

5 a. What are the assumptions made in the theory of pure bending? **(06 Marks)**

b. A T – section of flange 150mm wide and 15mm thick and overall depth of 200mm, with 15mm web thickness is loaded such that, the section has a moment of 25kN – m and shear force of 150kN. Sketch the bending and shear stress distribution diagram indicating salient values. **(14 Marks)**

6 a. Distinguish between slope and deflection. Explain with examples of a simply supported beam and a cantilever beam. **(08 Marks)**

b. A beam AB of 6m span is simply supported at the ends and is loaded with a point load of 10kN at the centre of the span and a udℓ of 5kN/m for the first half span of the beam. Find
i) deflection under point load ii) max deflection.
Take E = 200 GPa , $I = 25 \times 10^6\ mm^4$. **(12 Marks)**

7 a. Prove that a hollow shaft is stronger and stiffer than solid shaft of the same material, length and weight. **(08 Marks)**

b. A solid circular shafts transmits 294 kW at 300 rpm. If the maximum shear stress should be less than $42MN/m^2$ and the angle of twist in a length of 3m, should not exceed 1 degree. Find the diameter of the shaft. Take $C = 80\ GN/m^2$. **(12 Marks)**

8 a. List the assumptions made in the Euler's theory of columns. **(06 Marks)**

b. A cast iron column with 100mm external diameter and 80mm internal diameter is 3m long. Calculate the safe load using Rankine's formula if
i) Both ends are fixed ii) Both ends are hinged.
Take $\sigma_c = 600N/mm^2$, $\alpha = 1/1600$. Adopt a factor of safety = 3. **(14 Marks)**

*****

CBCS Scheme

USN 1SV14CV007

15CV/CT32

Third Semester B.E. Degree Examination, June/July 2018
Strength of Materials

Time: 3 hrs. Max. Marks: 80

Note: *1.* Answer any FIVE full questions, choosing ONE full question from each module.
2. Missing data, if any, may be suitably assumed.

Module-1

1 a. For a bar of uniform section derive an expression for elongation due to self weight. **(06 Marks)**

b. Evaluate the deformation of the bar, given, $E_1 = E_2 = E_3 = 200$GPa, refer Fig.Q1(b). **(10 Marks)**

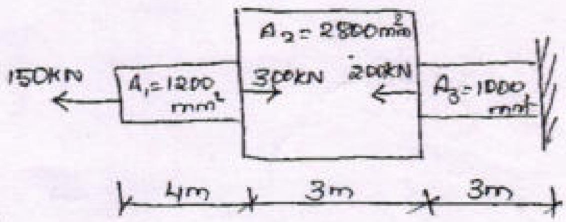


Fig.Q1(b)

OR

2 a. Derive an expression between Young's modulus, Modulus of rigidity and Poisson's ratio. **(10 Marks)**

b. A circular rod of dia 200mm and 500mm long is subjected to a tensile force of 45kN modulus of elasticity = 200 kN/mm^2, Find stress, strain and elongation of bar due to applied load. **(06 Marks)**

Module-2

3 At a certain point in a stressed body, the principal stresses are σ_x = 80 MPa and σ_y = –40MPa. Determine σ and τ on the planes whose normal's are at +30° and +120° with x – axis. **(16 Marks)**

OR

4 a. Derive an expression of tangential stress and longitudinal stress of thin walled pressure vessels. **(08 Marks)**

b. A rectangular block of material is subjected to a tensile stress of 100N/mm^2 on one plane and a tensile stress of 50N/mm^2 on a plane at right angles together with shear stress of 60 N/mm^2 on same planes, find : i) direction of the principal plane ii) magnitude of the principal plane iii) magnitude of greatest shear stress. **(08 Marks)**

Module-3

5 a. Define : i) bending moment ii) shear force iii) shear force diagram iv) bending moment diagram. **(08 Marks)**

b. Draw SFD and BMD for the cantilever beam shown in Fig.Q5(b). **(08 Marks)**

Fig.Q5(b)

OR

6 a. Derive the relation between load intensity, bending moment and shear force. **(06 Marks)**

b. .A beam ABC, 8m long has supplied at A and B, it is long between A and B. The beam carries an udl of 10kN/m between A and B. At free end point C, a point load of 15 kN acts. Draw BMD and locate point of contra-flexure, if any. **(10 Marks)**

Module-4

7 a. Explain pure bending with an suitable example and mention the assumptions of pure bending. **(06 Marks)**

b. A cast iron beam section shown in Fig.Q7(b) is freely supported on a span of 5m. IF the tensile stress is not to exceed 20 N/mm^2. Find the safe UDL which the beam can carry. Find also the maximum compressive stress. **(10 Marks)**

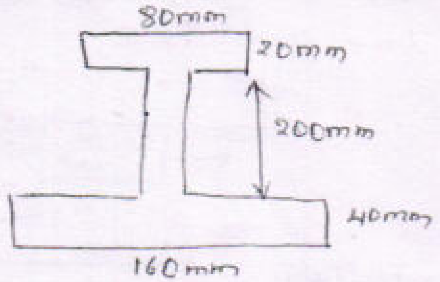


Fig.Q7(b)

OR

8 a. Derive an Euler's crippling load when both ends of the column are pinned. **(08 Marks)**

b. A hollow cylindrical cost iron column is 4m long both ends being, fixed. Design the column to carry a axial load of 250 kN. Use Rankine's formula and factor of safety = 5. The internal diameter may be taken as 0.80 time the external diameter. Take $E_C = 550$ N/mm^2 and $\alpha = \frac{1}{1600}$. **(08 Marks)**

Module-5

9 a. Derive torsional equation for circular shaft. **(08 Marks)**

b. A steel shaft transmits 105kN at 160 rpm. If the shaft is 100mm in diameter. Find the torque on the shaft and the maximum sharing stress induced. **(08 Marks)**

OR

10 a. Define pure torsion, polar modulus and torsional rigidity. **(06 Marks)**

b. A solid shaft is subjected to a torque of 15 kN-m. Find the necessary diameter of the shaft if the allowable shearing stress is $60N/mm^2$ and the allowable twist is 1 degree in a length of 20 diameters of the shaft. Take $C = 8 \times 10^4$ N/mm^2. **(10 Marks)**

* * * * *

CBCS Scheme

USN | | | | | | | | | | | **15CV/CT32**

Third Semester B.E. Degree Examination, June/July 2017
Strength of Materials

Time: 3 hrs. Max. Marks: 80

Note: *Answer FIVE full questions, choosing one full question from each module.*

Module-1

1 a. Define : (i) Poisson's ratio (ii) Volumetric strain (iii) Temperature stresses **(06 Marks)**

b. A steel bar of 20 mm diameter is subjected to tensile load test. Determine stress, strain, Young's modulus, % elongation from the following data:
Gauge length – 200 mm, Extension at a load of 100 kN – 0.147 mm, Total elongation 50 mm. Also determine the % decrease in cross sectional area of the specimen if the diameter of the rod at failure is 16 mm. **(10 Marks)**

OR

2 a. Derive the relationship between Young's modulus and shear modulus with usual notation. **(06 Marks)**

b. A steel tube 45 mm external diameter and 3 mm thick encloses centrally a solid copper bar 30 mm diameter. The bar and the tube are rigidly connected together at their ends at a temperature of 30°C. Find the stresses developed in each material when heated to 180°C. Take $E_S = 200\,GPa$, $\alpha_S = 10.8\times10^{-6}/^\circ C$; $E_C = 110\,GPa$, $\alpha_C = 17\times10^{-6}/^\circ C$ **(10 Marks)**

Module-2

3 a. Derive Lami's equation for thick cylinders. **(06 Marks)**

b. The state of stress at a point in a strained material is as shown in the Fig. Q3 (b)
Determine (i) Principal stresses and principal planes (ii) Max shear stress and its plane
(iii) Sketch the stress diagram showing stresses and planes. **(10 Marks)**

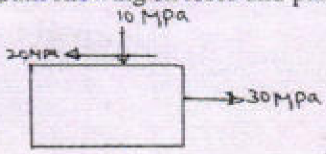


Fig. Q3(b)

OR

4 a. Derive expressions for normal stress and tangential stress for a member subject to uniaxial loading. **(06 Marks)**

b. A shell 3.25 m long, 1 m diameter is subjected to internal fluid pressure of 1 MPa. If the thickness of the shell is 10 mm. Find Hoop stress, longitudinal stress, max shear stress and change in diameter and length. Take $E = 2\times10^5$ MPa, $\frac{1}{m} = 0.3$. **(10 Marks)**

Module-3

5 a. Derive the relationship between load intensity, shear force and bending moment. **(06 Marks)**

b. A simply supported beam is subject to a point load of 15 kN together with udl of 15 kN/m applied as shown in Fig. Q5 (b). Draw SFD and BMD. Find also point of zero shear and its corresponding BM. **(10 Marks)**

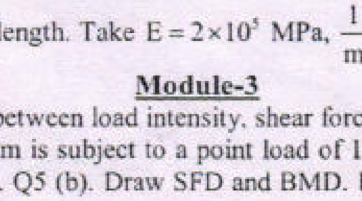


Fig. Q5 (b)

OR

6 a. Show that max BM for a simply supported beam of length l carrying udl of intensity W/unit length is $\frac{Wl^2}{8}$. **(06 Marks)**

b. Draw SFD and BMD for the load diagram, shown in Fig. Q6 (b). Mark the values at salient points. **(10 Marks)**

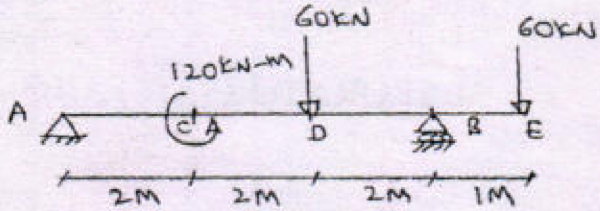


Fig. Q6 (b)

Module-4

7 a. Derive the bending equation, $\frac{M}{I} = \frac{f}{y} = \frac{E}{R}$ with usual notation. **(06 Marks)**

b. A hallow tube of 6 m length with external diameter 60 mm and thickness 10 mm is subject to minimum crippling load. Find Euler's critical load for this column : (i) When both ends are fixed. (ii) When one end fixed other end hinged. Assume E = 200 GPa. **(10 Marks)**

OR

8 a. Derive expression for crippling load for a long column when both ends are hinged. **(06 Marks)**

b. A circular pipe of external diameter 70 mm and thickness 8 mm is used as a simply supported beam over an effective span of 2.5 m. Find the max concentrated load that can be applied at the centre of the span if permissible stress in the tube is 150 N/mm^2. **(10 Marks)**

Module-5

9 a. Derive the torque equation $\frac{T}{I_p} = \frac{f_s}{R} = \frac{C_\theta}{l}$ with usual notation. **(06 Marks)**

b. State the theories of failures. Explain briefly any two of the theories. **(10 Marks)**

OR

10 a. State the assumption made in the theory of pure torsion. **(06 Marks)**

b. A hallow shaft has to transmit 600 kW power at 80 rpm. The maximum torque developed may exceed the mean torque by 40%. Design a suitable section if the working stress is 90 MPa. Take diameter ratio as 0.8. What will be the angular twist measured over a length of 2 m if C = 84 GPa? **(10 Marks)**

* * * * *

CBCS Scheme

USN | | | | | | | | | | **15CV/CT32**

Third Semester B.E. Degree Examination, Dec.2016/Jan.2017
Strength of Materials

Time: 3 hrs. Max. Marks: 80

Note: Answer any FIVE full questions, choosing one full question from each module.

Module-1

1 a. Name and define the four elastic constants. **(06 Marks)**

b. Determine the value of "P" and the total deformation of the stepped bar.
Take $E = 2.1 \times 10^5$ N/mm^2. Refer fig.Q1(b). **(10 Marks)**

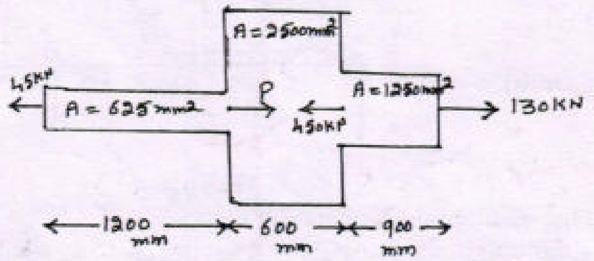


Fig.Q1(b)

OR

2 a. Derive the relationship between Young's modulus and bulk modulus. **(06 Marks)**

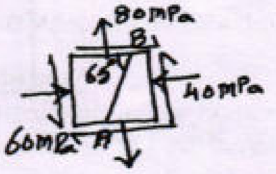
b. A steel bar is placed between two copper bars, each having same area and length as the steel bar. These are rigidly connected together at a temperature of 25^{0}C. When the temperature is raised to 325^{0}C, the length of the bar is increased by 1.5mm. Compute the original length and final stresses in each bar. Take E_{steel} = 210GPa and E_{copper} = 100GPa ;
$\alpha_{steel} = 12 \times 10^{-6}/^0C$ and $\alpha_{copper} = 17.5 \times 10^{-6}/^0C$. **(10 Marks)**

Module-2

3 a. Explain the procedure to construct Mohr's circle and to find principal stresses and their planes. **(04 Marks)**

b. The stresses acting at a point in a two dimensional stress system is as shown in fig.Q3(b). Determine : i) Principal stresses ii) Normal and tangential stress on the plane AB
iii) Maximum shear stress. **(12 Marks)**

Fig.Q3(b)



OR

4 a. Derive an expression for hoop stress in thin cylinder. **(04 Marks)**

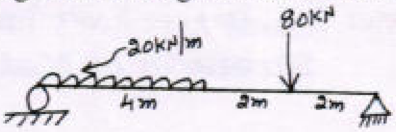
b. Find the thickness of the metal necessary for a steel cylindrical shell of internal dia 150mm to withstand an internal pressure of 50N/mm^2. The maximum hoop stress in the section not to exceed 150N/mm^2. If the thickness is found using the cylinder analysis, what is the percentage error? **(12 Marks)**

Module-3

5 a. Derive the relationship between intensity of load, shear force and bending moment. **(06 Marks)**

b. Draw shear force and bending moment diagrams for the beam shown in fig.Q5(b). **(10 Marks)**

Fig.Q5(b)

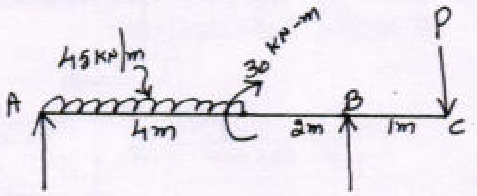


OR

6 a. Define i) Shear force ii) Bending moment and iii) Point of contra flexure. **(03 Marks)**

b. For the beam AC shown in fig.Q6(b), determine the magnitude of the load 'P' acting at C, such that the reaction at supports A and B are equal. Also draw SF and BM diagrams, locate the point of contra flexure if any. **(13 Marks)**

Fig.Q6(b)

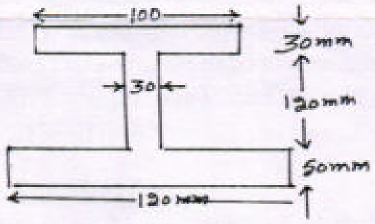


Module-4

7 a. What are the assumptions in bending theory? **(04 Marks)**

b. A beam simply supported at ends and having cross section as shown in fig.Q7(b) is loaded with a udℓ over a span of 8m. The allowable bending stress in tension is $30 N/mm^2$ and that in compression is $45 N/mm^2$. Determine the maximum value of udℓ, the beam can carry. **(12 Marks)**

Fig.Q7(b)



OR

8 a. Differentiate between short and long columns. **(04 Marks)**

b. What are the limitations of Euler's theory? **(04 Marks)**

c. A column 6m long has both of its ends fixed and has a timber section of 150mm × 200mm. Determine the crippling load on the column. Take $E = 17.5 \times 10^3 N/mm^2$. **(08 Marks)**

Module-5

9 a. Derive the torsion equation with usual notations. **(08 Marks)**

b. A hollow shaft of external dia 120mm transmits 300KW power at 200rpm. Determine the maximum internal dia, if the maximum shear stress in the shaft is not to exceed $60 N/mm^2$. **(08 Marks)**

OR

10 a. Explain Maximum Principal Stress theory. **(04 Marks)**

b. A solid circular shaft is subjected to a bending moment of 9000N-m and a twisting moment of 12000 N-m. In a simple uniaxial tensile test of the same material, it gave the following particulars : Stress at yield point = $300 N/mm^2$; $E = 200 GN/mm^2$.
Estimate the least dia required using i) Maximum principal stress theory ii) Maximum shear stress theory. Take FOS = 3 and $\mu = 0.25$. **(12 Marks)**

* * * * *

CBCS Scheme

Shridevi Institute of Engineering and Technology, Tumkur – 06

III Semester: I Internal Assessment Test: 05/09/2019

18CV32 – Strength of Materials

Time: 75 Minutes Max Marks: 30

Note: 1. ***Answer any two full questions***

2. All questions carry equal marks

1 a) Define the following terms: i) Poisson's ratio iii) Modulus of Rigidity **(04 Marks)**

b) Derive the expression for the extension of uniformly tapering circular bar subjected to a load 'P' with usual notations **(05 Marks)**

c) An axial pull of 40kN is acting on a bar consisting of three sections of length 300mm, 250mm and 200mm and of diameters 20mm, 40mm and 50mm respectively. If the Young's modulus = $2 \times 10^5 N/mm^2$, determine (i) Stress in each section (ii) total extension of bar. **(06 Marks)**

OR

2 a) Establish the relationship between E, C and Poisson's ratio. **(07 Marks)**

b) A bar 30 mm diameter was subjected to tensile load of 54kN and the measured extension on a 300mm gauge length was 0.112mm and change in diameter was 0.00366mm. Calculate Poisson's ratio and the values of three modulii. **(08 Marks)**

3 Explain the terms i) Shear force ii) Point of contraflexure **(06 Marks)**

b) Draw SFD and BMD for the beam loaded as shown in fig 3(b) and also locate the point of contraflexure. **(09 Marks)**

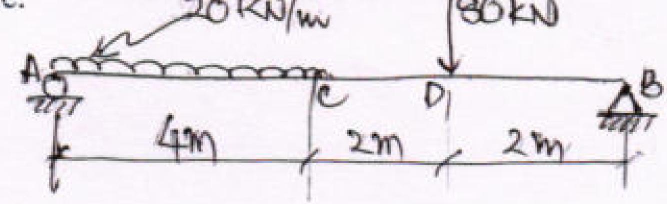


Fig 3(b)

OR

4 a) Derive the relationship between load, SF and BM. **(06 Marks)**

b) Draw SFD and BMD for the beam loaded as shown in fig 4(b). Indicate the point of contraflexure if any. **(09 Marks)**

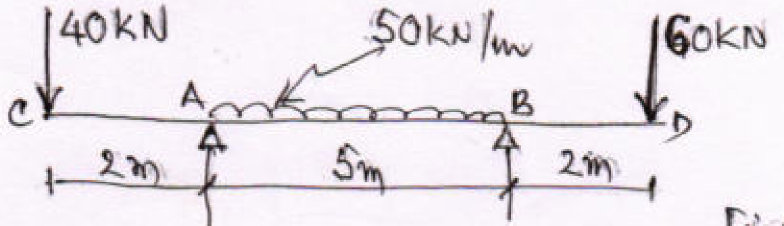


Fig 4(b)

C. Nagaraju

Course Instructor: C. Nagaraja

Strength of Materials – 18CV32

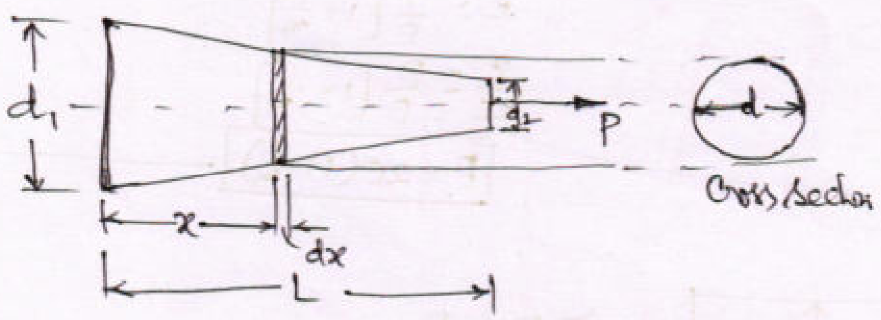
Scheme of Valuation

I Internal Assessment Test – 05/09/2019

1 a) Definition of i) Poisson's ratio $= \dfrac{e_{lateral}}{e_{longitudinal}}$ — 02 m

ii) Modulus of rigidity $= C = \dfrac{\text{Shear Stress}}{\text{Shear Strain}} = \dfrac{\tau}{\phi}$ — 02 m

b)



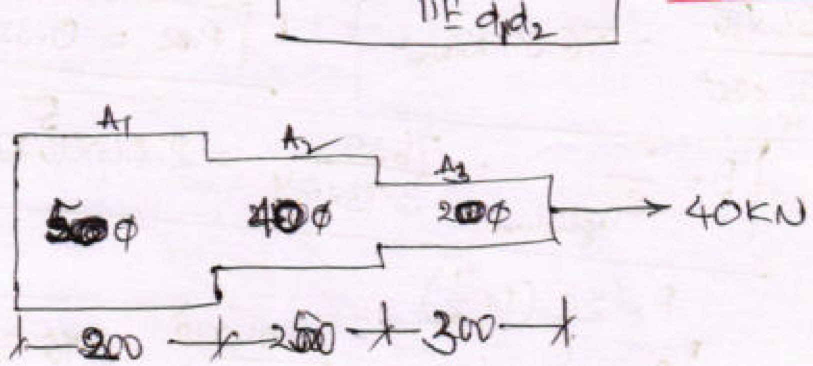
Cross sectional area $= \frac{\pi}{4}(d_1 - kx)^2$ where $k = \frac{d_1 - d_2}{L}$

Extension of bar $= \dfrac{P.dx}{\frac{\pi}{4}(d_1-kx)^2 \cdot E}$

$= \dfrac{4P}{\pi E}\int_0^L \dfrac{dx}{(d_1-kx)^2}$

$$\boxed{\delta l = \frac{4PL}{\pi E\, d_1 d_2}}$$ — 05 m

c)



$\delta l = \dfrac{Pl_1}{A_1E} + \dfrac{Pl_2}{A_2E} + \dfrac{Pl_3}{A_3E} = \dfrac{P}{E}\left[\dfrac{l_1}{A_1} + \dfrac{l_2}{A_2} + \dfrac{l_3}{A_3}\right]$

$A_1 = 1963.495\ mm^2$

$A_2 = 1256.637\ mm^2$

$A_3 = 314.159\ mm^2$

$\delta l = \dfrac{40\times10^3}{2\times10^5}\left[\dfrac{200}{1963.495} + \dfrac{250}{1256.637} + \dfrac{300}{314.159}\right]$

$\boxed{\delta l = 0.2511\ mm}$ — 3 m

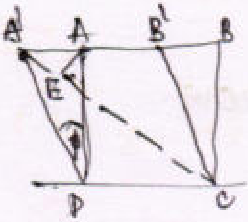
$\sigma_1 = 20.37\ N/mm^2$ — 1 m

$\sigma_2 = 31.83\ N/mm^2$ — 1 m

$\sigma_3 = 127.32\ N/mm^2$ — 1 m

2

a) $E = 2C\left[1+\frac{1}{m}\right]$



$\phi = \frac{AA'}{AD}$

$\phi = \frac{2A'E}{EC} = 2e.$

Shear strain = 2 x longitudinal strain $\qquad C = \frac{\tau}{\phi}$

$\frac{\tau}{C} = 2e \quad \therefore e = \frac{\tau}{2C}$

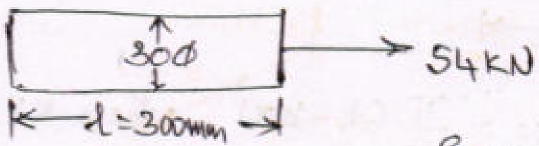
Net strain in diagonal AC: $e = \frac{\tau}{E} + \frac{1}{m}\frac{\tau}{E}$

$e = \frac{\tau}{E}\left[1+\frac{1}{m}\right]$

$\frac{\tau}{2C} = \frac{\tau}{E}\left[1+\frac{1}{m}\right]$

$\boxed{E = 2C(1+\frac{1}{m})}$ —— 07 m

b)



$\delta l = 0.112$ mm

$e_{lateral} = \frac{0.00366}{30}$

$\boxed{e_{lateral} = 1.22\times10^{-4}}$ —— 1 m

$\boxed{e_{longitudinal} = \frac{0.112}{300} = 3.73\times10^{-4}}$ —— 1 m

$\boxed{\text{Poisson's ratio} = \frac{1}{m} = \frac{e_{lateral}}{e_{longitudinal}} = \frac{1.22\times10^{-4}}{3.73\times10^{-4}}}$ —— 1 m

$\boxed{\sigma = \frac{54\times10^3}{\frac{\pi}{4}\times30^2} = 76.39\ N/mm^2}$ $\quad \boxed{P.R = 0.327}$ —— 1 m

$\boxed{E = \frac{\sigma}{e_{longitudinal}} = \frac{76.39}{3.73\times10^{-4}} = 2.04\times10^5\ N/mm^2}$ —— 1 m

$E = 2C\left(1+\frac{1}{m}\right)$

$\boxed{C = \frac{E}{2(1+\frac{1}{m})} = \frac{2.04\times10^5}{2(1.327)} = 76.86\times10^3\ N/mm^2}$ —— 1 m

$E = 3K\left[1-2.\frac{1}{m}\right]$

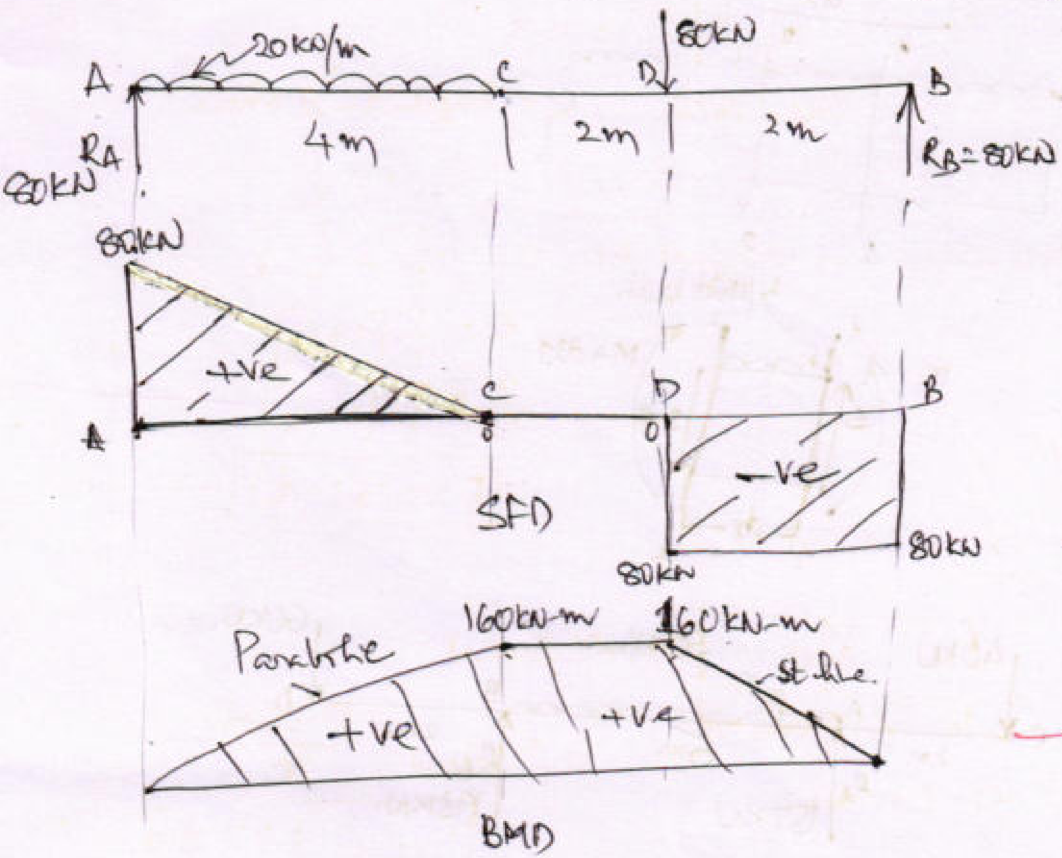
$\boxed{K = \frac{E}{3\left[1-2.\frac{1}{m}\right]} = \frac{2.04\times10^5}{3\times[1-2(0.327)]} = 196.53\times10^3\ N/mm^2}$ —— 2 m

3

a) Explanation of i) Shear force — Sum of all vertical forces to any one side of a section considered is known as SF (3) marks

ii) Point of Contraflexure ———— (3) marks
Point where SF changes its sign from +ve to -ve or vice versa. The value of BM is zero at P.C.

b)



Soln

— (2) m

— (2) m

$\Sigma M_A = 0$ $\quad -R_B \times 8 + 80\times 6 + 80\times 2 = 0$ $\quad \boxed{R_B = 80kN}$

$\Sigma V = 0$ $\quad \boxed{R_A = 80kN}$ — (2) m

Calculate SF & BM:

B&D

$SF_{1-1} = -80kN$

Constant between B&D.

$M_{1-1} = +80 \cdot x$

$x = 0 \quad M = +0$

$x = 2 \quad M = +160 kN\text{-}m$

C&D

$SF_{2-2} = -80 + 80 = 0$

Constant between C&D

$M_{2-2} = +80 \cdot x - 80(x-2)$

$x = 2 \quad M = +160 kN\text{-}m$

$x = 4 \quad M = +160 kN\text{-}m$

A&C

$SF_{3-3} = -80 + 80 \pm 20(x-4)$

$x = 4 \quad SF = 0$

$x = 8 \quad = +80kN$

$M_{3-3} = +80x - 80(x-2) - 20\frac{(x-4)^2}{2}$

$x = 4 \quad M = +160kN\text{-}m$

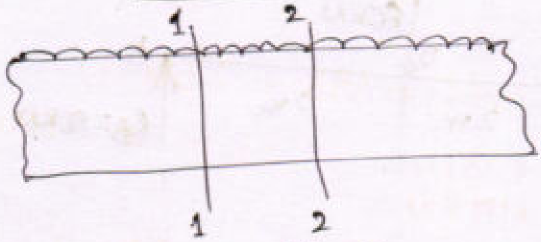
$x = 8 \quad M = 0kN$

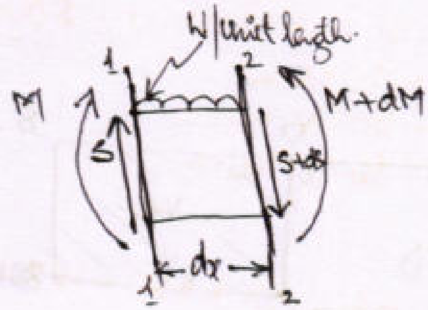
— (3) m

4

a) $\boxed{W = \frac{dS}{dx}}$ Derivation of relationship

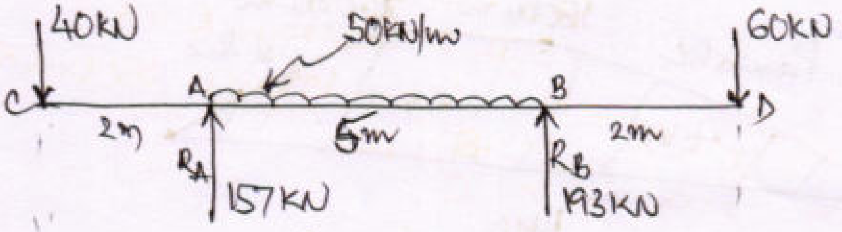
$\boxed{S = \frac{dM}{dx}}$

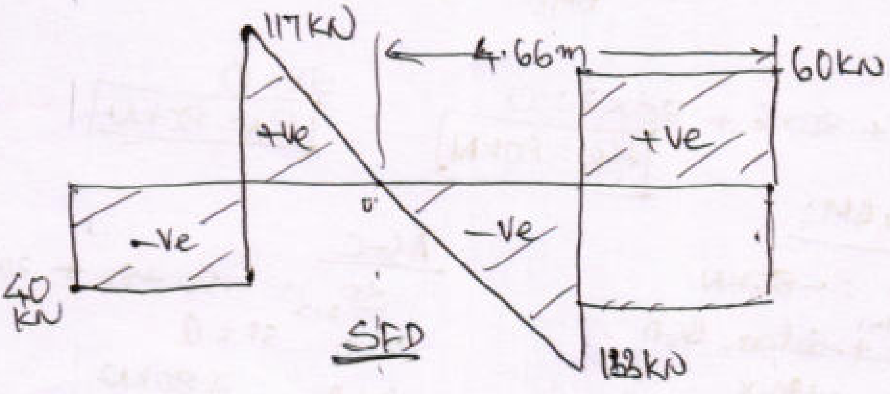




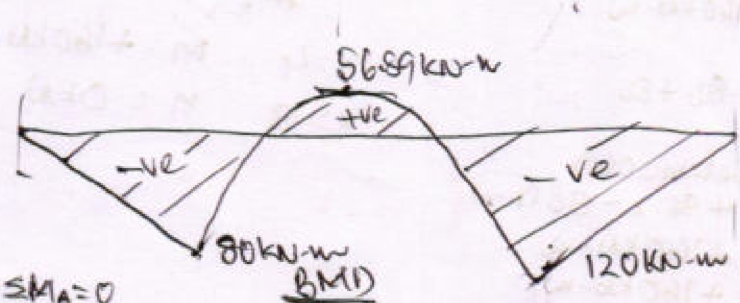
06 m

b)





02 m



3 m

$\Sigma M_A = 0$

$-R_B \times 5 + 60 \times 7 + 250 \times 2.50 - 40 \times 2 = 0$

$\boxed{R_B = 193 \text{ KN}}$

$\Sigma F_y = 0$

$-40 + R_A - 250 + 193 - 60 = 0$

$\boxed{R_A = 157 \text{ KN}}$

2 m

SF & BM

Between B & D

$SF = +60 kN$

Constant between B & D

$M = -60 \cdot x.$

$x = 0$ $M = 0$

$x = 2$ $M = -120 kN\text{-}m$

B & A

$SF = +60 - 193 + 50(x-2)$

$x = 2$ $SF = -133 kN$ $SF = 0$ at $50(x-2) = 133$

$x = 7$ $SF = +117 kN$ $x = 4.66 m$

$M = -60 \cdot x + 193(x-2) - \frac{50(x-2)^2}{2}$

$x = 2$ $M = -120 kN\text{-}m$

$x = 7$ $M = -80 kN\text{-}m$

$x = 4.66$ $M = 56.89 kN\text{-}m$

C & A

$SF = +60 - 193 + 250 - 157$

$SF = -40 kN\text{-}m$

Constant between C & A

$M = -60x + 193(x-2) - 250(x-4.5) + 157(x-7)$

$x = 7$ $M = -80 kN$

$x = 9$ $M = 0$

Q3

CBCS Scheme

Shridevi Institute of Engineering and Technology, Tumkur – 06
III Semester: II Internal Assessment Test: 19/10/2019
18CV32 – Strength of Materials

Time: 75 Minutes Max Marks: 30

Note: 1. ***Answer any two full questions***
2. All questions carry equal marks

1 a) Derive the Bernoulii – Euler's equation M/I = f/Y = E/R. **(07 Marks)**

b) A T section is having a flange of 250 mm x 100 mm. The web is also 250 mm x 100 mm. It is subjected to a bending moment of 20kN - m. Draw the bending stress distribution across the cross section indicating the salient values. **(08 Marks)**

OR

2 a) What are the assumptions in the theory of pure bending? **(05 Marks)**

B) A circular pipe of external diameter 70 mm and thickness 8mm is used as a simply supported beam over an effective span of 2.5 m. Find the maximum concentrated load that can be applied at the centre of span if the permissible stress in the tube is 150 N/mm^2. **(10 Marks)**

3 Derive the torsion equation $T/J = q_r/r = C\theta/l$. **(07 Marks)**

b) Determine the diameter of a solid circular shaft which will transmit 400 kW at 300 rpm. The maximum shear stress should not exceed 32 N/mm^2. The twist should not exceed 1° in a length of 2 m. Assume modulus of rigidity as 90 kN/mm^2. **(08 Marks)**

OR

4 a) Using Euler's theory, obtain the expression for the crippling load of a long column pinned at both ends. **(06 Marks)**

b) The external and internal diameter of a hollow cast iron column are 45 mm and 35 mm. If the length of this column is 3.5 m and both of its are fixed, determine the crippling load using Rankine's formula. Take the values $\sigma_c = 550 N/mm^2$ and $\alpha = 1/1600$ in Rankine's formula. **(09 Marks)**

Course Instructor: C. Nagaraja

Scheme of Valuation

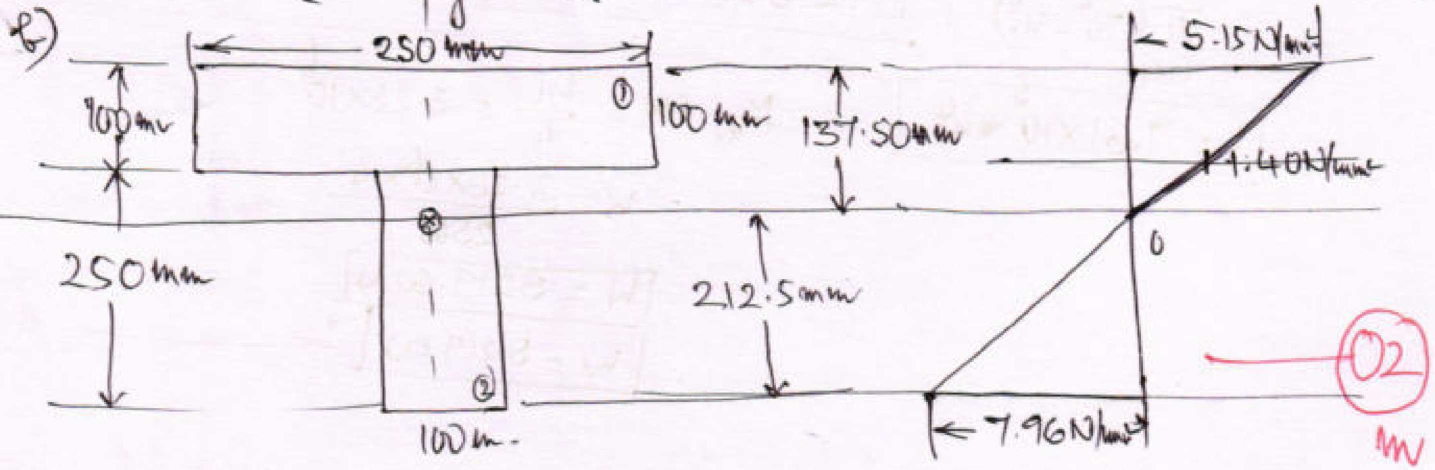
III Semester : Strength of Materials – 18CV32

II Internal Assessment Test

1 a) Derivation of Bernoulli-Euler's equation

$$\frac{M}{I} = \frac{f}{y} = \frac{E}{R}$$ — 07 M

b)



Bending Stress Distribution

$$\bar{y} = \frac{(250\times100)(300) + (250\times100)(125)}{(250\times100)2}$$

$$\boxed{\bar{y} = 212.5\ mm}$$ — 02 M

$$I = \left[\left(\frac{250\times100^3}{12}\right) + (250\times100)(87.50)^2 + \left(\frac{100\times250^3}{12}\right) + (100\times250)(87.5)^2\right]$$

$$\boxed{I = 533.85\times10^6\ mm^4}$$ — 02 M

$$f = \frac{M}{I}\times y$$

$$f_{top\ fibre} = \frac{20\times10^6}{533.85\times10^6}\times 137.50 = 5.15\ N/mm^2$$

$$f_{junction\ of\ web\ \&\ flange} = \frac{20\times10^6}{533.85\times10^6}\times 37.50 = 1.40\ N/mm^2$$

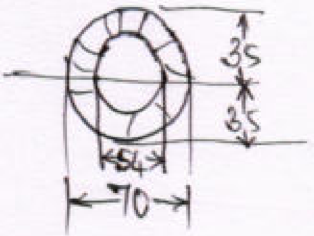
$$f_{NA} = 0$$

$$f_{bottom} = \frac{20\times10^6}{533.85\times10^6}\times 212.50 = 7.96\ N/mm^2$$

— 02 M

2 a) Assumptions in the theory of pure bending — 05 M

b)



$M = \frac{f}{y} \cdot I$

$= \frac{150}{35} \times 7.61 \times 10^5$

$\boxed{M = 3.26 \times 10^6 \text{ N-mm}}$ — 04 M

$I = \frac{\pi}{64}(70^4 - 54^4)$

$\boxed{I = 7.61 \times 10^5 \text{ mm}^4}$ — 02 M

M_{ax} BM $= \frac{WL}{4} = 3.26 \times 10^6$

$W = \frac{3.26 \times 10^6 \times 4}{2500}$

$\boxed{W = 5219.62 \text{N}}$

$\boxed{W = 5.219 \text{ kN}}$ — 04 M

3 a) Derivation of torsion equation $\frac{T}{J} = \frac{q}{r} = \frac{C\theta}{L}$ — 07 M

b)

$P = 400 \text{ kW} = 400 \times 10^3$ N-m/sec $= 400 \times 10^6$ N-mm/sec

$N = 300$ rpm

$q = 32 \text{ N/mm}^2$

$\theta = 1^\circ = \frac{\pi}{180}$

$l = 2000$ mm.

$C = 90 \text{ kN/mm}^2$

$P = \frac{2\pi NT}{60}$

$T = \frac{60 P}{2\pi N}$

$T = \frac{60 \times 400 \times 1000 \times 1000}{2\pi \times 300}$

$\boxed{T = 12.73 \times 10^6 \text{ N-mm}}$ — 02 M

From shear stress considerati

$T = \frac{J}{r} \cdot q$

$T = \frac{\frac{\pi d^4}{32}}{\frac{d}{2}} \cdot q$

$T = \frac{\pi d^3}{16} \cdot q$

$d = \sqrt[3]{\frac{16 \times 12.73 \times 10^6}{\pi \times 32}}$

$\boxed{d = 126.5 \text{mm}}$

From angle of twist consideratin

$\frac{T}{J} = \frac{C\theta}{l}$

$J = \frac{Tl}{C\theta}$

$\frac{\pi d^4}{32} = \frac{12.73 \times 10^6 \times 2000 \times 180}{90 \times 10^3 \times \pi}$

$d = \sqrt[4]{\frac{(16.20 \times 10^6) \times 32}{\pi}}$

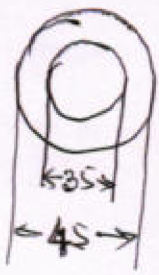
$\boxed{d = 113.35 \text{ mm}}$ — 03 M

Considering the maximum value $\boxed{d = 113.35 \text{mm}}$ — 01 — 02 M

4 a) Derivation of expression for Euler's Crippling load of long Column pinned at both ends:

$$\boxed{P_{cr} = \frac{\pi^2 EI}{l^2}}$$ 06 m

b)



$$I = \frac{\pi (45^4 - 35^4)}{64}$$

$$\boxed{I = 127.62 \times 10^3 \ mm^4}$$ 02 m

$$A = \frac{\pi}{4}(45^2 - 35^2)$$

$$\boxed{A = 628.31 \ mm^2}$$ 01 m

$\sigma_c = 550 N/mm^2$

$\alpha = \frac{1}{1600}$

$l_e = \frac{l}{2} = 1750 mm$

fixed

$$P_R = \frac{\sigma_c \cdot A}{1 + \alpha \left(\frac{l_e}{k}\right)^2}$$

$$k = \sqrt{\frac{I}{A}}$$

$$\boxed{k = 14.25 \ mm}$$ 02 m

$$= \frac{550 \times 628.31}{1 + \frac{1}{1600}\left(\frac{1750}{14.25}\right)^2}$$

$$\boxed{P_R = 33.14 \times 10^3 \ N}$$ 04 m

Shridevi Institute of Engineering and Technology, Tumkur – 06
III Semester: III Internal Assessment Test: 25/11/2019
18CV32 – Strength of Materials

Time: 75 Minutes — Max Marks: 30

Note: 1. ***Answer any two full questions***
2. All questions carry equal marks

1 a) Derive an expressions for Euler's crippling load for a column with one end fixed and the other end free. **(06 Marks)**

b) Compare the crippling loads given by Euler's and Rankine's formula formula for a tubular steel column2.5 m long having outer and inner dia as 45 and 35 mm respectively loaded through pin jointed ends. Take yield stress = 320 N/mm^2, α = 1/1750 and E = 210 GPa. For what length of the column, this cross section ceases to apply? **(09 Marks)**

OR

2 a) Derive an expression with usual notations for the maximum deflection in a simply supported beam subjected to point load at the mid span **(06 Marks)**

b) Find the maximum deflection and the maximum slope for the beam loaded as shown in fig 2 (b). Take flexural rigidity EI = 15 X 10^9 k N/mm^2. **(09 Marks)**

3 Derive the expression for $EI.d^2y/dx^2 = M$ with usual notations. **(06 Marks)**

b) Determine the deflection under the loads in the beam shown in fig 3(b). Take flexural rigidity as constant throughout. **(09 Marks)**

OR

4 a) Derive the expressions for hoop stress, longitudinal stress and maximum shear stress for thin cylinder. **(06 Marks)**

b) A pipe of 400 mm internal diameter and 120 mm thickness contains a fluid at a pressure of 90 N/mm^2. Find the maximum and minimum hoop stresses across the section. Also sketch radial and hoop stress distribution across the section. **(09 Marks)**

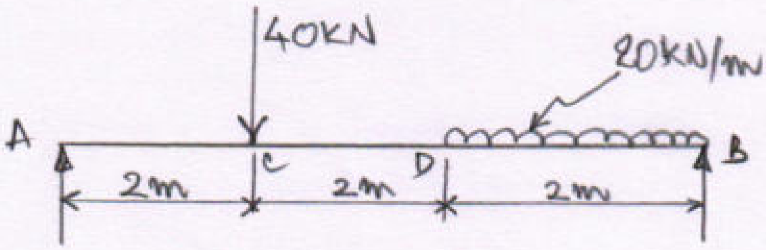


Fig 2(b)

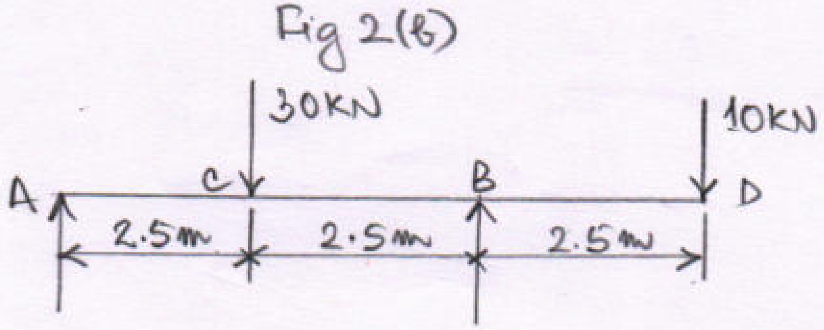


Fig 3(b)

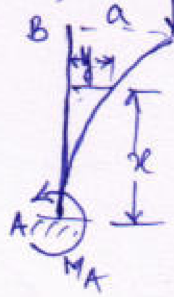
Scheme of Valuation

III Semester : II Internal Assessment test : 25/11/2019

18CV32 - Strength of Materials

1 a) Expression of Euler's crippling load for a column with one end fixed and other end free.

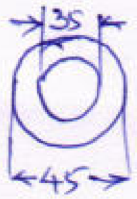
$$\boxed{P_{cr} = \frac{\pi^2 EI}{4l^2}}$$



06 m

b)

$l = 2.5m$



$I = \frac{\pi}{64}(45^4 - 35^4)$

$\boxed{I = 127.62 \times 10^3\ mm^4}$

$A = \frac{\pi(45^2 - 35^2)}{4}$

$\boxed{A = 628.31\ mm^2}$

Euler's formula:

$$(P_{cr})_E = \frac{\pi^2 EI}{l^2} = \frac{\pi^2 \times 210 \times 10^3 \times 127.62 \times 10^3}{(2500)^2}$$

$$\boxed{(P_{cr})_E = 42.32 \times 10^3\ N}$$

3 m

Rankine's formula

$$(P_{cr})_R = \frac{f_c A}{1 + \alpha\left(\frac{l_e}{k}\right)^2}$$

$k = \sqrt{\frac{127.62 \times 10^3}{628.31}}$

$k = 14.25mm$

$l_e = l = 2500$ for pin jointed ends

$$(P_{cr})_R = \frac{320 \times 628.31}{1 + \frac{1}{7500}\left(\frac{2500}{14.25}\right)^2}$$

$$\boxed{(P_{cr})_R = 10.81 \times 10^3\ N}$$

3 m

Then

$$\frac{\pi^2 EI}{l^2} = P_E = \sigma_y . A = 320 \times A$$

$$\frac{\pi^2 E . AK^2}{l^2} = 320 \times A$$

$$\frac{\pi^2 E}{\left(\frac{l_e}{k}\right)^2} = 320$$

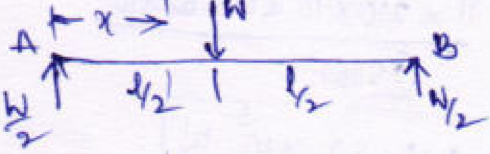
$$\left(\frac{l_e}{k}\right)^2 = \frac{\pi^2 E}{320}$$

$$\frac{l_e}{k} = \sqrt{\frac{\pi^2 \times 210 \times 10^3}{320}} = 80.47$$

$$\boxed{l_e = 80.47 \times 14.25 = 1146.83 m}$$ — 03 m

2

a) Maximum deflection in a simply supported beam subjected to point load at mid span



$$M_x = \frac{W}{2} . x$$

$$EI \frac{d^2y}{dx^2} = \frac{Wx}{2}$$

$$EI \frac{dy}{dx} = \frac{Wx^2}{4} + C_1$$

when $x = \frac{l}{2}$, $\frac{dy}{dx} = 0$ $\therefore C_1 = -\frac{Wl^2}{16}$

$$EI \frac{dy}{dx} = \frac{Wx^2}{4} - \frac{Wl^2}{16} \quad \text{—— (1)}$$

$$EI . y = \frac{Wx^3}{12} - \frac{Wl^2}{16} . x + C_2$$

when $x = 0$, $y = 0$, $C_2 = 0$

$$EI\, y = \frac{Wx^3}{12} - \frac{Wl^2}{16} . x$$

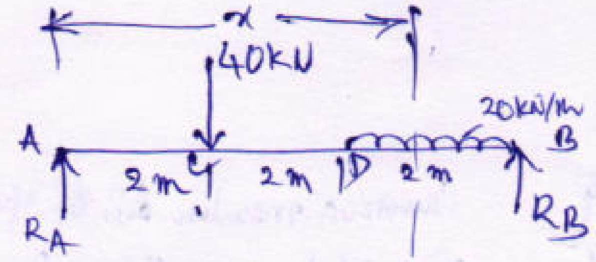
At $x = l/2$

$$y_{max} = \frac{1}{EI}\left[\frac{W . l^3}{96} - \frac{Wl^3}{32}\right]$$

$$\boxed{y_{max} = \frac{Wl^3}{48EI}}$$

— 06 m

2 b)



Soln: $\Sigma M_B = 0$

$$+R_A \times 6 - 40\times 4 - 40\times 1 = 0$$

$$\boxed{R_A = 33.33\ \text{kN}}$$ —— 1 m

$$M_x = 33.33.x - 40(x-2) - 20\frac{(x-4)^2}{2}$$

$$EI\frac{d^2y}{dx^2} = 33.33.x - 40(x-2) - \frac{20(x-4)^2}{2}\ \text{kN-m}$$

$$EI\frac{dy}{dx} = 33.33\frac{x^2}{2} - \frac{40(x-2)^2}{2} - \frac{20(x-4)^3}{6} + C_1\ \text{kN-m}^2$$

$$EI\,y = 33.33\frac{x^3}{6} - \frac{40(x-2)^3}{6} - \frac{20(x-4)^4}{24} + C_1x + C_2\ \text{kN-m}^3$$

At $x=0$, $y=0$, $C_2=0$ —— 2 m

$x=6$ $y=0$ $C_1 = -126.66$ —— 02 m

Assuming maximum deflection in portion CD

$$\frac{dy}{dx} = 0, \rightarrow 33.33\frac{x^2}{2} - 20(x-2)^2 - 126.66 = 0$$

$$33.33\frac{x^2}{2} - 20x^2 + 80x - 80 - 126.66 = 0$$

$$x^2 - 24x + 62 = 0$$

$\boxed{x = 2.945\text{m}}$ Hence assumption is correct. —— 02 m

Then

$$EI\,y_{max} = 33.33\times\frac{(2.945)^3}{6} - \frac{40}{6}(0.945)^3 - 126.66\times 2.945$$

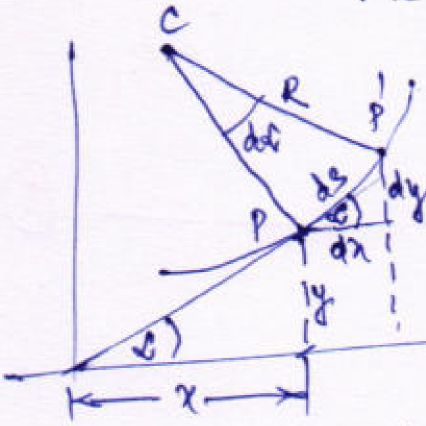
$$y = -231.05\ \text{kN-m}^3$$

$$y_{max} = \frac{-231.05\times 1000^3}{15\times 10^9}$$

$$\boxed{y_{max} = -15.40\ \text{mm}}$$ —— 02 m

3 a) Derivation of expression

$$M = EI\frac{d^2y}{dx^2}$$



Consider a member AB of span 'l' Subjected to a uniform bending moment M so that the member is bent into a Circular shape. Let 'R' be the radius of curvature of bent member.

Consider an elemental length $PP' = ds$ of the curve. As the distance 'ds' is very small,

$$ds^2 = dx^2 + dy^2$$

from fig,

$$ds = R.d\alpha = \sqrt{(dx^2 + dy^2)} \quad \text{—— (1)}$$

$$\frac{1}{R} = \frac{d\alpha}{\sqrt{(dx^2+dy^2)}}$$

$$\tan\alpha = \frac{dy}{dx}.$$

Differentiate

$$\sec^2\alpha \cdot d\alpha = \frac{d^2y}{dx^2}\cdot dx$$

$$(1+\tan^2\alpha)\cdot d\alpha = \frac{d^2y}{dx^2}\cdot dx$$

$$\left(1+\left(\frac{dy}{dx}\right)^2\right)\cdot d\alpha = \frac{d^2y}{dx^2}\cdot dx$$

$$d\alpha = \frac{\left(\frac{d^2y}{dx^2}\right)\cdot dx}{\left(1+\left(\frac{dy}{dx}\right)^2\right)} \quad \text{—— (2)}$$

Substitute (2) in (1),

$$\frac{R.\left(\frac{d^2y}{dx^2}\right)\cdot dx}{\left[1+\left(\frac{dy}{dx}\right)^2\right]} = \sqrt{(dx^2+dy^2)}$$

$$\frac{1}{R} = \frac{\frac{d^2y}{dx^2}}{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}$$

Thus

$$\frac{1}{R} = \frac{d^2y}{dx^2}$$

From bending Equation

$$\frac{M}{I} = \frac{f}{y} = \frac{E}{R}$$

$$M = \frac{1}{R}\cdot EI$$

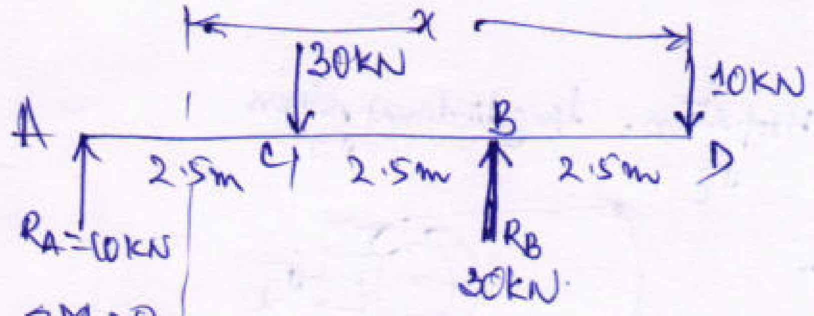
$$\therefore \boxed{M = EI\frac{d^2y}{dx^2}}$$

This is the differential Equation of deflected curve.

06

Since dy and dx are very small quantities their squares are much smaller. Hence, the denominator is taken as 1.

3 b)



Soln:

$\Sigma M_A = 0$

$$-R_B \times 5 + 10 \times 7.5 + 30 \times 2.5 = 0$$

$$\boxed{R_B = 30KN}$$

$\Sigma V = 0$

$$R_A + R_B = 40$$

$$\boxed{R_A = 10KN}$$ 02

$$M_x = -10 \cdot x + 30(x-2.5) - 30(x-5)$$

$$EI\frac{dy}{dx} = -10 \cdot \frac{x^2}{2} + 30\frac{(x-2.5)^2}{2} - 30\frac{(x-5)^2}{2} + C_1$$

$$EI\,y = -10\frac{x^3}{6} + \frac{30}{6}(x-2.5)^3 - \frac{30}{6}(x-5)^3 + C_1x + C_2$$

When $x = 2.5m$ $y = 0$

$$-\frac{10(2.5)^3}{6} + 2.5C_1 + C_2 = 0$$

$$\boxed{2.5C_1 + C_2 = 26.04} \text{ — (1)}$$

When $x = 7.5$ $y = 0$

$$-\frac{10(7.5)^3}{6} + \frac{30}{6}(5)^3 - \frac{30}{6}(2.5)^3 + 7.5C_1 + C_2 = 0$$

$$\boxed{7.5C_1 + C_2 = 156.25} \text{ — (2)}$$ 02

Solving (1) and (2)

$$C_2 = 26.04 - 2.5C_1$$

$$C_2 = 156.25 - 7.5C_1$$

$$156.25 - 7.5C_1 = 26.04 - 2.5C_1$$

$$\boxed{C_1 = 26.04}$$

$$\boxed{C_2 = -39.06}$$ 03

Deflectn Equatn

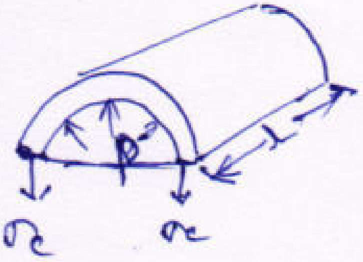
$$EI\,y = -10\frac{x^3}{6} + \frac{30}{6}(x-2.5)^3 - \frac{30}{6}(x-5)^3 + 26.04x - 39.06$$

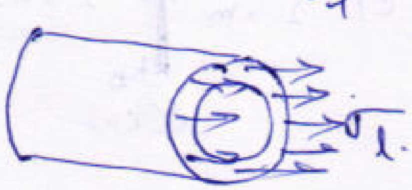
$x = 5m$ $y_C = -\frac{10 \times 5^3}{6} + 26.05 \times 5 - 39.06 = \frac{-117.14}{EI}$ mm (Downward)

$x = 0$ $y_D = -\frac{39.06}{EI}$ mm (Downward) 03

4 a) Expressions for Hoop stress, longitudinal stress

σ_c σ_l





$$p \times \frac{\pi d^2}{4} = \sigma_l \, \pi \cdot d \cdot t$$

$$p \cdot d\, l = \cancel{2}(2t \cdot l)\, \sigma_c$$

Hoop stress: $\boxed{\sigma_c = \frac{p \cdot d}{2t}}$

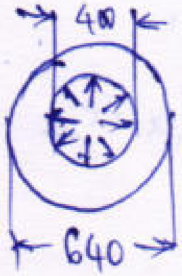
$$\boxed{\sigma_l = \frac{p \cdot d}{4 \cdot t}}$$

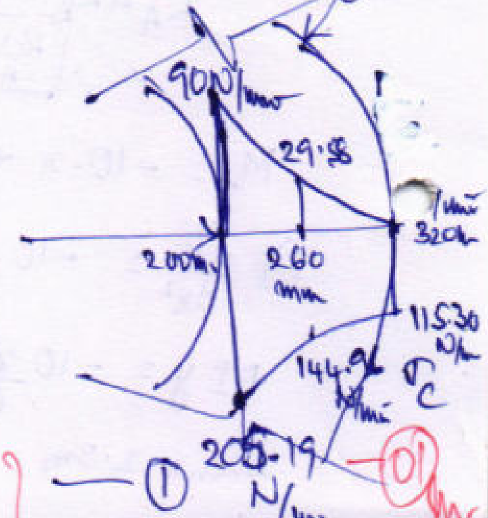
06 M

Cylinder

$$\boxed{\sigma_{max} = \frac{p \cdot d}{8t}}$$

b)





01 M

Stress Distribution

From Lame's eqn

$$P_x = \frac{b}{x^2} - a$$

$$\left.\begin{aligned} 90 &= \frac{b}{200^2} - a \quad \text{— (1)} \\ 0 &= \frac{b}{320^2} - a \quad \text{— (2)} \end{aligned}\right\}$$

$r_1 = 200$ mm

$r_2 = 200 + 120 = 320$ mm

$P_1 = 90$ N/mm²

$P_2 = 0$

02 M

Solving

$$90 = \frac{b}{200^2} - \frac{b}{320^2}$$

$$\boxed{b = 5.90 \times 10^6}$$ 01 M

$$\boxed{a = 57.69}$$ 01 M

Radial stress:

At $x = r_1 = 200$ mm

$x = 260$ mm

$x = 320$ mm

$$P_x = \frac{5.90 \times 10^6}{200^2} - 57.69 = 90 \text{ N/mm}^2$$

$$P_x = \frac{5.9 \times 10^6}{260^2} - 57.69 = 29.58 \text{ N/mm}^2$$

$$P_x = \quad = 0 \text{ N/mm}^2$$

02 M

Hoop stress:

At $x = r_1 = 200$ mm

$x = 260$ mm

$x = 320$ mm

$$\sigma_c = \frac{5.90 \times 10^6}{200^2} + 57.69 = 205.19 \text{ N/mm}^2$$

$$\sigma_c = \quad = 144.96 \text{ N/mm}^2$$

$$\sigma_c = \quad = 115.307 \text{ N/mm}^2$$

02 M